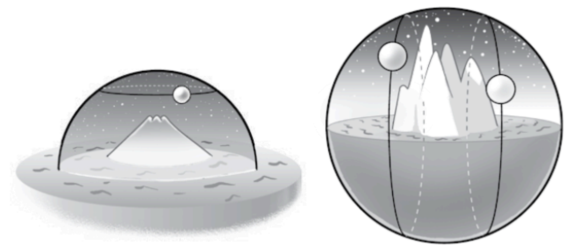
LA GRAVITÉ SELON NEWTON

PGC-09

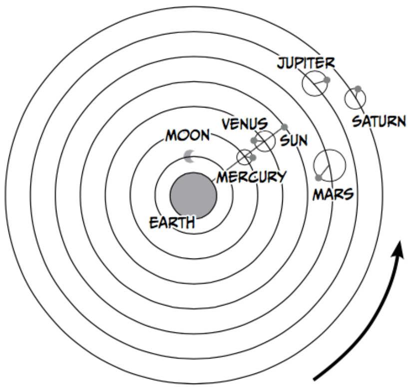
UN PEU D'HISTOIRE

~ 400	Grèce antique
BC	
AC	
~150	Ptolémée
1543	Copernic 'De Revolutionibus'
1609 1619	Kepler 'Astronomia nova' 'Harmonices Mundi'
1609 1632	Galilée Téléscope 'Dialogo sopra i due massimi sistemi del mondo'

