# L’ÉNERGIE

PGC-05

# PUISSANCE

$$\text{Puissance} = \frac{\text{Travail}}{\text{Int. Temps}} \Rightarrow P_m = \frac{\Delta W}{\Delta t}$$

$$\Delta t \to 0 \qquad P = \frac{dW}{dt} \qquad [P] = \frac{J}{s} = \text{Watt } (W)$$

$$P = \frac{dW}{dt} = \frac{d(\vec{F} \cdot \vec{x})}{dt} = \vec{F} \cdot \frac{d\vec{x}}{dt} = \vec{F} \cdot \vec{v}$$

pour Force const.

# EXEMPLE

La vitesse moyenne de l'ascenseur express de la tour Sears à Chicago est de 548.6 m/min. Quelle est la puissance moyenne délivrée par son moteur lors de la montée d'une charge totale de $1.0 \times 10^3$ kg au 103e étage à 408.4m au-dessus du sol ?

$m = 1 \times 10^3$ kg

$g = 10 \; m/s^2$

$h = 408.4 m$

$V_m = 548.6 \; m/min = \frac{548.6 m}{60 s} = 9.14 \frac{m}{s}$

$P = \frac{\Delta W}{\Delta t}$

$\Delta W = F \cdot h = mgh = 4 \times 10^6 J$

$\Delta t = \frac{h}{V_m} = 44.6 \; s$

$P = \frac{\Delta W}{\Delta t} = 90 \; kW$

h

h=0

# QUELQUES EXEMPLES

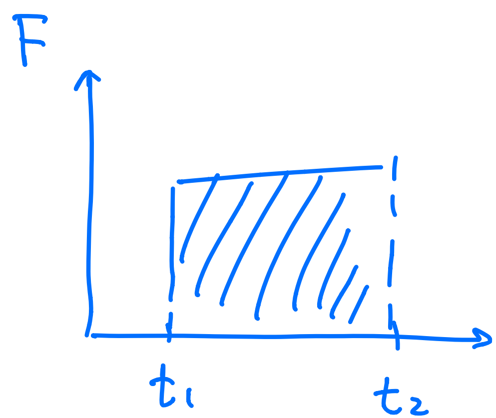
- La puissance nécessaire pour faire monter 1 kg de 1m par seconde  est  9.81 W
- La puissance consommée par une forte ampoule électrique est 50-100 W
- La puissance consommée par un aspirateur est de 400-2000 W
- La puissance produite par une éolienne de 1 à 7 MW
- La puissance produite par un réacteur nucléaire est environ $1{,}5\ 10^{9}$ W = 1.5 GW (une centrale en comporte plusieurs)

# IMPACT DE FORCE

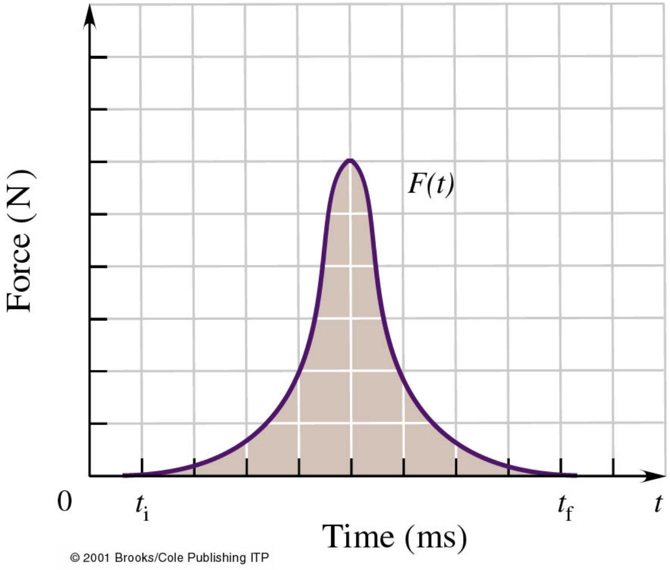
$$\vec{F}_m = \frac{\Delta \vec{P}}{\Delta t} \Rightarrow \Delta \vec{P} = \vec{F}_m \cdot \Delta t$$

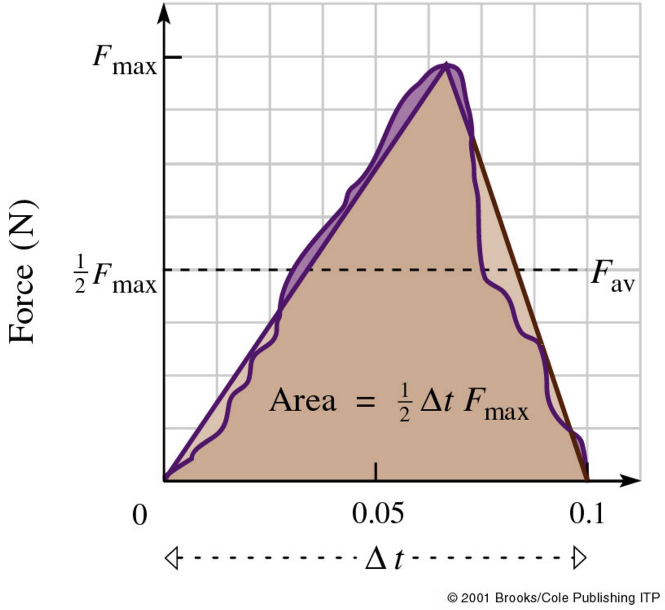
$$\Delta P = \underbrace{F_m \cdot \Delta t}_{\text{impact}}$$

