# ÉQUILIBRE

PGC-04

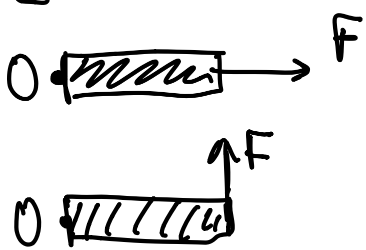
# LE MOMENT DE FORCE

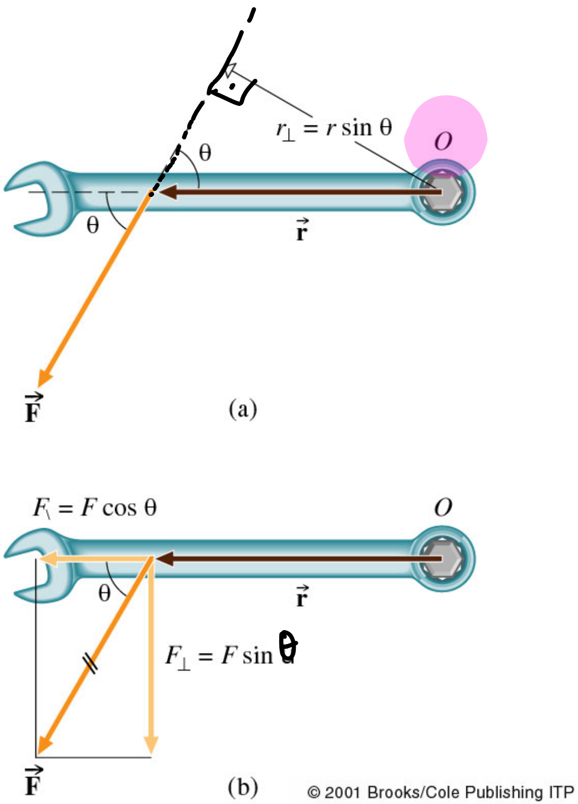
$$\tau_O = r_\perp \cdot F = r \cdot \sin\theta \cdot F$$