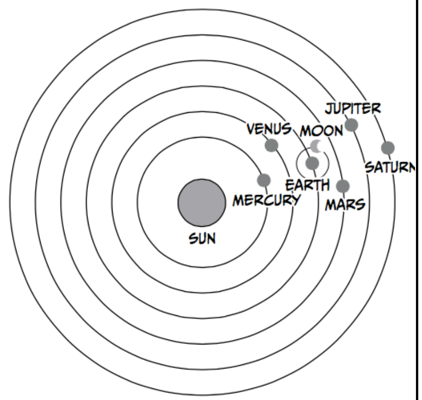


Gai Tian:
A cosmology positing a hemispherical dome over Earth

Hun Tian:
A spherical cosmology



Modèle géocentique

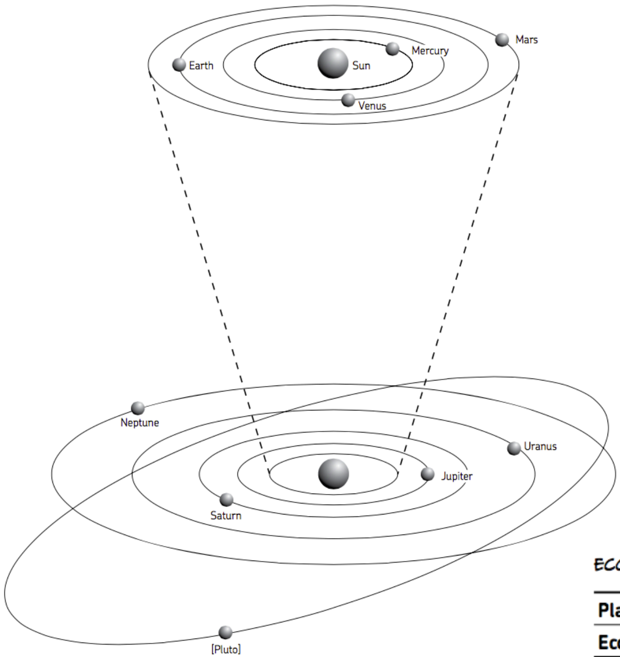


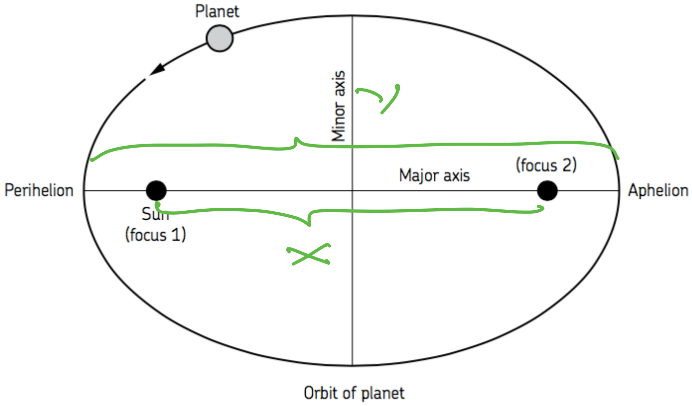
Modèle heliocentrique

Dessins: The Manga Guide To Astronomy

LES TROIS LOIS DE KEPLER

1.





Orbit of a planet according to Kepler's First Law

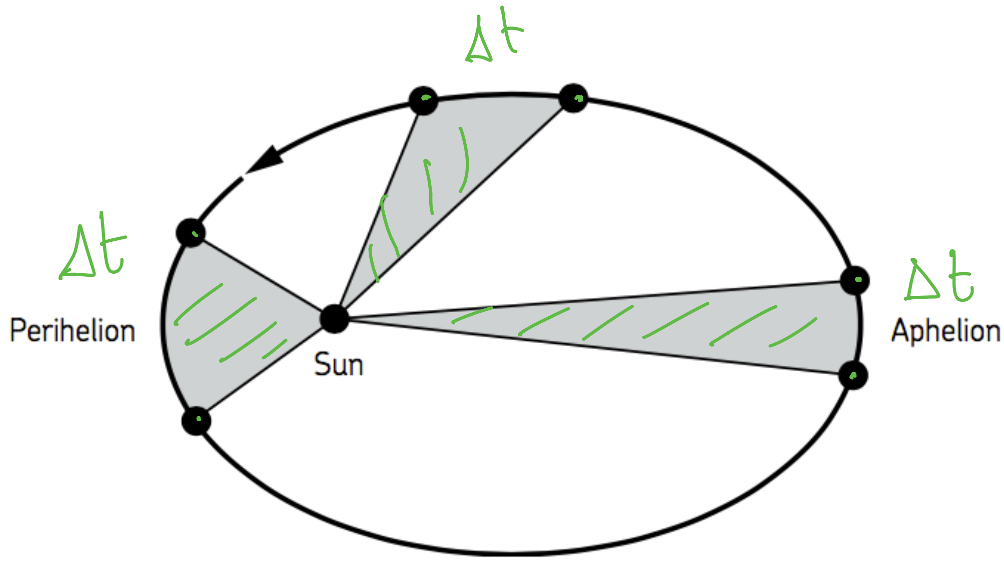
$$ecc = \frac{x}{y}$$

ECCENTRICITY OF EACH PLANET IN THE SOLAR SYSTEM

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Eccentricity	0.2056	0.0068	0.0167	0.0934	0.0485	0.0555	0.0463	0.0090

LES TROIS LOIS DE KEPLER

2.



Orbit of a planet according to Kepler's Second Law

LES TROIS LOIS DE KEPLER

3.