$$\Delta P = \int_{t_1}^{t_2} F_m \, dt$$

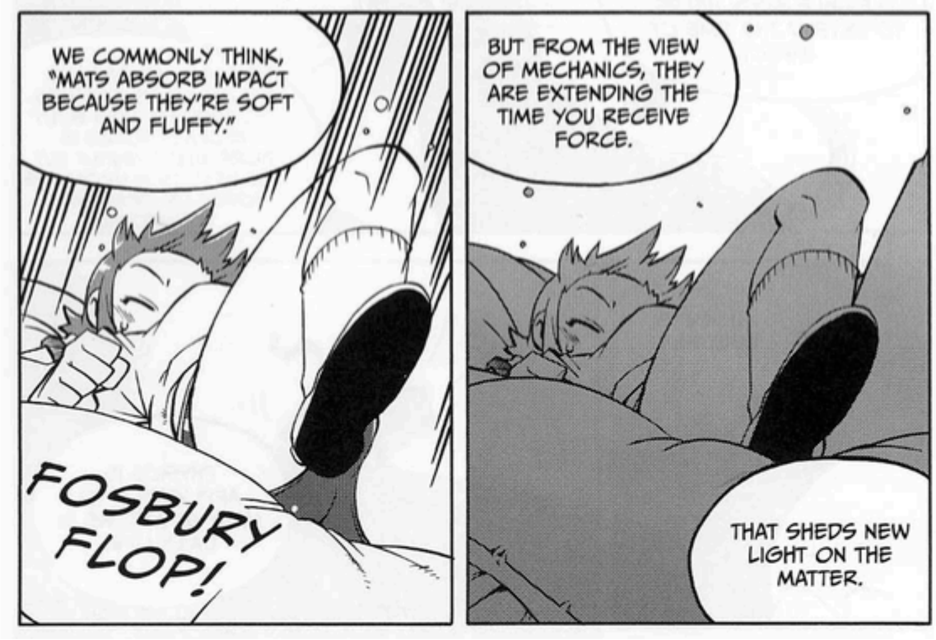


# IMPACT ET FORCE VARIABLE



# IMPACT ET FORCE VARIABLE

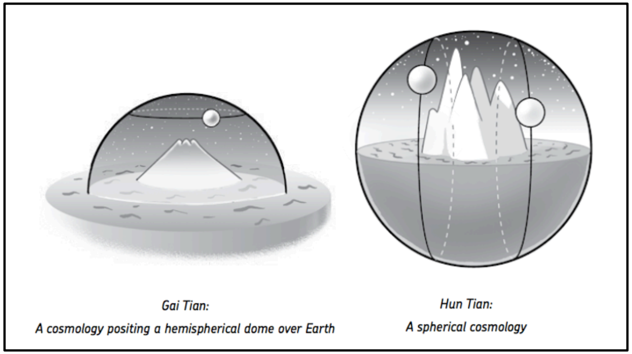


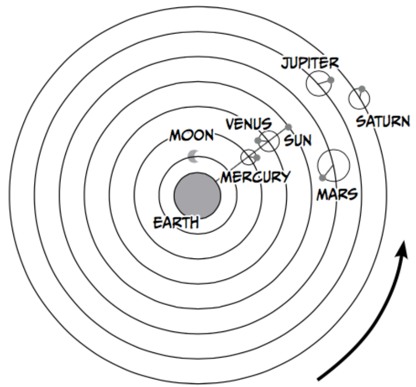
# LA GRAVITÉ SELON NEWTON

PGC-06

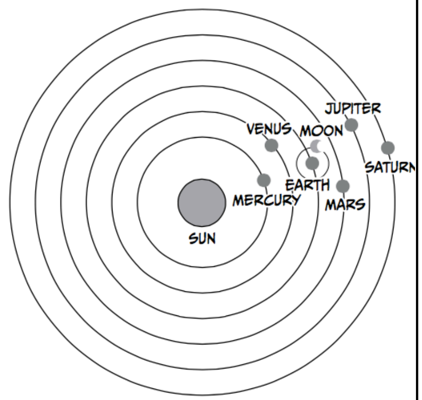
# UN PEU D'HISTOIRE

| | |
|---|---|
| ~ 400 | Grèce antique |
| BC | |
| AC | |
| ~150 | Ptolémée |
| 1543 | Copernic<br>'De Revolutionibus' |
| 1609<br>1619 | Kepler<br>'Astronomia nova'<br>'Harmonices Mundi' |
| 1609<br>1632 | Galilée<br>Téléscope<br>'Dialogo sopra i due massimi sistemi del mondo' |



