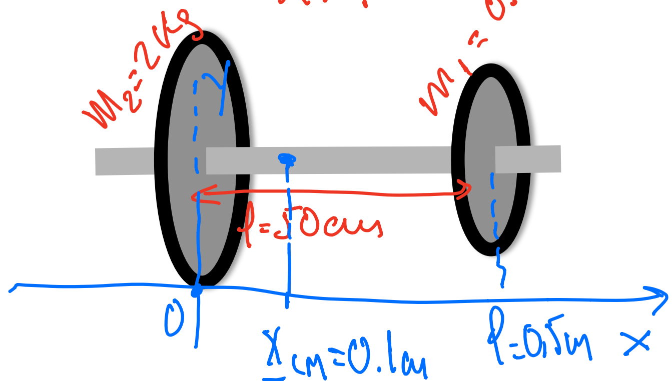
DYNAMIQUE DE LA ROTATION

PGC-10

LE CENTRE DE MASSE - RAPPEL

Un haltère se compose d'un disque de 500 gr d'une coté et d'un disque de 2 kg de l'autre coté. On considère la barre qui les connecte sans masse et le longueur 50 cm. Calculer le centre de masse.

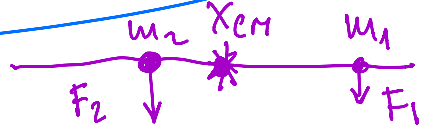
$$x_{CM} = \frac{\sum_i F_i x_i}{\sum F_i} \qquad F = mg \qquad x_{CM} = \frac{\sum_i m_i x_i}{\sum m_i}$$



$$x_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{m_2 \cdot 0 + m_1 \cdot l}{m_1 + m_2} \Rightarrow$$

$$\Rightarrow x_{CM} = 0.1\,m$$

$$\sum T_{CM} = 0$$

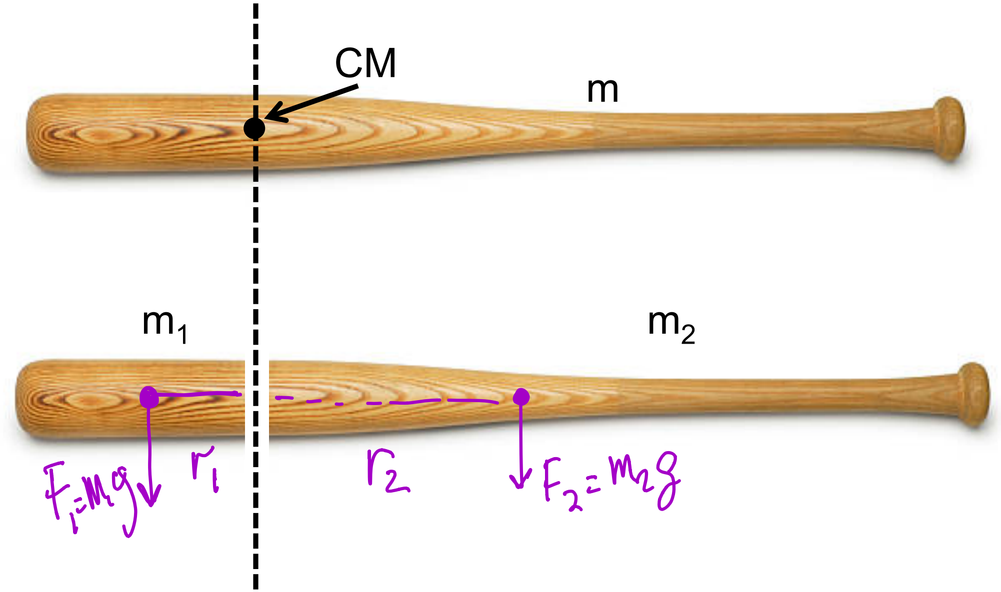


LE CENTRE DE MASSE - QUESTION

(a) $m_1 > m_2$

(b) $m_1 = m_2$

(c) $m_1 < m_2$



$F_1 \cdot r_1 = F_2 \cdot r_2$

DYNAMIQUE DE ROTATION - INTRO

a rectiligne ⟺ F — masse "Résistance" — $F = ma$

a angulaire ⟺ τ — masse — moment d'inertie — $\tau = I \cdot a_{ang}$.