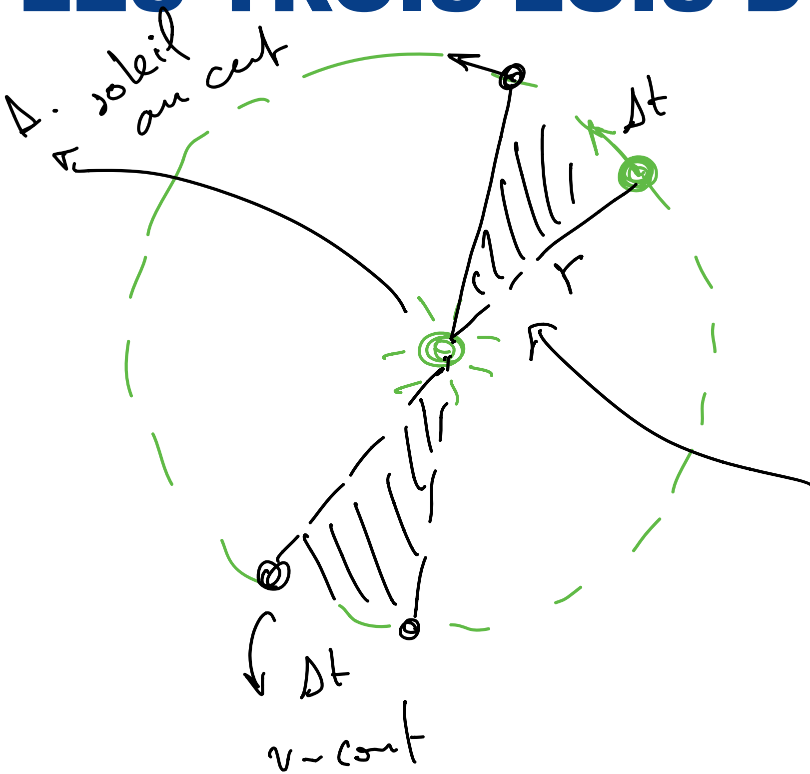
$$\frac{r^3}{T^2} = \text{constante}$$

SEMIMAJOR AXIS OF A PLANET'S ORBIT AND ORBITAL PERIOD

Planet	Semimajor axis of orbit *a* (AUs)	a^3	Orbital period relative to the fixed star's *P* (solar years)	P^2	a^3/P^2
Mercury	0.3871	0.05800555	0.2409	0.05803281	0.9995
Venus	0.7233	0.37840372	0.6152	0.37847104	0.9998
Earth	1.0000	1	1.0000	1	1.0000
Mars	1.5237	3.53751592	1.8809	3.53778481	0.9999
Jupiter	5.2026	140.819017	11.8620	150.707044	1.0008
Saturn	9.5549	872.32524	29.4580	867.773764	1.0052
Uranus	19.2184	7098.25644	84.0220	7049.69648	1.0055
Neptune	30.1104	27299.1783	164.7740	27150.4711	1.0055

LES TROIS LOIS DE KEPLER

Approx. à cercle



3. $T^2 \propto r^3$

1609 : télescope ... voir/observer les lois de Kepler !

NEWTON 1642-1727

g_{TL}

(F_{TL})

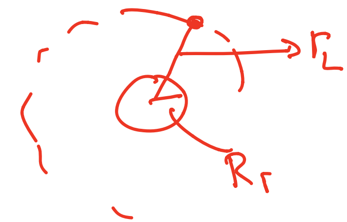
$$a_c = g_{TL} = \frac{v_L^2}{r_L}$$

$$v_L = \frac{2\pi r_L}{T_L}$$

$$\Rightarrow g_{TL} = \frac{4\pi^2 r_L}{T_L^2} = 0.00272 \ m/s^2$$

$$\frac{g_{TL}}{g_T} = \frac{0.00272}{9.8} \approx \frac{1}{3600}$$

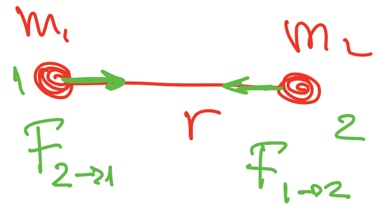
$$\frac{r_L}{R_T} \approx \frac{3.84 \times 10^8 m}{6.37 \times 10^6 m} \approx 60$$