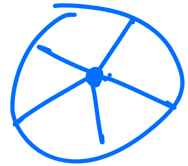
Modèle géocentique

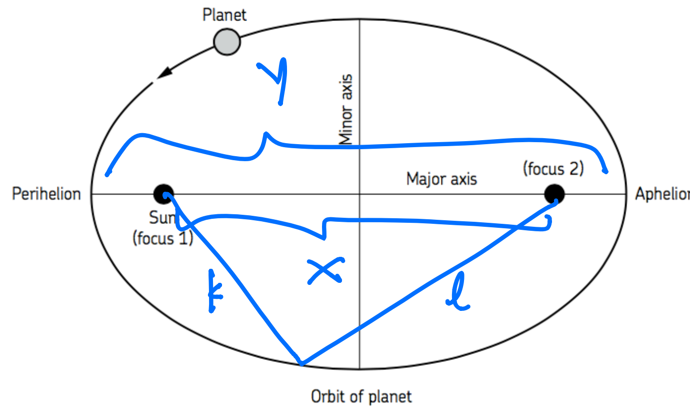


Modèle heliocentrique

# LES TROIS LOIS DE KEPLER

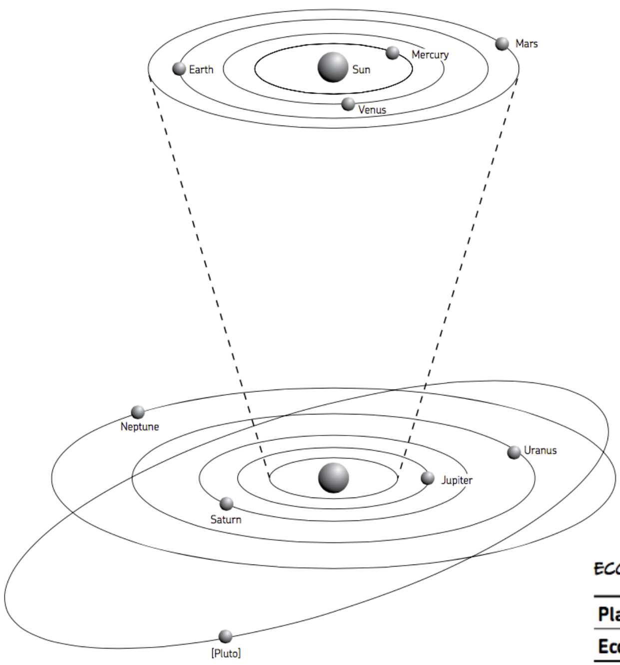
## 1.






*Orbit of a planet according to Kepler's First Law*

$$k + l = \text{const}$$



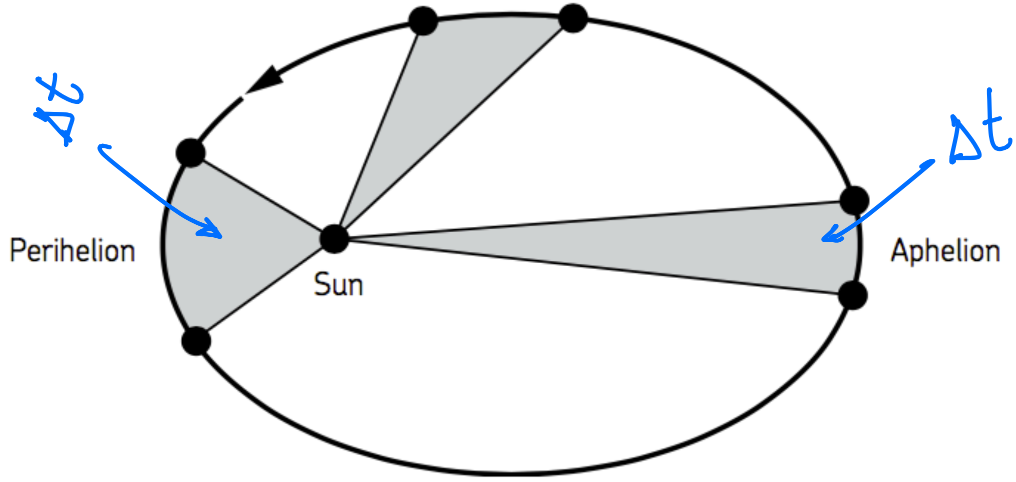

$$\to \frac{x}{y}$$

ECCENTRICITY OF EACH PLANET IN THE SOLAR SYSTEM

| **Planet** | Mercury | Venus | Earth | Mars | Jupiter | Saturn | Uranus | Neptune |
|---|---|---|---|---|---|---|---|---|
| **Eccentricity** | 0.2056 | 0.0068 | 0.0167 | 0.0934 | 0.0485 | 0.0555 | 0.0463 | 0.0090 |

