LA CINÉMATIQUE - MRUA

PGC-02

L'ACCÉLÉRATION

$$\vec{a} = \frac{d\vec{v}}{dt}$$

taux de variation.

$\vec{a} = \vec{0}$ $\quad$ v = constante en direction et module !

$\vec{a}$ et $\vec{v}$ $\parallel$ $\Rightarrow$ $\vec{v}$ ne change pas de direction.

accélération moyenne

$$\vec{a}_m = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

instantanée

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{S}}{dt^2}$$

$$[a] = \frac{m}{s^2} \text{ en S.I.}$$

mouvement 1-D

$\vec{z}, \vec{v}, \vec{a}$ //

MRUA

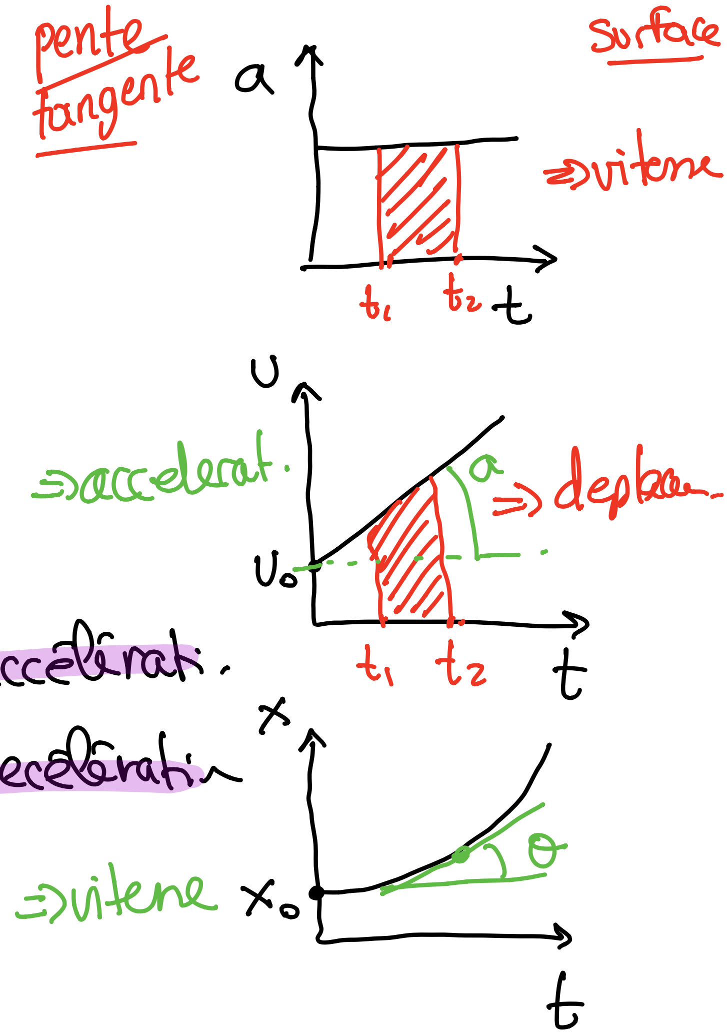
$a = \text{constante}$

$$\underline{a} = \frac{dv}{dt} \Rightarrow v = \int_0^t a\,dt = \underline{at + v_0}$$

$$x = \int_0^t v\,dt = \frac{1}{2}at^2 + v_0 t + x_0$$

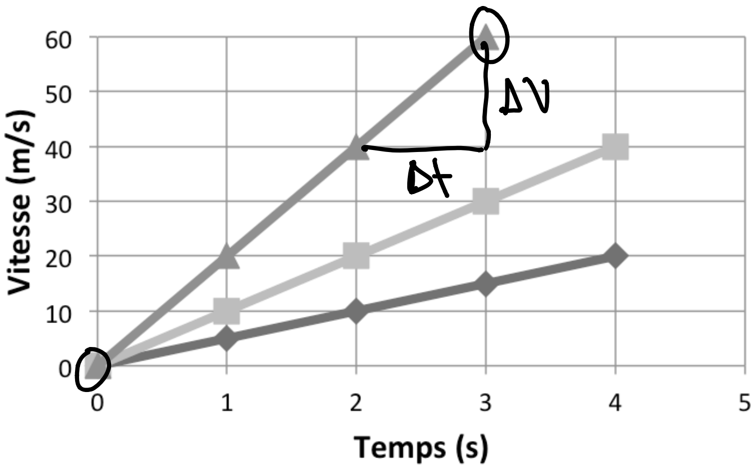
$\vec{a}$ et $\vec{v}$ // $\quad |\vec{a}| > 0 \Rightarrow v > v_0$ accélérati.