$$= r \cdot F_\perp = r \cdot F \cdot \sin\theta$$

bras de levier

$$[\tau] = [F][r] = N\,m$$



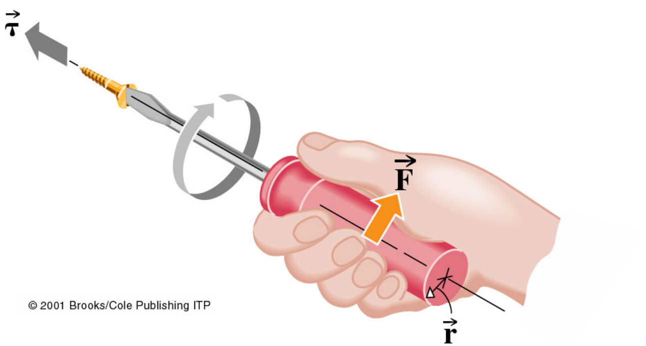



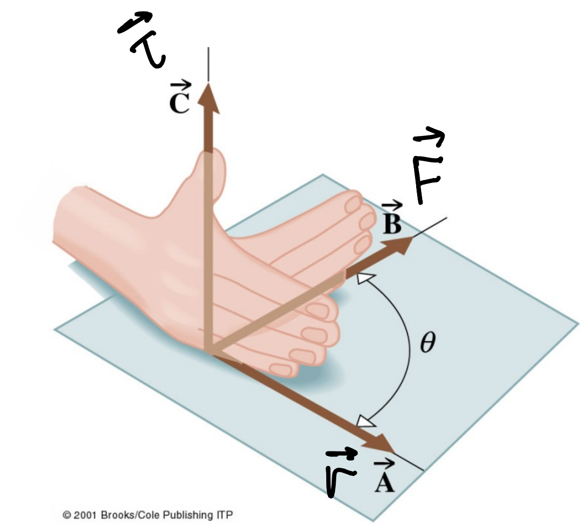
# LE MOMENT DE FORCE

$\vec{\tau}$ : vecteur

$\tau_o = r \sin\theta \, F$

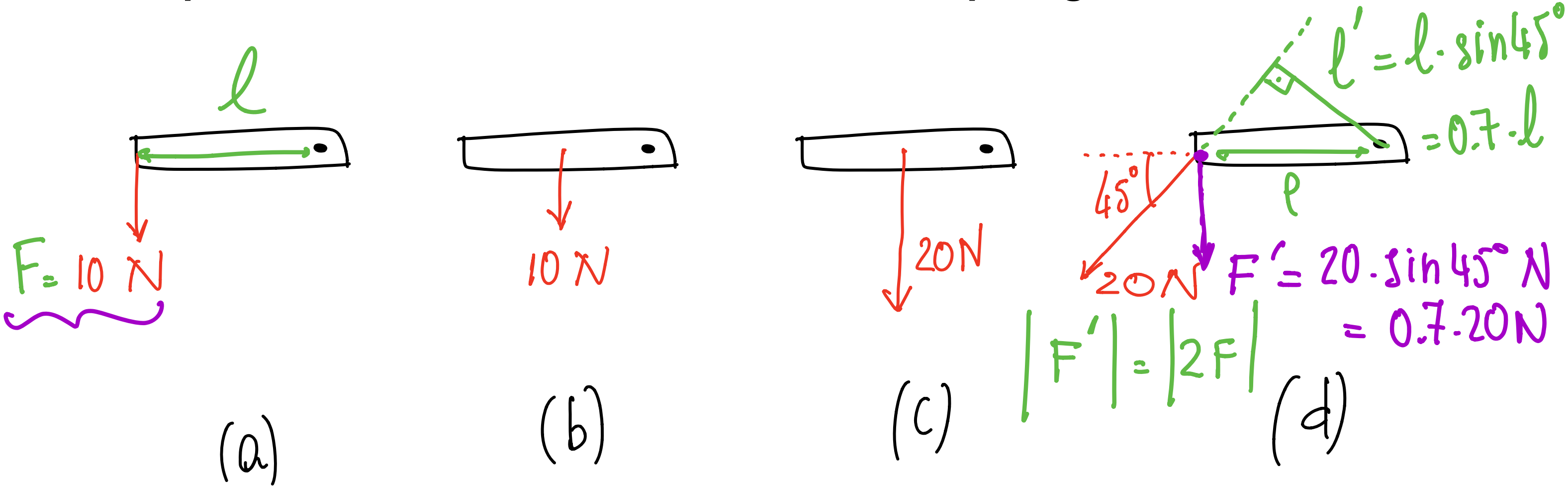
$\Rightarrow \quad \vec{\tau}_o = \vec{r} \times \vec{F}$



# QUESTION

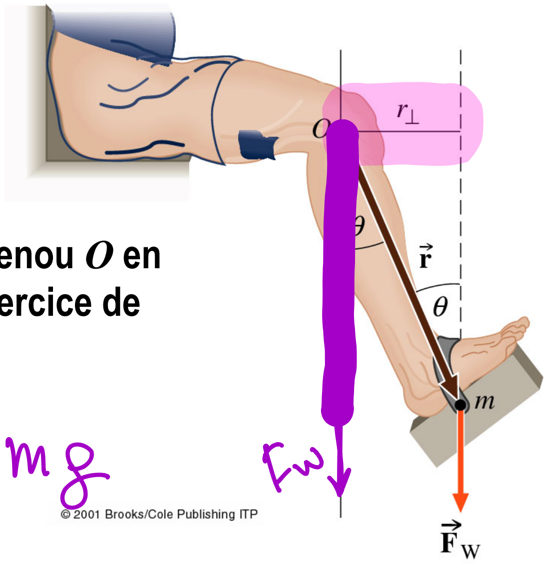
**Dans quelle situation le moment de force est-il plus grand?**




$$\tau = F \cdot l$$

$$\tau = 2F \cdot 0.7 \cdot l = 1.4 \cdot F \cdot l$$

# EXEMPLE




**QUESTION: Exprimez le moment des forces par rapport au genou *O* en fonction de *θ*, *m*, et la distance *r* du genou au talon dans l'exercice de musculation ci-contre. Négligez la masse de la jambe.**