MOMENT D'INERTIE

$$\left.\begin{array}{l} \tau_0 = rF = r \cdot m_\bullet\, a_t \\ a_t = r\, a_{ang} \end{array}\right\} \Rightarrow \tau_0 = \overbrace{m_\bullet r^2}^{I_0}\, a_{ang}.$$

$$F = m\, a$$

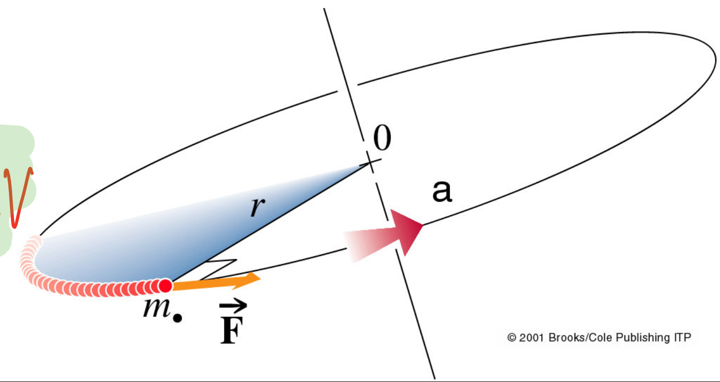
$$\tau_0 = I_0\, a_{ang}$$

$$\tau_0^{TOT} = \sum \tau_0 = \sum_i (m_i r_i^2)\, a_{ang}$$

$$I_0 = \sum_i m_i r_i^2$$

dm

$$\left.\begin{array}{l} I = \int r^2\, dm \\ dm = \rho\, dV \end{array}\right\} \Rightarrow I = \int r^2 \rho\, dV$$