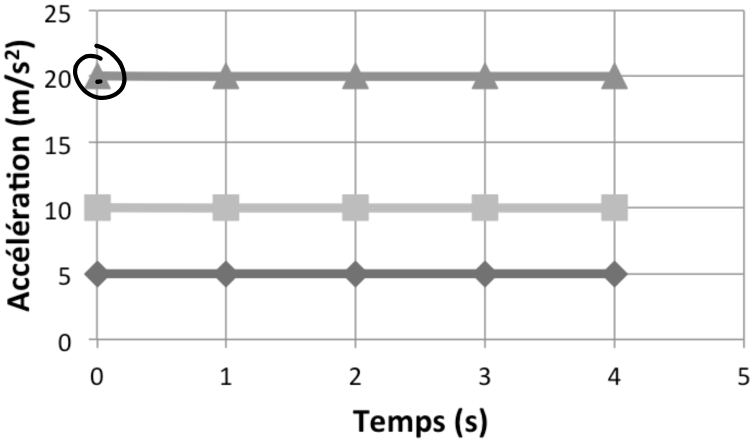
$|\vec{a}| < 0 \Rightarrow v < v_0$ décélération



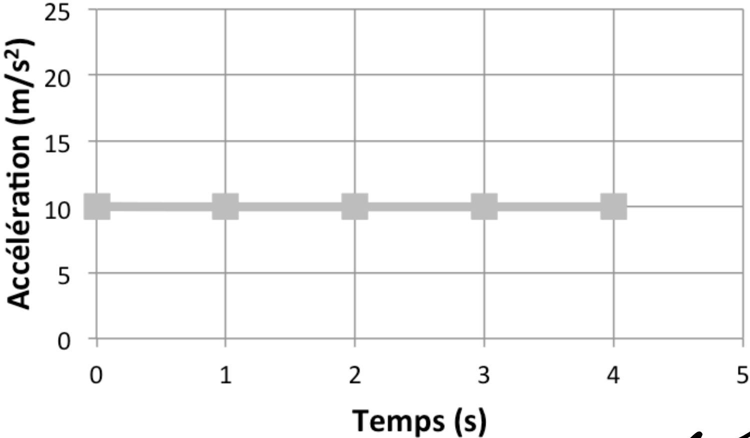
MRUA

$$a = \frac{20\ m/s}{1\ s} \Rightarrow a = 20\ m/s^2$$



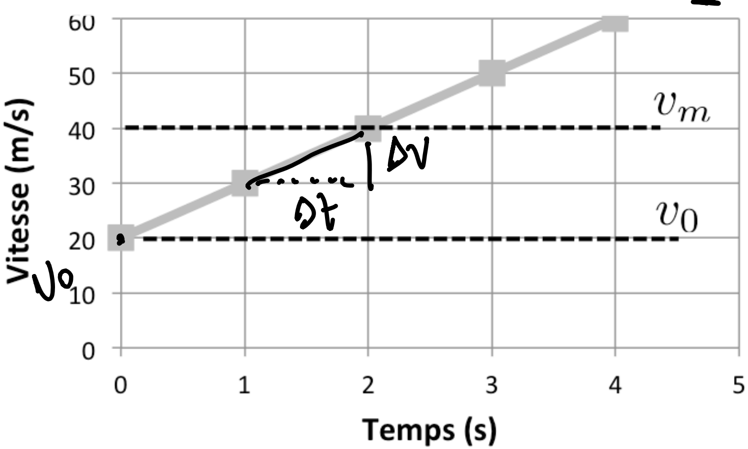


MRUA

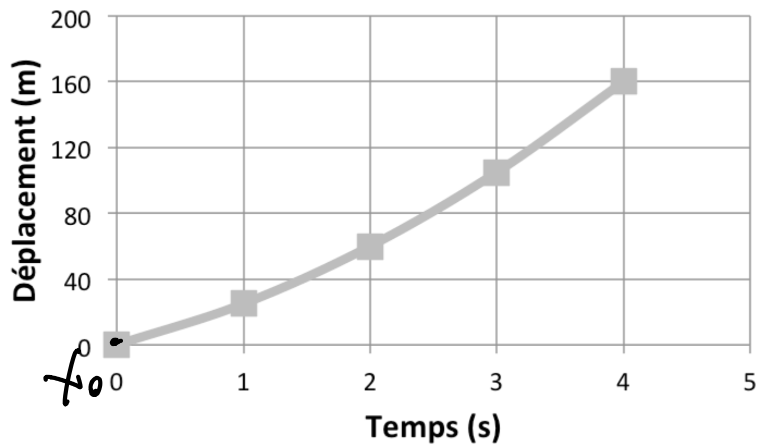


$a = 10\ m/s^2$

$v = at + v_0$



$x = \frac{1}{2}at^2 + v_0 t + \cancel{x_0} \rightarrow 0$



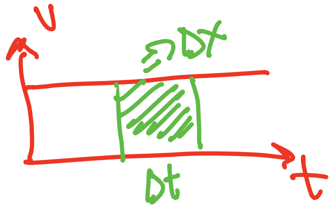
RESUMÉ

vitesse

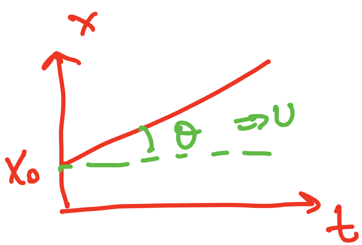
MRU, $a=0$

$$v = \frac{dx}{dt} = \text{constante}$$

$$x = v_0 t + x_0$$



$$[v] = \frac{m}{s}$$