$$\frac{g_T}{g_{TL}} = \left(\frac{1/R_T}{1/r_L}\right)^2$$

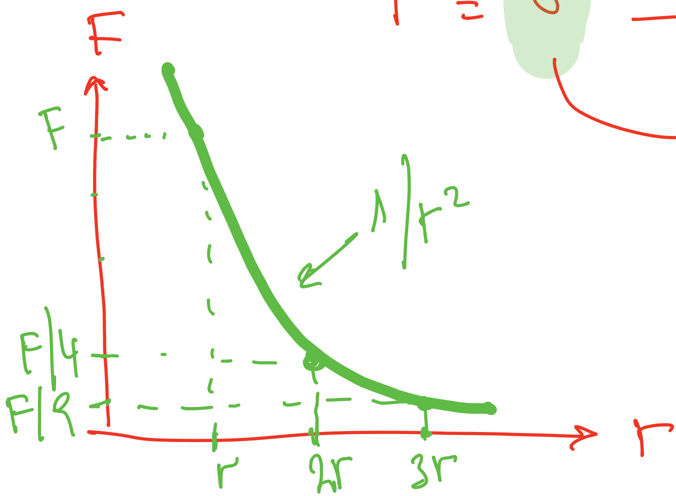
LOI DE GRAVITÉ DE NEWTON

$$F \propto \frac{m_1 m_2}{r^2}$$



$$F = G \frac{m_1 m_2}{r^2}$$

$G \rightarrow 6.67 \times 10^{-11}\ Nm^2/kg^2$



GRANDEUR DE LA FORCE GRAVITATIONNELLE

$m_1 = 1\,kg$

$m_2 = 1\,kg$

$r = 1\,m$

$$F = G\frac{m_1 m_2}{r^2} = 6.67\times10^{-11}\,N\frac{\cancel{m^2}}{\cancel{kg^2}}\cdot\frac{\cancel{kg^2}}{\cancel{m^2}} = 6.67\times10^{-11}\,N$$

$m_1 = M_T = 6\times10^{24}\,kg$

$m_2 = 1\,kg$

$r = R_T = 6.4\times10^{6}\,m$

$$F = G\frac{\overset{M_T}{m_1}\, m_2}{r^2} = 6.67\times10^{-11}\cdot\frac{6\times10^{24}\cdot 1}{(6.4\times10^{6})^2}\,N = 9.8\,N$$

$\Downarrow$

le poids !

PAS PAR COINSIDENCE !