# LES TROIS LOIS DE KEPLER

## 2.




*Orbit of a planet according to Kepler's Second Law*

# LES TROIS LOIS DE KEPLER

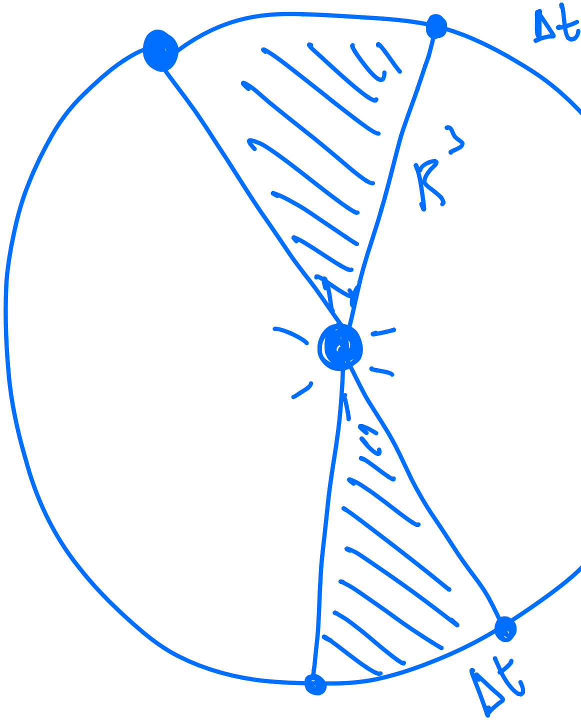
3.

$$\frac{R^3}{T^2} = \text{comtante}$$

SEMIMAJOR AXIS OF A PLANET'S ORBIT AND ORBITAL PERIOD

| Planet | Semimajor axis of orbit $a$ (AUs) | $a^3$ | Orbital period relative to the fixed star's $P$ (solar years) | $P^2$ | $a^3/P^2$ |
|---|---|---|---|---|---|
| Mercury | 0.3871 | 0.05800555 | 0.2409 | 0.05803281 | 0.9995 |
| Venus | 0.7233 | 0.37840372 | 0.6152 | 0.37847104 | 0.9998 |
| Earth | 1.0000 | 1 | 1.0000 | 1 | 1.0000 |
| Mars | 1.5237 | 3.53751592 | 1.8809 | 3.53778481 | 0.9999 |
| Jupiter | 5.2026 | 140.819017 | 11.8620 | 150.707044 | 1.0008 |
| Saturn | 9.5549 | 872.32524 | 29.4580 | 867.773764 | 1.0052 |
| Uranus | 19.2184 | 7098.25644 | 84.0220 | 7049.69648 | 1.0055 |
| Neptune | 30.1104 | 27299.1783 | 164.7740 | 27150.4711 | 1.0055 |

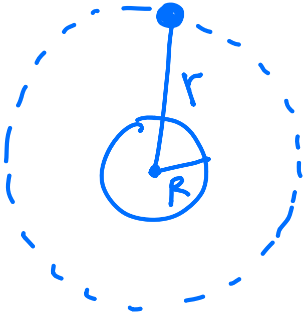
# LES TROIS LOIS DE KEPLER




$$T^2 \propto R^3$$

1609 : Télescope

# NEWTON 1642-1727




$$\frac{g_T}{g_{TL}} = \left( \frac{1/R}{1/r} \right)^2$$

$$F \sim \frac{1}{r^2}$$

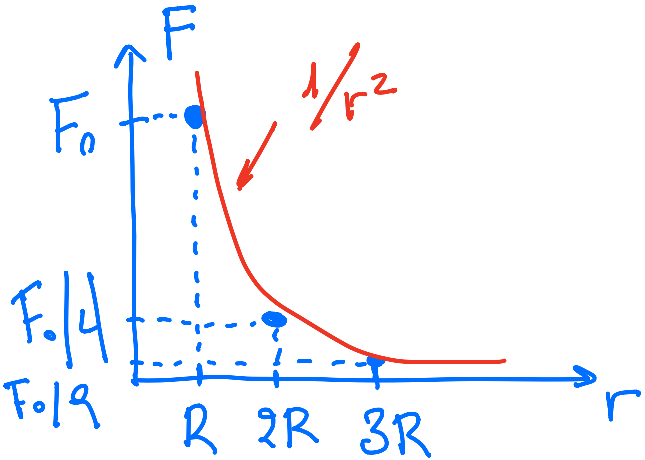
# LOI DE GRAVITÉ DE NEWTON

$$F \propto \frac{m_1 \cdot m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

$m_1$ $m_2$

$$-F_{2\to1} = F_{1\to2}$$



# GRANDEUR DE LA FORCE GRAVITATIONNELLE

$$\left.\begin{array}{l} m_1 = 1\,kg \\ m_2 = 1\,kg \\ r = 1\,m \end{array}\right\} \Rightarrow \boxed{F = G\,\frac{m_1 m_2}{r^2}} = \underbrace{6.67 \times 10^{11}}\,N$$

$$\left.\begin{array}{l} m_1 = M_T = 6 \times 10^{24}\,kg \\ \boxed{m_2 = 1\,kg} \\ r = R_T = 6.4 \times 10^{6}\,m \end{array}\right\} \Rightarrow F = 9.8\,N$$

# PRINCIPE DE L'ÉQUIVALENCE

$$m_{\text{inertie}} = \frac{F}{a}$$ ⟹ pas de gravité

$$m_{\text{gravitationelle}} = \frac{r^2 F}{G M_T}$$ ⟹ pas d'acceler.