MOMENT D'INERTIE DES CORPS SIMPLES

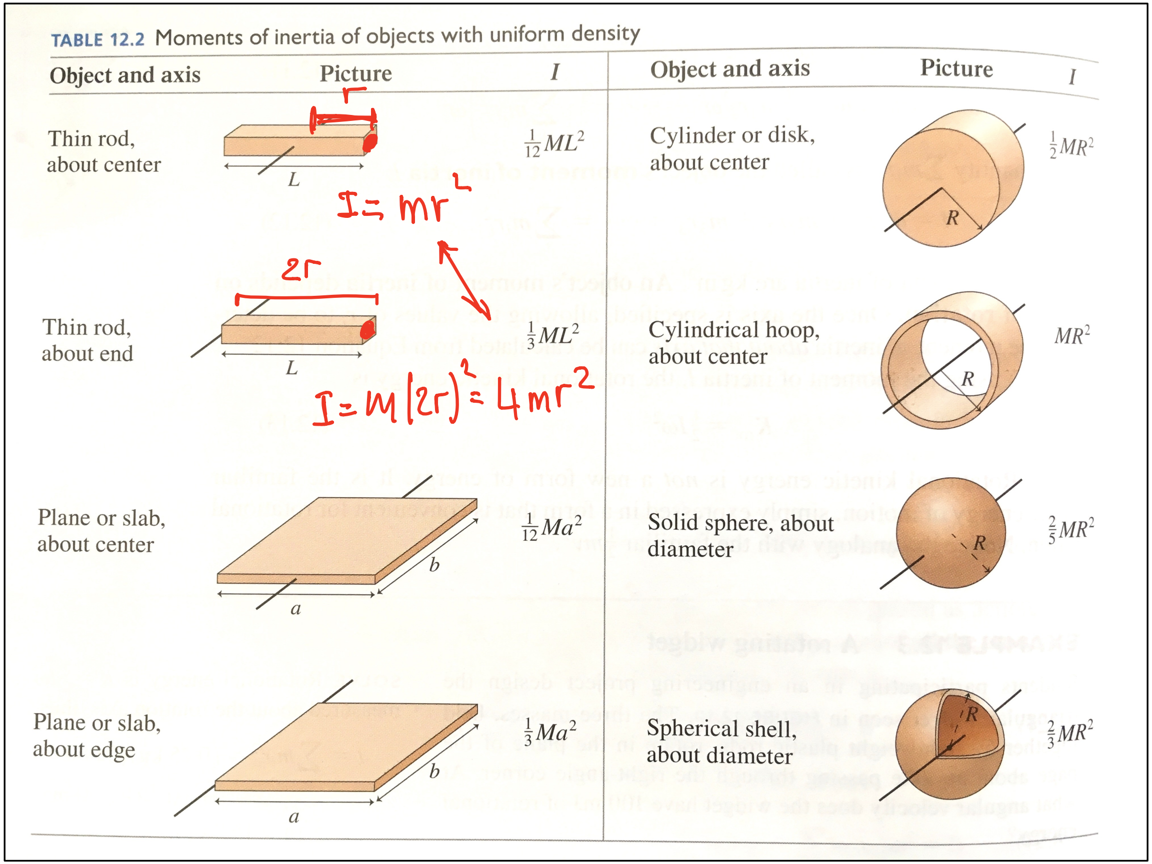


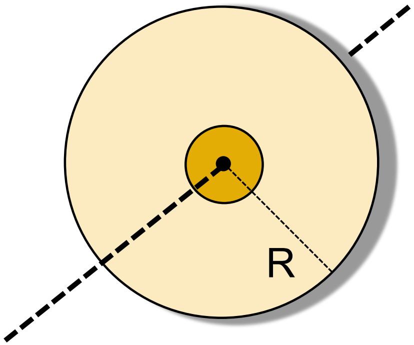
TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Image prise du livre de Randall Kight

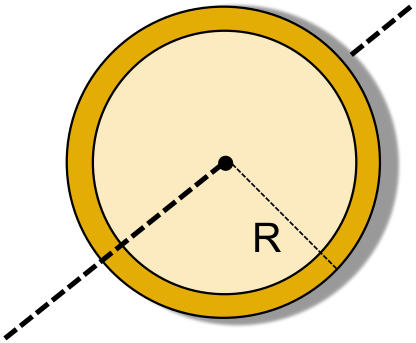
MOMENT D’INERTIE

Masse M



Objet 1

Masse M



Objet 2

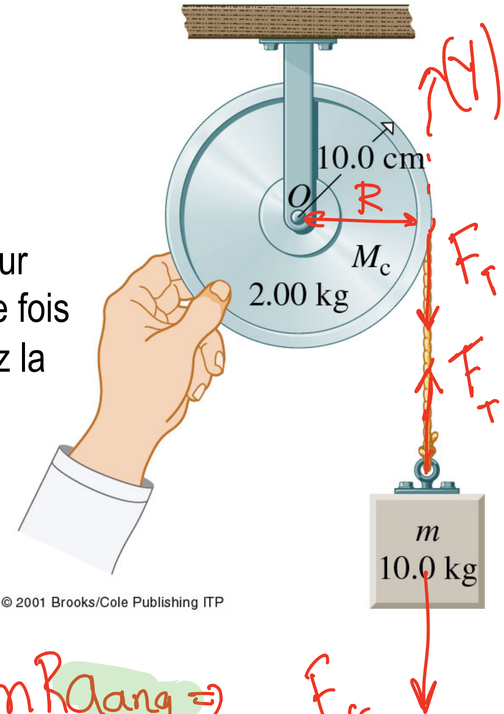
(a) $I_1 > I_2$

(b) $I_1 = I_2$

(c) $I_1 < I_2$

EXEMPLE

Une masse $m = 10.0$ kg est suspendue à une corde enroulée autour d'un cylindre de rayon $R = 10.0$ cm et de masse $M_c = 2.00$ kg. Une fois lâché, le cylindre est libre de tourner autour de son axe. Déterminez la tension de la corde, et les accélérations du cylindre et de la masse.



$$\sum \tau_o = r \cdot F_T \cdot R = I\, a_{ang} \Rightarrow F_T = \frac{I\, a_{ang}}{R}$$