$F = mg$

$m = 1\,kg$

$g = 9.8\,m/s^2$

$\Big\}\; F = 9.8\,N.$

PRINCIPE DE L'ÉQUIVALENCE

$$m_{inertie} = \frac{F}{a} \rightsquigarrow \cancel{G}$$

$$m_{gravit} = \frac{r^2 F}{GM} \rightsquigarrow \cancel{a}$$

$$m_{inertie} = m_{gravitation}$$

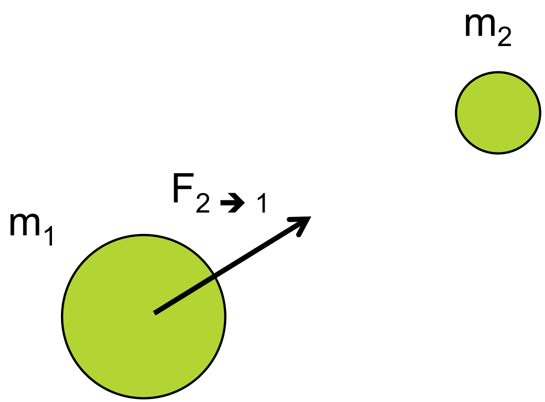
THÉORIE DE GRAVITÉ DE NEWTON

1. $F = G \dfrac{m_1 m_2}{R^2}$
2. PRINCIPE D'ÉQUIVALENCE
3. 3 lois DE NEWTON

QUESTION

$$F = G\frac{m_1 m_2}{r^2}$$

$$F_{2\to 1} = F_{1\to 2}$$



$m_2 = 2\, m_1$

(a) $F_{1\to 2} = F_{2\to 1}$

(b) $F_{1\to 2} = 2\, F_{2\to 1}$

(c) $2\, F_{1\to 2} = F_{2\to 1}$

(d) $F_{1\to 2} = 4\, F_{2\to 1}$

(e) $4\, F_{1\to 2} = F_{2\to 1}$

GRAND G ET PETIT g

$$F_G = \frac{GMm}{r^2}$$

$$F_D = mg$$

$$g_0 = \frac{GM}{r^2}$$

$$g_{Mars} = 3.8\, m/s^2$$

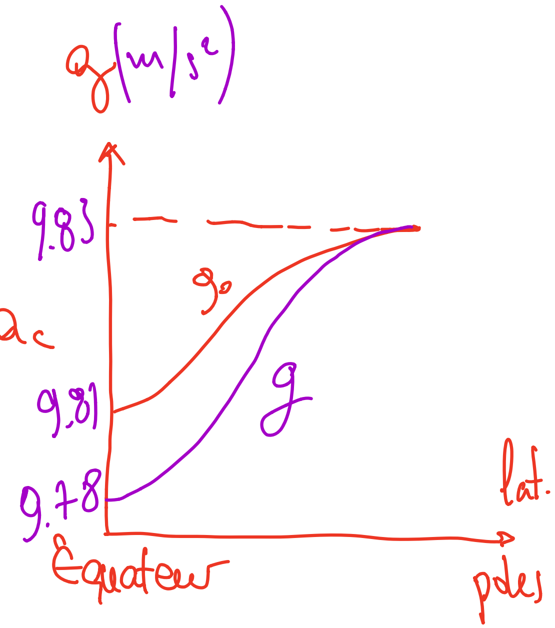
La Rotation.

$$\Sigma F = m a_c \Rightarrow F_G - F_D = m a_c$$

$$\Rightarrow \frac{G\cancel{m}M}{r^2} - \cancel{m}g = \cancel{m}a_c$$

$$\Rightarrow g = \frac{GM}{R^2} - a_c$$

$$g < g_0$$

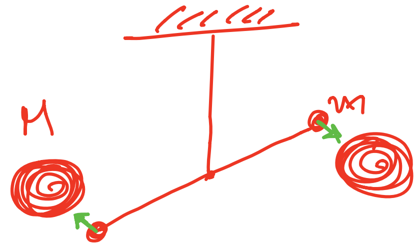


LA MASSE DE LA TERRE?

M? G?

Henry Cavendish

$$G = \frac{F_{Mm}\, r^2}{M m}$$



$m = 10\ \text{gr}$

$M = 1\ \text{kg}$

G

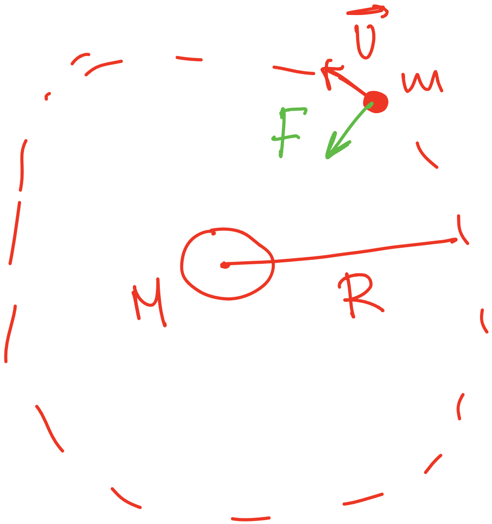
$$M_T = \frac{g R_T^2}{G}$$

g : cinematique

G : Cavendish

R_T : techniques surveillance

ORBITE DE SATELLITE



$$F = \frac{GmM}{R^2} = m\,a_c = \frac{mv^2}{R}$$

$$\frac{GmM}{R^2} = \frac{mv^2}{R} \Rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$$

$$v = \frac{circ}{per} = \boxed{\frac{2\pi R}{T}}$$

3ème Kepler!