$$m_{\text{inertie}} = m_{\text{gravitationnelle}}$$

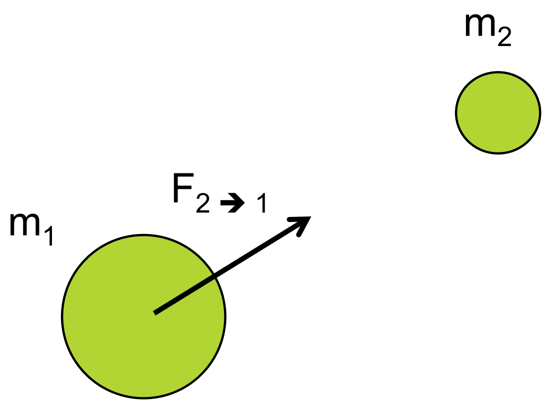
# THÉORIE DE GRAVITÉ DE NEWTON

1. $F = G \dfrac{m_1 m_2}{r^2}$

2. principe d'équivalence

3. 3 lois de Newton

# QUESTION




$m_{\cancel{2}1} = 2\ m_{\cancel{1}2}$

(a) $F_{1 \rightarrow 2} = F_{2 \rightarrow 1}$

(b) $F_{1 \rightarrow 2} = 2\ F_{2 \rightarrow 1}$

(c) $2\ F_{1 \rightarrow 2} = F_{2 \rightarrow 1}$

(d) $F_{1 \rightarrow 2} = 4\ F_{2 \rightarrow 1}$

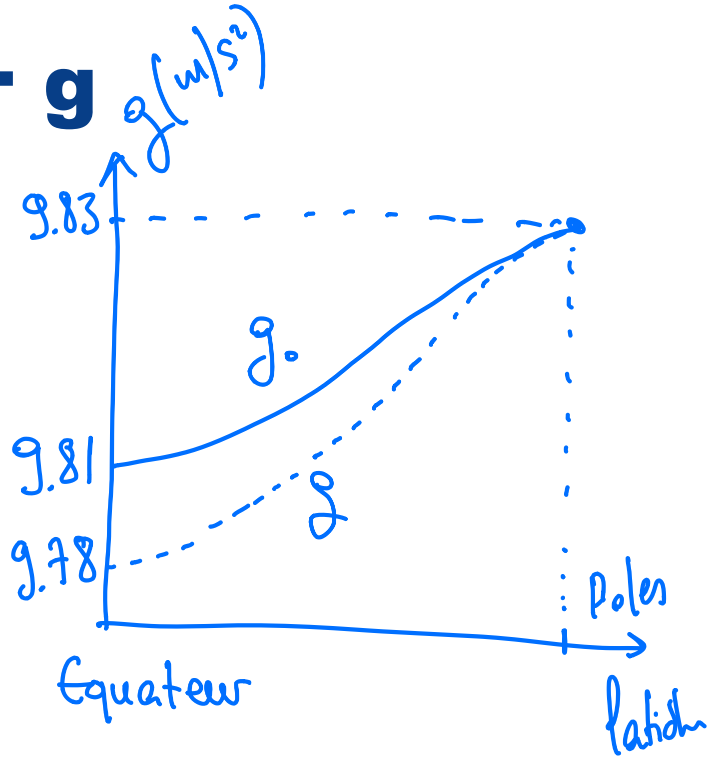
(e) $4\ F_{1 \rightarrow 2} = F_{2 \rightarrow 1}$

# GRAND G ET PETIT g

$$F_G = G\frac{Mm}{R^2}$$

$$F_D = mg$$

$$\Rightarrow \boxed{g_0 = \frac{GM}{R^2}}$$




$$g_T = 9.8\ m/s^2$$

$$g_{Mars} = 3.8\ m/s^2$$

$$\Sigma F = m a_c \Rightarrow F_G - F_D = m a_c \Rightarrow$$

$$\Rightarrow g = \frac{GM}{R^2} - a_c \quad \left(g = g_0 - a_c\right)$$

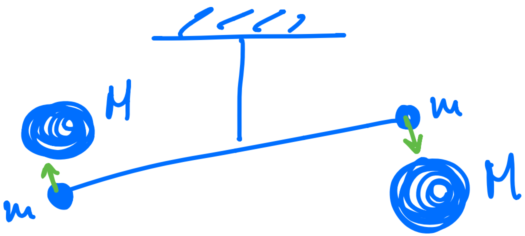
# LA MASSE DE LA TERRE?

$$g = \frac{G M_T}{R^2}$$

1%

HENRY CAVENDISH

$$G = \frac{F R^2}{M m}$$




g : cinematique

R : techniques
surveillance

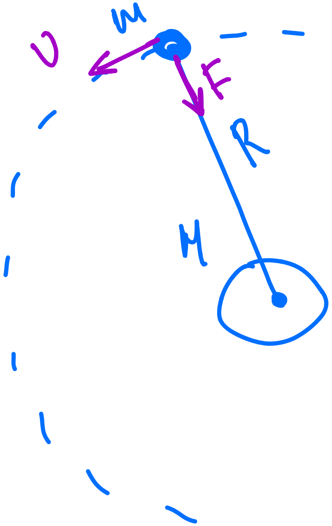
G : Exp. Cavendish

m

$$\Downarrow$$

$$M_T = \frac{g R^2}{G}$$

# ORBITE DE SATELLITE




$$F_c = m a_c = F_G = G\frac{mM}{R^2} \Rightarrow$$

$$\Rightarrow \frac{\cancel{m}v^2}{R} = G\frac{\cancel{m}M}{R^2} \Rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$$