$$\tau_O = r_\perp \cdot F = r_\perp \cdot mg = r \sin\theta \, mg$$

$\tau_O$ min $\quad \theta : 0° \quad F_W \parallel r$

$\tau_O$ max $\quad \theta : 90° \quad F_W \perp r$

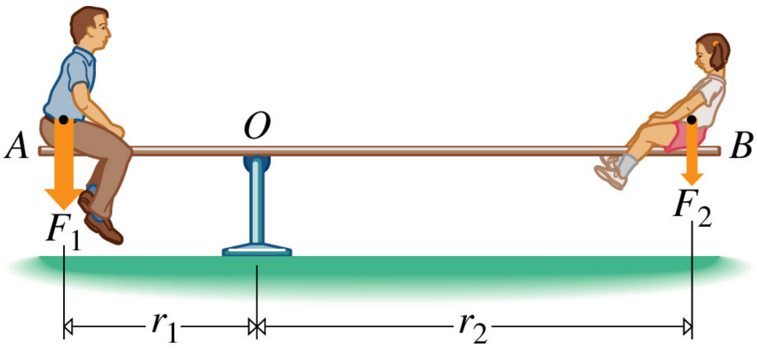
# SECONDE CONDITION D'ÉQUILIBRE

$\sum \vec{F} = 0 \Leftrightarrow \Delta \vec{v} = 0$

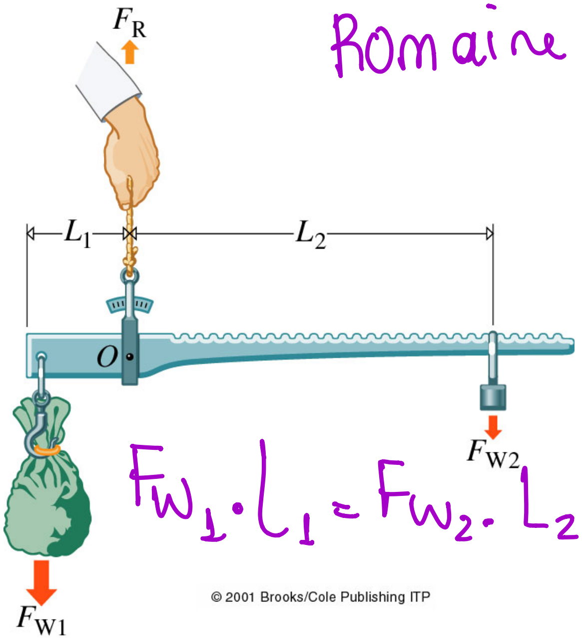
$\sum \vec{T} = 0 \Leftrightarrow \Delta \omega = 0$