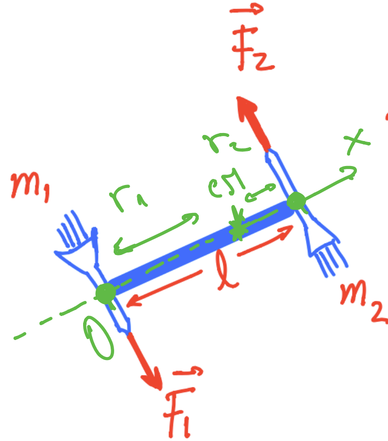
$$\sum F = m a = m R \cdot a_{ang}$$

$$mg - F_T = m \cdot R \cdot a_{ang} \Rightarrow mg - \frac{I \cdot a_{ang}}{R} = m R a_{ang} \Rightarrow$$

$$\Rightarrow a_{ang} = \frac{mg}{I/R + mR} \qquad \Rightarrow a_{ang} = \frac{mg}{\frac{1}{2} M_c R + mR} = \dots = 89.2 \text{ rad/s}^2$$

$$I = \frac{1}{2} M_c R^2 \qquad a = R \cdot a_{ang} = 8.92 \text{ m/s}^2 < g$$

EXEMPLE



$m_1 = 100{,}000\ kg$

$m_2 = 200{,}000\ kg$

$l = 90m$

$F_1 = F_2 = 50{,}000\ N$

ω at $\Delta t = 30s$?

$r_1 = 60\ m$

$r_2 = 30\ m$

$$\tau = a_{ang} \cdot I$$

$$X_{CM} = \frac{m_1 \cdot X_1 + m_2 \cdot X_2}{m_1 + m_2} = 60m$$

$$I = m_1 r_1^2 + m_2 r_2^2 = 540.000.000\ kg\, m^2$$

$$\tau = F_1 \cdot r_1 + F_2 \cdot r_2 = 4.500.000\ Nm$$

$$a_{ang} = 0.00833\ rad/s^2$$

$$\omega = a_{ang}\,\Delta t \Rightarrow \omega = 0.25\ rad/s$$

EXEMPLE

$\tau = I a_{ang}.$

Où a_{ang} max?