MRUA — accélération

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$\boxed{v = at + v_0} \quad (1) \quad v = f(t)$$

$$\boxed{x = \frac{1}{2}at^2 + v_0 t + x_0} \quad (2) \quad x = f(t)$$

$$[a] = \frac{m}{s^2} \quad SI$$

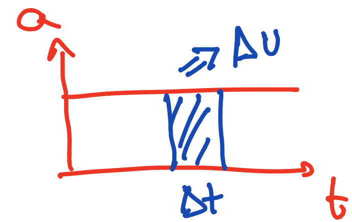
$(1) \Rightarrow t = \dfrac{v - v_0}{a}$

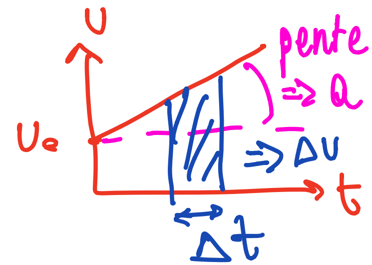
$(2) \Rightarrow \boxed{v^2 = 2ax + v_0^2} \quad (3)$

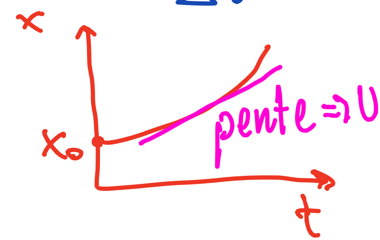
$v = f(x)$

$x = f(v)$

pour $x_0 = 0$



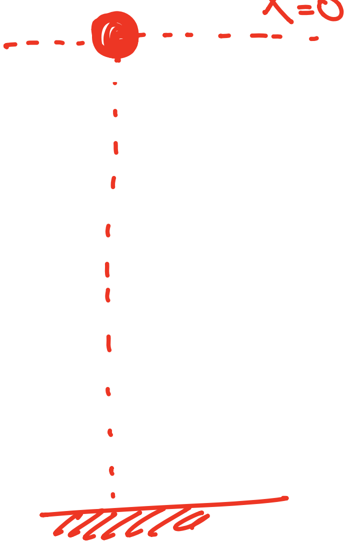




LA CHUTE LIBRE

GRAVITÉ Responsable pour accélération.
(g = 9.81 m/s²)