Balane Romaine

$\sum \vec{T}_0 = 0 \Rightarrow F_1 \cdot r_1 = F_2 \cdot r_2$






$F_{W_1} \cdot L_1 = F_{W_2} \cdot L_2$

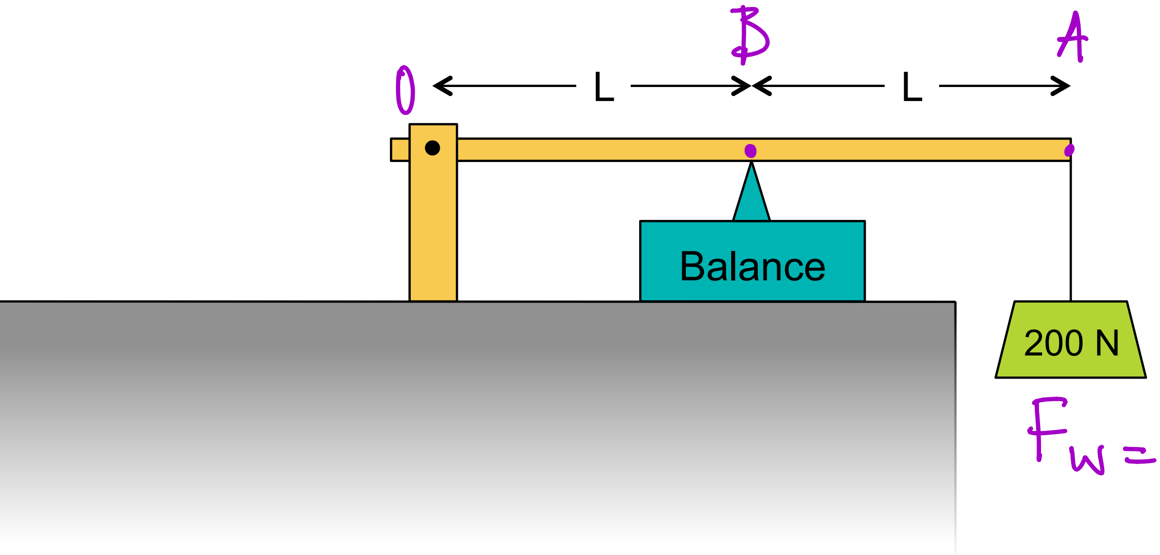
# QUESTION

Quelle est l'indication de la balance?

$$\sum \vec{\tau} = 0$$

$$\left.\begin{aligned} \tau_{A_O} &= \tau_{B_O} \\ \tau_B &= L \cdot F_{Balance} \\ \tau_A &= 2 \cdot L \cdot F_W \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow F_{Balance} = 2 F_W$$



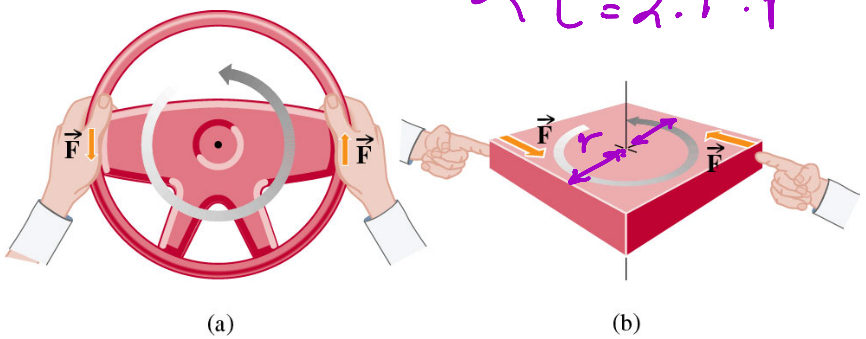

$$F_W = 200\,N$$

# FORCES NON-CONCOURANTES

Un couple des forces

Non collinear $\sum \vec{F} = 0$ mais $\sum \vec{\tau} \neq 0$

$\sum \tau = 2.F.r$

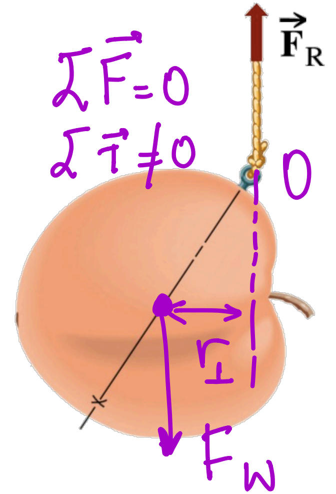


# CENTRE DE MASSE

$x_{c.g.}$ : point qui concentre toute masse du solide.