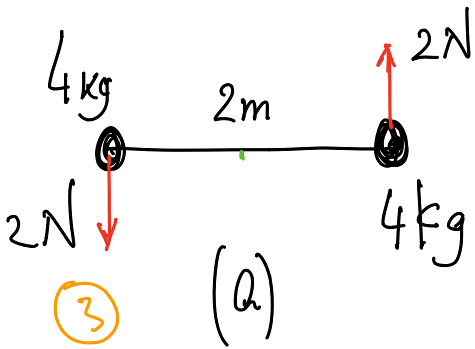
$a_{ang} = \dfrac{\tau}{I}$

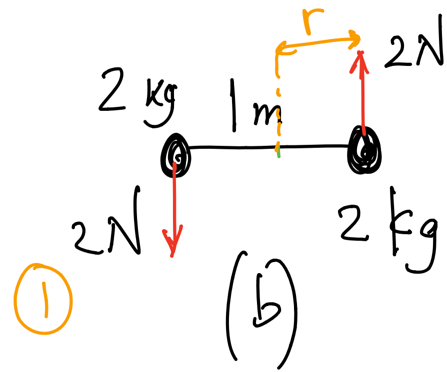
$I = 2mr^2$

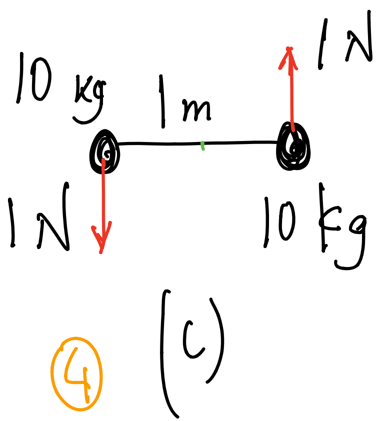
$I \propto mr^2$

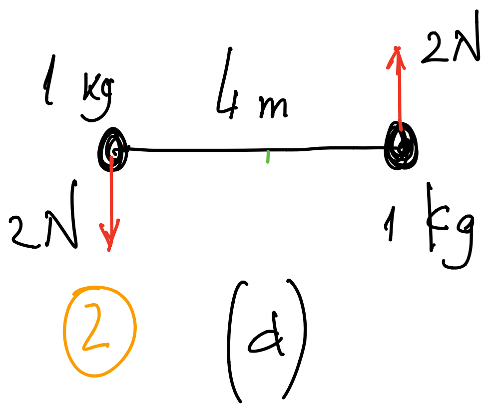
$\tau \propto Fr$

$a_{ang} \propto \dfrac{Fr}{mr^2} = \dfrac{F}{mr}$









MOMENT CINÉTIQUE

Éq. rotat. $\vec{F}$: $\vec{\tau}$

$\vec{p}$: $\vec{L}$ moment cinétique

$$\boxed{\vec{L} = I\vec{\omega}}$$

$$\boxed{\vec{p} = m\vec{v}}$$

$\vec{\tau} = \vec{r} \times \vec{F}$

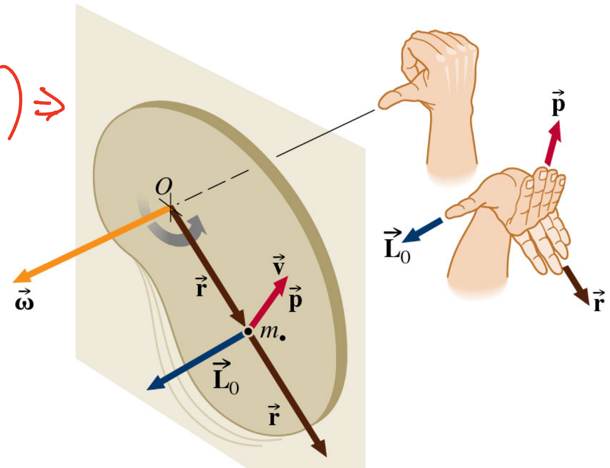
$\vec{L} = \vec{r} \times \vec{p}$ $\Rightarrow \vec{L} = m\vec{r} \times \vec{v}$

$\vec{p} = m\vec{v}$ $L = \sum(rmv) = \sum(rmr\omega) \Rightarrow$

$v = r\omega$

I

$\Rightarrow L = \sum mr^2 \omega \rightsquigarrow \vec{L} = I\vec{\omega}$



CONSERVATION DU MOMENT CINÉTIQUE

$$\vec{\tau} = I \cdot \vec{a}_{ang} = I \cdot \frac{\Delta \vec{\omega}}{\Delta t}$$

$$\Delta \vec{L} = I \Delta \vec{\omega} \Rightarrow \Delta \vec{\omega} = \frac{\Delta \vec{L}}{I}$$

$$\Rightarrow \vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

$$\Delta t \to 0$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\text{Si } \vec{\tau} = 0 \Rightarrow \boxed{d\vec{L} = 0}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

En absence de moment de force, le moment cinétique est CONSERVÉ !

RECAP

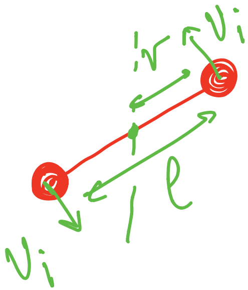
Symétrie entre les lois de translation et celles de la rotation

$$\vec{F} = m\vec{a} \qquad \vec{\tau} = I\,\vec{a}_{ang}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I\,\vec{\omega}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$

EXEMPLE



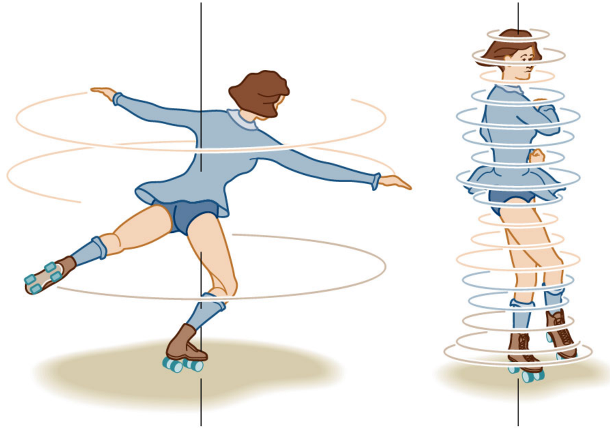
$r = l/2$

$l_i = 50\,cm$

$\omega_i = 2\,rev/s$

$l_f = 160\,cm$

$\omega_f = ?$



(a) (b)