$$\frac{2\pi R}{T} = \sqrt{\frac{GM}{R}} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3 \Rightarrow \boxed{\frac{r^3}{T^2} = \frac{GM}{4\pi^2}}$$

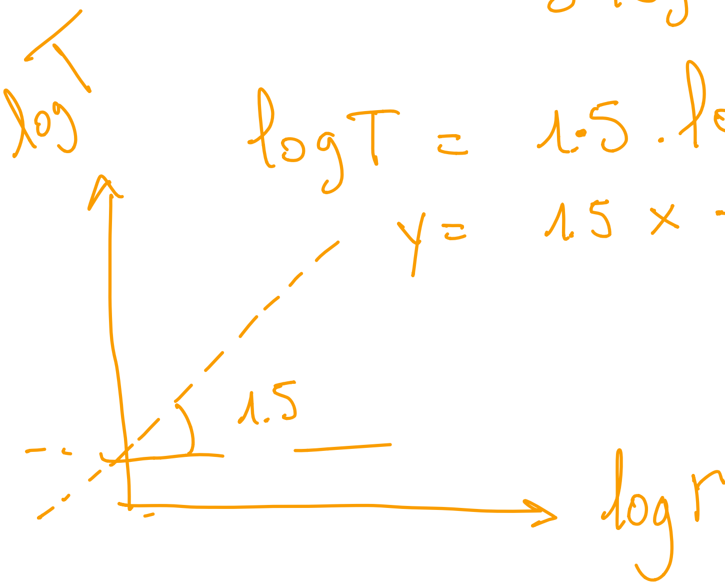
ORBITES GEOSTATIONNAIRES

$$\frac{r^3}{T^2} = \text{const} \Rightarrow \log r^3 - \log T^2 = \text{const}$$