$\sum \vec{F} = 0$

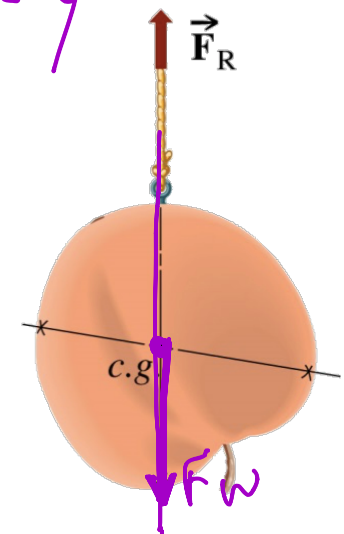
$\sum \vec{\tau} \neq 0$




$$x_{c.g.} = \frac{\sum_{i=1}^{N} F_{W_i} \cdot x_i}{\sum_{i=1}^{N} F_{W_i}}$$

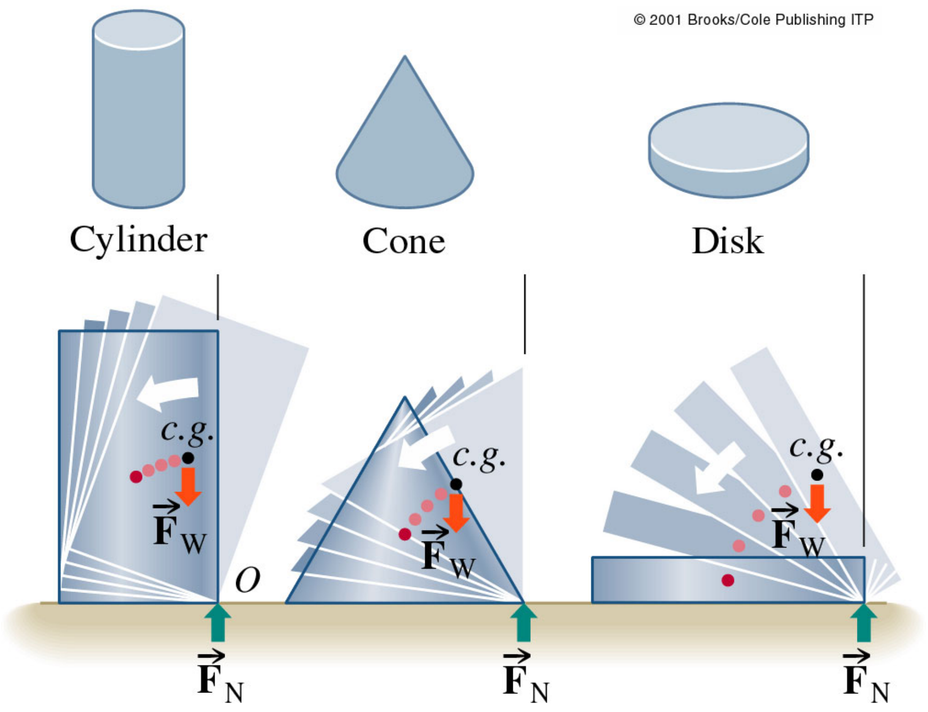
pareil pour y

bras du levier




"Le point où s'applique la résultante force $\Sigma F$".

# STABILITÉ ET ÉQUILIBRE



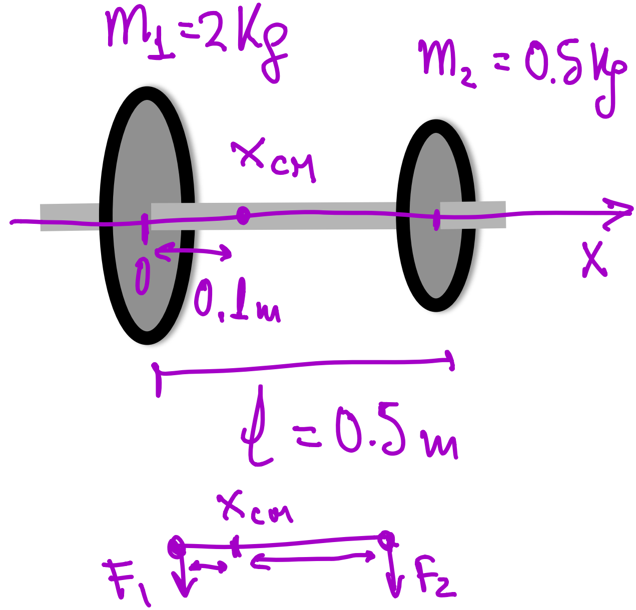
# LE CENTRE DE MASSE

Un haltère se compose d'un disque de 500 gr d'une coté et d'un disque de 2 kg de l'autre coté. On considère la barre qui les connecte sans masse et le longueur 50 cm. Calculer le centre de masse.