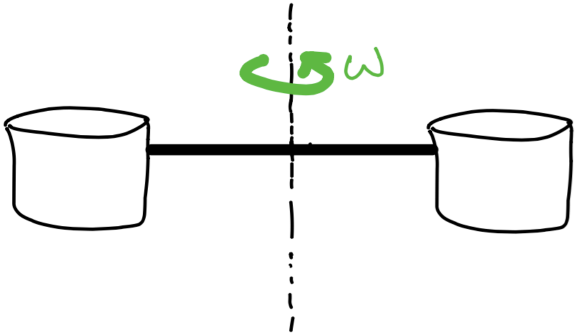
$$L_i = I_i\,\omega_i = 2m\cdot r_i^2\cdot \omega_i$$

$$L_f = 2m\, r_f^2\cdot \omega_f$$

$$L_i = L_f \Rightarrow r_i^2\,\omega_i = r_f^2\,\omega_f$$

$$\Rightarrow \omega_f = \frac{r_i^2\,\omega_i}{r_f^2} \simeq 0.2\,rev/s$$

QUESTION



$$L_i = 2m_i r^2 \omega_i$$

$$L_f = 2m_f r^2 \omega_f$$

$$L_i = L_f$$

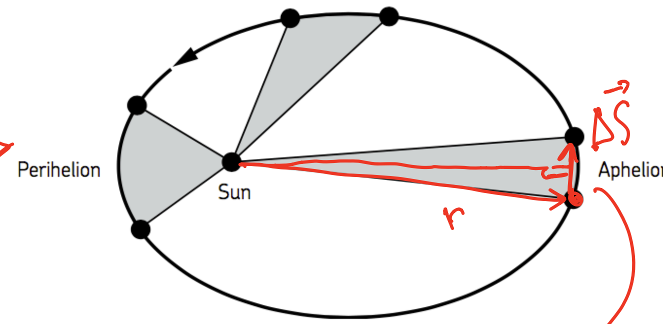
$$m_i \omega_i = m_f \cdot \omega_f$$

FLASH BACK: 2ÈME LOI DE KEPLER

$A = \frac{a \cdot b}{2}$

$\vec{L}_i = \vec{r} \times \vec{p} \Rightarrow L = rp\sin\theta$

$p = mv = m\frac{\Delta S}{\Delta t}$



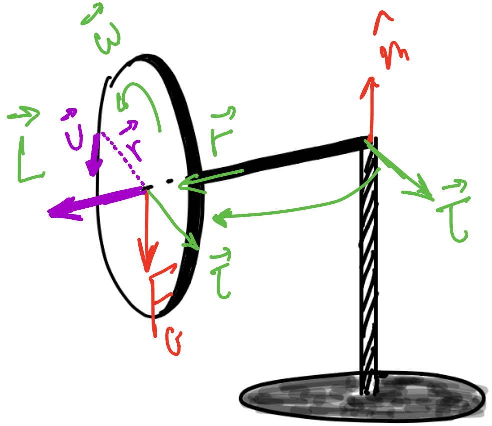
Orbit of a planet according to Kepler's Second Law

$\vec{L}_f = \vec{L}_i = r\,m\frac{\Delta S}{\Delta t}\sin\theta \Rightarrow \Delta A = \frac{\boxed{\vec{\Delta S} \cdot r \cdot \sin\theta}}{2}$

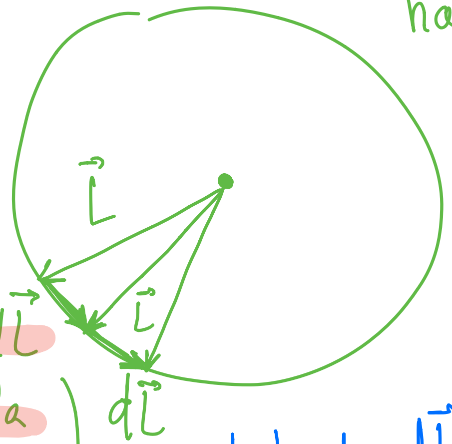
$\Rightarrow r \cdot \Delta S \cdot \sin\theta = \underline{const} \Rightarrow \underline{\Delta A = const}$

LE GYROSCOPE

$\vec{F}_G \Rightarrow \vec{T} \Rightarrow \vec{\tau} = \dfrac{d\vec{L}}{dt}$



vue depuis le haut



(dans la direction de $\vec{\tau}$)

– c'est de $d\vec{L}$ qui maintient le mouvement