$$V = \frac{\text{circ}}{\text{per}} = \frac{2\pi R}{T}$$

$$\Rightarrow$$

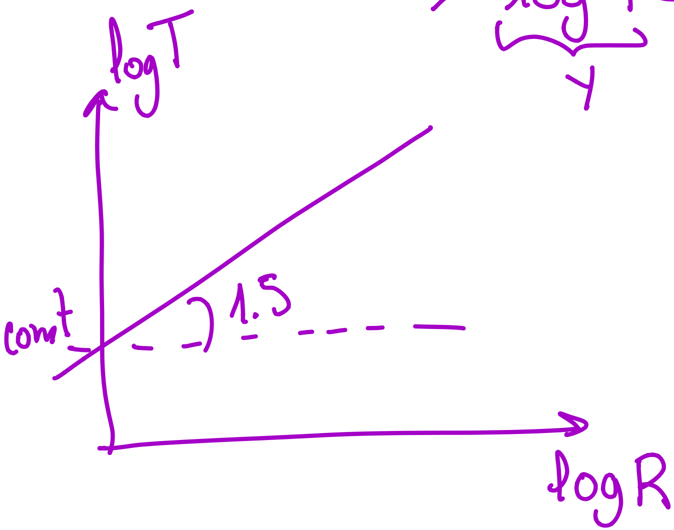
$$\frac{2\pi R}{T} = \sqrt{\frac{GM}{R}} \Rightarrow T^2 = \frac{4\pi^2}{GM}R^3 \Rightarrow \boxed{\frac{R^3}{T^2} = \frac{GM}{4\pi^2}} = \text{comtante}$$

# ORBITES GEOSTATIONNAIRES

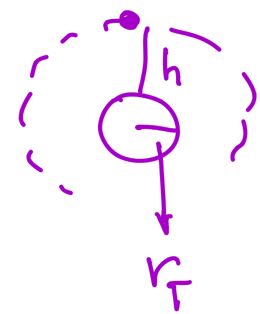
$$\frac{R^3}{T^2} = \text{cont} \Rightarrow \log\left(\frac{R^3}{T^2}\right) = \text{cont} \Rightarrow \log R^3 - \log T^2 = \text{cont}$$

$$\Rightarrow 3\log R - 2\log T = \text{cont} \Rightarrow$$

$$\Rightarrow \underbrace{\log T}_{Y} = 1.5\,\underbrace{\log R}_{X} + \text{cont}$$