$t=0$: $x_0 = 0$, $v_0 = 0$, $a = g$



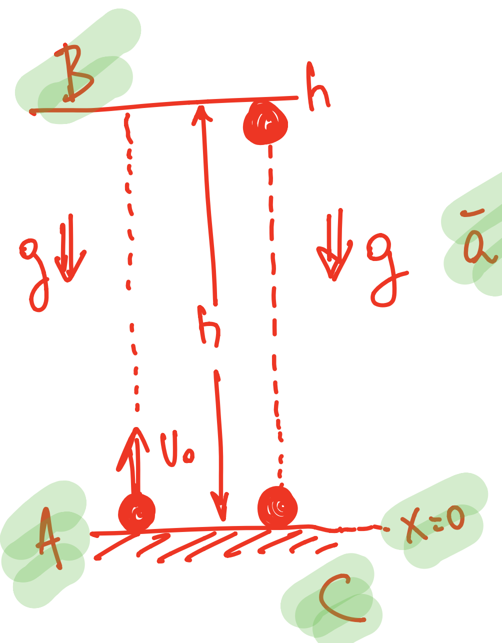
Donc (2) $\Rightarrow$ $x = \frac{1}{2} g t^2$

ou $t = \sqrt{\frac{2x}{g}}$

pas de masse!!

DEMO Newton tube.

MOUVEMENT VERTICAL



à A : $t=0$: $S_0=0$, v_0 , $a=-$

Donc : (2) $\Rightarrow x(t) = v_0 t - \frac{1}{2} g t^2$

(1) $\Rightarrow v(t) = v_0 - gt < v_0$!

à B : $V=0$
$x=h$

Donc (3) $\Rightarrow v_0^2 = 2gh \Rightarrow \boxed{h = \frac{v_0^2}{2g}}$

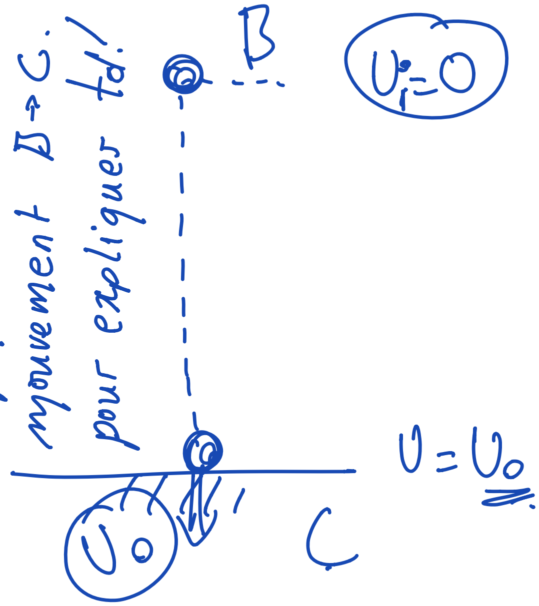
et (1) $\Rightarrow v_0 = g t_m \Rightarrow \boxed{t_m = \frac{v_0}{g}}$

à C : $x=0$ (3) $\Rightarrow v^2 = 2gh \Rightarrow v = \sqrt{2gh}$

qui est égal à v_0 !

$t_d \Rightarrow v_0 = g t_d \Rightarrow \boxed{t_d = \frac{v_0}{g}}$

analytiquement le mouvement B→C pour expliquer td!