$$x_{cM} = \frac{\sum_i F_i x_i}{\sum_i F_i} \qquad F_i = m_i g$$

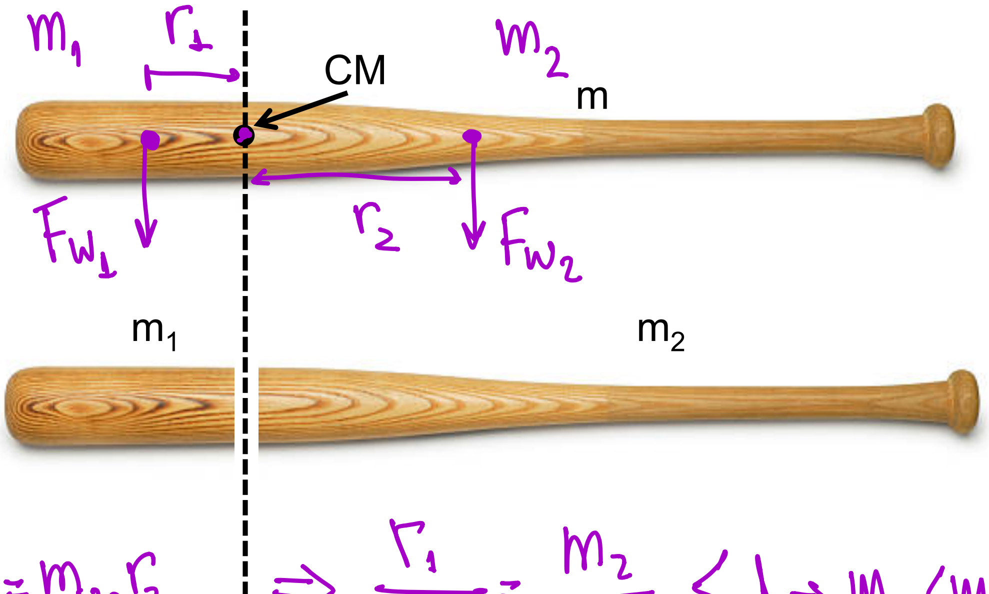
$$x_{cM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 \cdot x_1 + m_2 \cdot x_2}{m_1 + m_2} =$$

$$= \frac{\cancel{2 \cdot 0} + 0.5 \cdot 0.5}{2 + 0.5}\, m = 0.1\, m$$



# LE CENTRE DE MASSE - QUESTION

$$\frac{r_1}{r_2} < 1$$




(a) $m_1 > m_2$

(b) $m_1 = m_2$

(c) $m_1 < m_2$

$$\sum \vec{\tau}_0 = 0 \Rightarrow m_1 \cdot r_1 = m_2 \cdot r_2 \Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1} < 1 \Rightarrow \underline{\underline{m_2 < m_1}}$$

# DYNAMIQUE DE ROTATION

PGC-04

# DYNAMIQUE DE ROTATION - INTRO

$a \Longleftrightarrow F$

$F = ma$

$m$ : inertie qui s' oppose au changement de son état. " Resistance "

$a_{ang} \Longleftrightarrow \tau$

$\tau = I \cdot a_{ang}$

$I$ : inertie
masse et comment elle est repartie autour de l'axe de rotation.
$\Rightarrow$ " moment d' inertie "

# MOMENT D'INERTIE

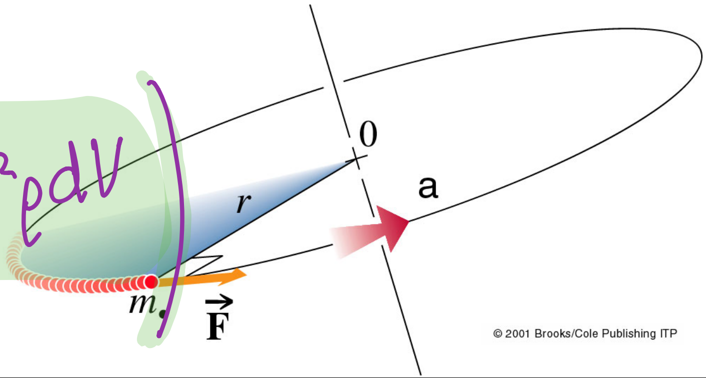
$$\left.\begin{aligned} \tau_0 &= r \cdot F = r \cdot m_0 \cdot a_t \\ a_t &= r \cdot a_{ang} \end{aligned}\right\} \Rightarrow \begin{aligned} \tau_0 &= m_0 \cdot r^2 \cdot a_{ang} \\ \tau_0 &= I_0 \, a_{ang} \\ &\Updownarrow \\ (F &= m \, a) \end{aligned}$$