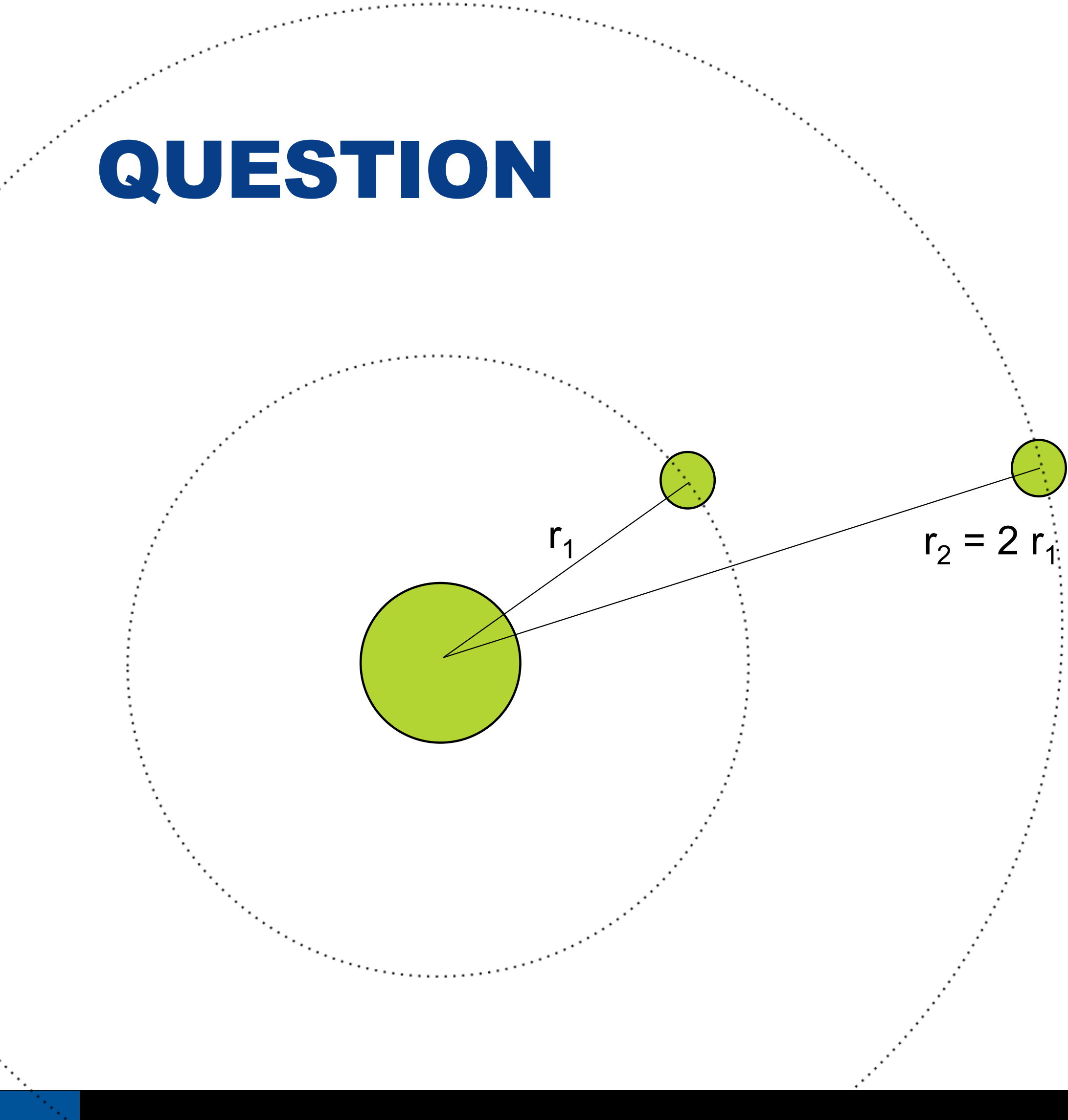

$$T = 24h$$




$$R = r_T + h$$

$$\Rightarrow h \simeq 5.6\, r_T$$

# QUESTION




$T_2$ ? $T_1$

(a) $T_2 = 2\ T_1$

(b) $T_2 = 2.8\ T_1$

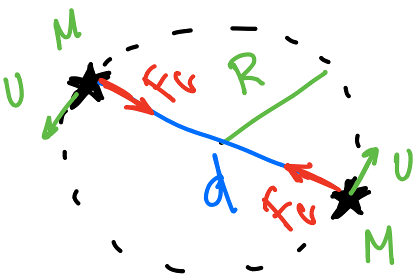
(c) $T_2 = 4\ T_1$

(d) $T_2 = 0.2\ T_1$

# EXEMPLE

Système binaire des étoiles. Masse de chacune: 2 x masse du soleil. Période de 90 jours. Quelle est la distance entre les deux étoiles?

$$F_G = \frac{G M_1 M_2}{d^2} = \frac{G M^2}{d^2} = M a_c \Rightarrow$$




$$\Rightarrow \left.\begin{aligned} \frac{G M^2}{4R^2} &= \frac{M v^2}{R} \\ v &= \frac{2\pi R}{T} \end{aligned}\right\} \Rightarrow R = \ldots$$

$M = 2 m_s$

$d = 2R$

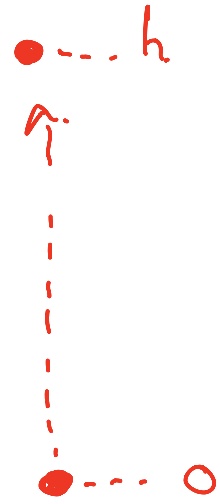
$m_s = 1.99 \times 10^{30}$ kg

# ÉNERGIE POTENTIELLE GRAVITATIONNELLE TERRESTRE

$\Delta E_M = 0 \Rightarrow \Delta E_c = \Delta E_p$

$\Delta E_c = W \Rightarrow \Delta E_p = W = F.h$

$\Rightarrow \Delta E_p = mgh$




Considerer Terre avec Rayon $R_T$

$- \, h \, -$ $E_p^h = ?$

# ÉNERGIE POTENTIELLE GRAVITATIONNELLE TERRESTRE

$$\Delta E_p = E_{p_f} - E_{p_i} = W_F = \int_{r_i}^{r_f} \vec{F}_G \, d\vec{r}$$

$$F_G = G\frac{Mm}{r^2}$$

$$\Delta E_p = \int_{r_i}^{r_f} \frac{GMm}{r^2} dr \Rightarrow \qquad \left( \int \frac{1}{r^2} dr = -\frac{1}{r} \right)$$

$$\Delta E_p = GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = GMm\left(\frac{1}{R_o} - \frac{1}{R_o+h}\right)$$