$$3 \log r - 2 \log T = \text{const}$$

$$\log T = 1.5 \cdot \log r + \text{const}$$

$$y = 1.5\, x + \text{const}$$

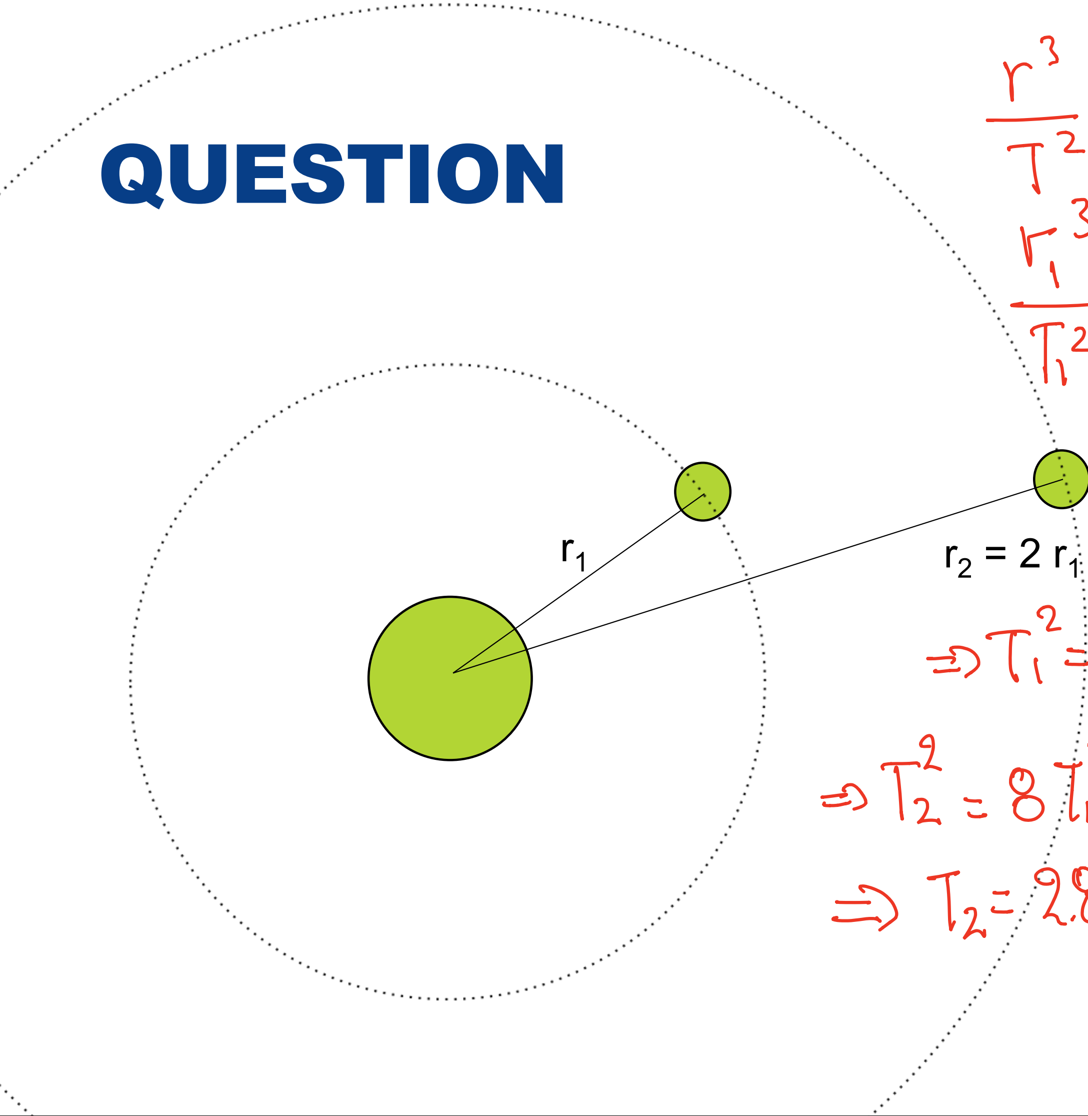


$T = 24h$

$r = 36.000$ km

$\simeq 5.6\, R_T$

QUESTION



$$\frac{r^3}{T^2} = \text{const}$$

$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2} \Rightarrow$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} = \frac{\cancel{r_1^3}}{2^3\,\cancel{r_1^3}}$$

$$\Rightarrow T_1^2 = \frac{T_2^2}{2^3} \Rightarrow$$

$$\Rightarrow T_2^2 = 8\,T_1^2$$

$$\Rightarrow T_2 = 2.8\,T_1$$

T_2 ? T_1

(a) $T_2 = 2\,T_1$

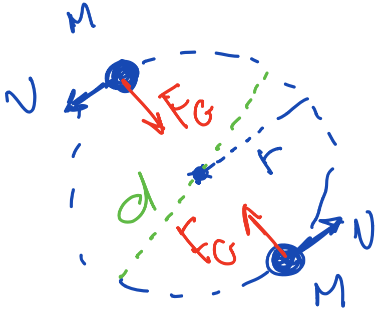
(b) $T_2 = 2.8\,T_1$

(c) $T_2 = 4\,T_1$

(d) $T_2 = 0.2\,T_1$

EXEMPLE

Système binaire des étoiles. Masse de chacune: 2 x masse du soleil. Période de 90 jours. Quelle est la distance entre les deux étoiles?



$$F = \frac{G M_1 M_2}{d^2} = \frac{G M^2}{d^2} = M a_c$$

$$\Rightarrow \frac{G M^2}{4r^2} = \frac{M v^2}{r} \Rightarrow r = \dots 4.67 \times 10^{10}\ m$$

$$v = \frac{2\pi r}{T}$$

$M = 2 m_s$

$d = 2r$

$m_s = 1.99 \times 10^{30}$ kg.