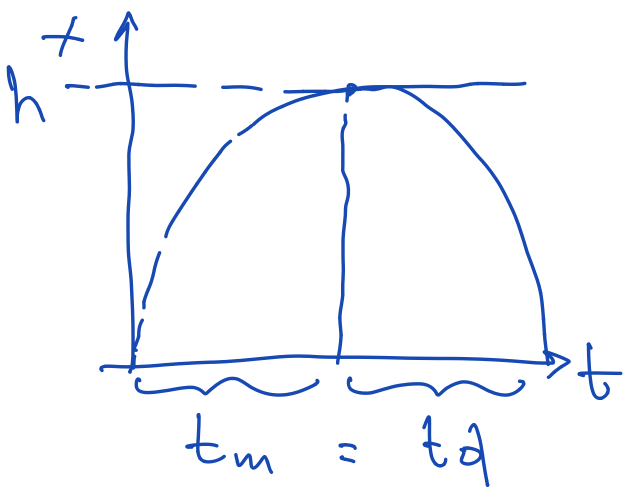


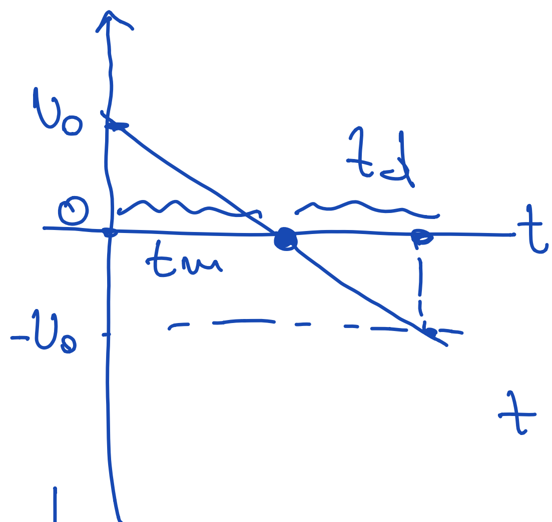
$$\boxed{v(t) = \cancel{v_i} + at}$$

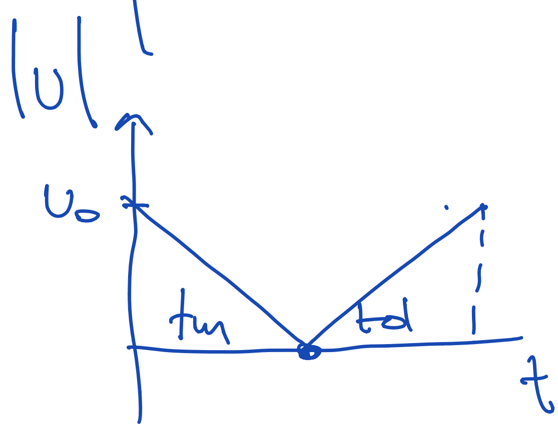
$a = g$

$v_0 = g\, t_d$

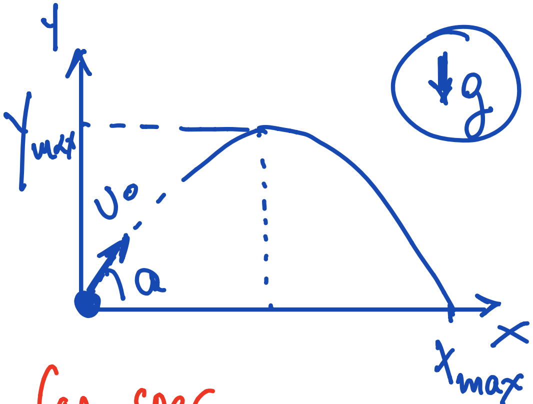
$\Rightarrow \underline{t_d} = \frac{v_0}{g}$







MOUVEMENT BALISTIQUE



$$V_0 = \begin{pmatrix} V_{0x} \\ V_{0y} \end{pmatrix} = \begin{pmatrix} V_0 \cos a \\ V_0 \sin a \end{pmatrix} \begin{matrix} \Rightarrow \text{MRU} \\ \Rightarrow \text{MRUA } (\downarrow g) \end{matrix}$$

$$r(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} V_{0x} \cdot t \\ -\frac{1}{2} g t^2 + V_{0y} t \end{pmatrix} = \begin{pmatrix} V_0 \cancel{\cos} a \cdot t \\ -\frac{1}{2} g t^2 + \cancel{V_0 \sin a \, t} \end{pmatrix}$$

$$v(t) = \begin{pmatrix} \frac{dx(t)}{dt} \\ \frac{dy(t)}{dt} \end{pmatrix} = \begin{pmatrix} v_0 \cancel{\cos} a t \\ v_0 \cancel{\sin a} - gt \end{pmatrix}$$

$$a(t) = \begin{pmatrix} \frac{d^2 x(t)}{dt^2} \\ \frac{d^2 y(t)}{dt^2} \end{pmatrix} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$$

Cas spec.

tir orizontal: $a = 0$

Pour tout autre a:

$$h = Y_{max} = \frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2 a}{2g}$$

$$t_T = \frac{2 V_0 \sin a}{g}$$

$$x_{max} = x(t_T) = \boxed{\frac{2 V_0^2}{g} \cos a \sin a}$$

$$x_{max} = \frac{2v_0^2}{g}\cos a \cdot \sin a$$

LE DESSIN EST-IT CORRECT?

Trois obus tirés d'un meme point sous des angles différents par rapport à l'horizontale: 30º, 45º et 60º. Leurs trajectoires sont representées sur le dessin suivant. Est-il correct?

Notez!

pour a et $90° - a$ le même x_{max}!