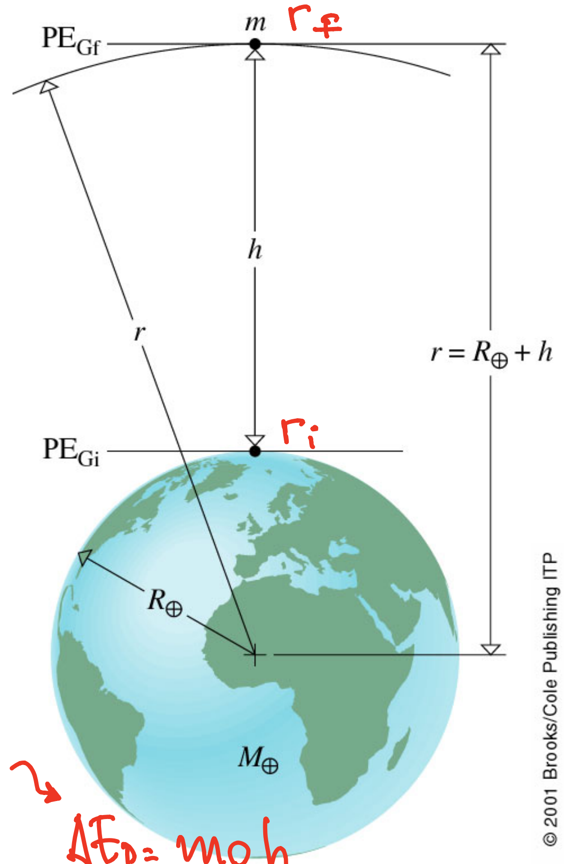
$$\Delta E_p = GmM\left(\frac{h}{R_o(R_o+h)}\right)$$

$$h \ll R_o \Rightarrow R_o + h \simeq R_o$$

$$\Rightarrow \Delta E_p = GmM\frac{h}{R_o^2}$$

$$g = \frac{GM}{R_o^2}$$

$$\Delta E_p = mgh.$$



# ÉNERGIE POTENTIELLE GRAVITATIONNELLE TERRESTRE

$$\Delta E_p = GmM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

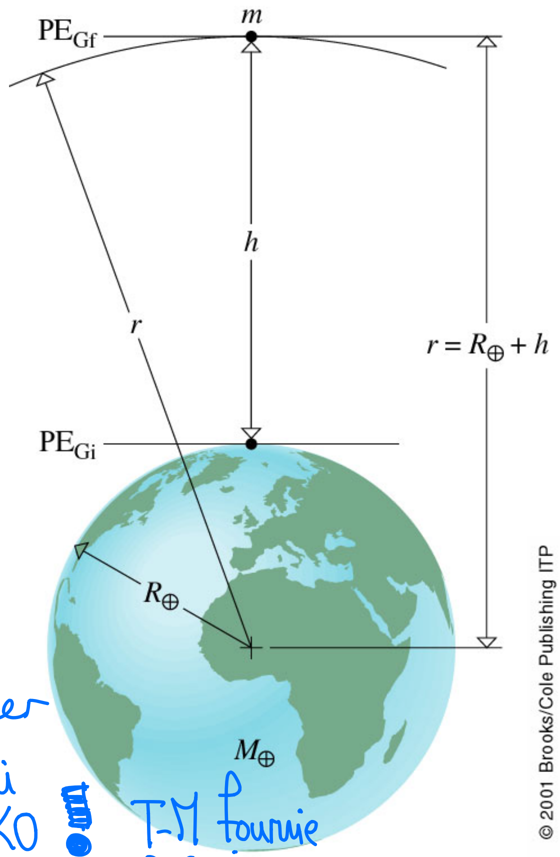
$E_p = 0$ @ $r_\infty \gg R_\circ$ $\quad r_f = R_\circ + h$

$$\Delta E_p = GmM\left(\frac{1}{r_\infty} - \frac{1}{R_\circ + h}\right) \Rightarrow$$

$$\Delta E_p = -\frac{GmM}{R_\circ + h} = -\frac{GmR}{r} \Rightarrow$$

$$\Delta E_p = -\frac{GmR}{r}$$

Travail pour approcher à la Terre une masse qui vient de l'infini <0 !




T-M fournie energie

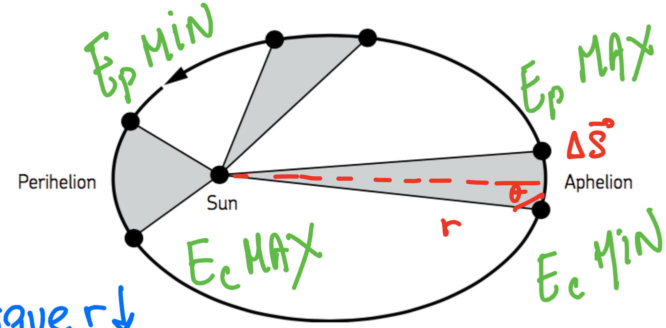
# EXEMPLE – ÉNERGIE GRAVITATIONNELLE

Pourquoi une planète accélère-t-elle sur son orbite en s'approchant du Soleil ?

Moment cinétique

$E_p + E_{CIN} = \text{constante}$

$E_p \propto -\frac{1}{r}$ Quand la planète s'approche $E_p \Downarrow$ puisque $r \downarrow$




*Orbit of a planet according to Kepler's Second Law*

$E_{CIN}$ MAX quand $E_p$ MIN !

$\Delta A = \frac{1}{2} \Delta S \cdot r \cdot \sin\theta$

$\vec{L} = \vec{r} \times \vec{p} \Rightarrow L = r p \sin\theta$

$L = r m \frac{\Delta S}{\Delta t} \sin\theta \Rightarrow L_f = L_i \Rightarrow r \, \Delta S \sin\theta = \text{const} \Rightarrow$

$\Rightarrow \Delta A = \text{const}$