$$\tau_0 = I_0 \cdot a_{ang}$$

$$\tau_0^{TOT} = \sum \tau_0 = \sum_i (m_i r_i^2) \cdot a_{ang}$$

$$I_0 = \sum_i (m_i r_i^2)$$

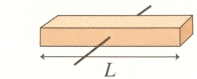
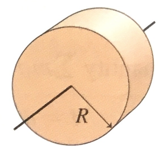
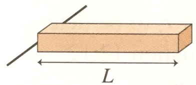
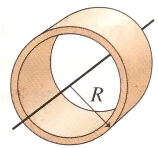
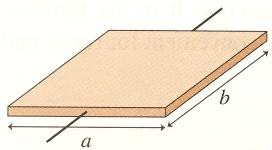
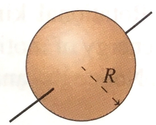
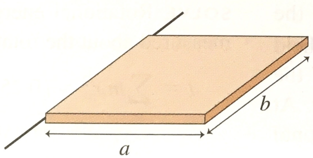
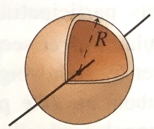
$$\left( \underline{\underline{dm}} \quad \left.\begin{aligned} I_0 &= \int r^2 dm \\ dm &= \rho \, dV \end{aligned}\right\} \Rightarrow I_0 = \int r^2 \rho \, dV \right)$$



# MOMENT D'INERTIE DES CORPS SIMPLES

$\tau = I \cdot a_{ang}$

**TABLE 12.2** Moments of inertia of objects with uniform density

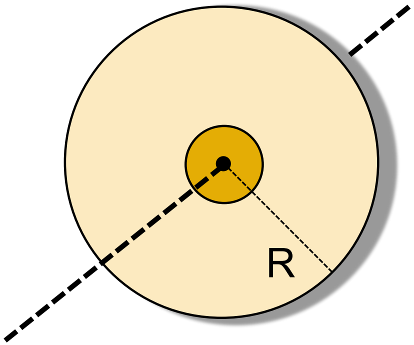
| Object and axis | Picture | $I$ | Object and axis | Picture | $I$ |
|---|---|---|---|---|---|
| Thin rod, about center |   | $\frac{1}{12}ML^2$ | Cylinder or disk, about center |   | $\frac{1}{2}MR^2$ |
| Thin rod, about end |   | $\frac{1}{3}ML^2$ | Cylindrical hoop, about center |   | $MR^2$ |
| Plane or slab, about center |   | $\frac{1}{12}Ma^2$ | Solid sphere, about diameter |   | $\frac{2}{5}MR^2$ |
| Plane or slab, about edge |   | $\frac{1}{3}Ma^2$ | Spherical shell, about diameter |   | $\frac{2}{3}MR^2$ |

Exemple

même masse, differente inertie ! ie resistance au mouvement !

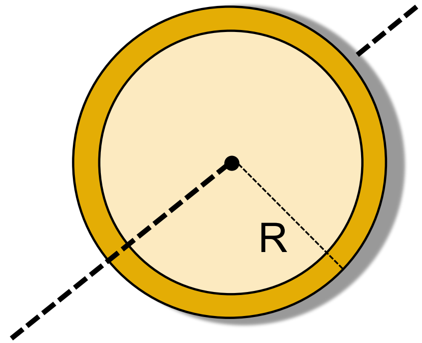
# MOMENT D'INERTIE

Masse M




Objet 1

Masse M




Objet 2

(a) $I_1 > I_2$

(b) $I_1 = I_2$

(c) $I_1 < I_2$