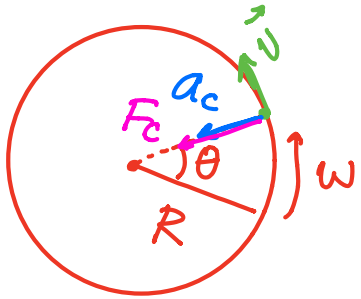
LE MOUVEMENT DE ROTATION

PGC-03

MOUVEMENT CURVILIGNE UNIFORME – RAPPEL

$$\omega = \frac{d\theta}{dt}$$

$v, \omega = \text{const}$



$$a_c = \frac{v^2}{R} = \omega R^2$$

$$F_c = m a_c = m\frac{v^2}{R} = m\omega^2 R$$

$\vec{a} \parallel \vec{R}$ ↑↓

a_c, F_c

QUESTION

Une balle roule du sommet d'une colline avec une vitesse *v*. À ce moment:

(a) $F_N > F_W$

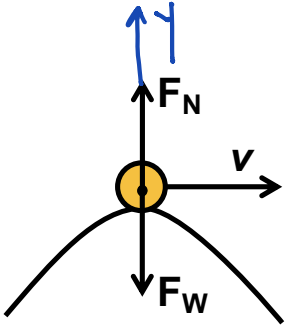
(b) $F_N = F_W$

(c) $F_N < F_W$

(d) On ne peut pas dire si on ne connait pas *v*

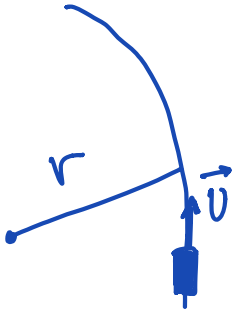
$\Sigma \vec{F} < 0$

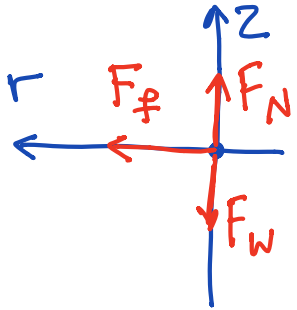
$F_N - F_W < 0 \Rightarrow F_N < F_W$

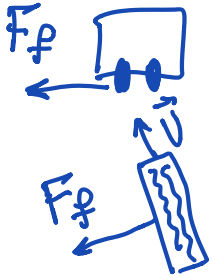


EXEMPLE – TOURNER AU COIN I

Quelle est la vitesse maximale d'une voiture de 1500 kg pour qu'elle prenne un virage de rayon r = 50 m sans glisser? Considerez $\mu_s = 1.0$.







$$\Sigma F_z = 0 \Rightarrow F_N = F_W = mg$$

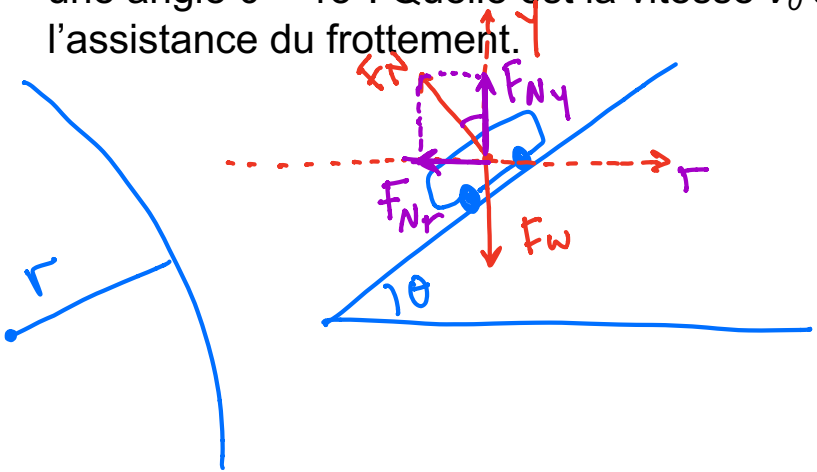
$$\Sigma F_r = m\frac{v^2}{r} = F_f^{max} = \mu_s F_N \Rightarrow$$

$$\Rightarrow \cancel{m}\frac{v^2}{r} = \mu_s \cancel{m} g \Rightarrow$$

$$\Rightarrow v = \sqrt{r \mu_s g} = \dots = 22 \text{ m/s}$$

EXEMPLE – TOURNER AU COIN II

Cette même voiture prend un virage de rayon r = 70 m à une autoroute relevée à une angle θ = 15°. Quelle est la vitesse v_0 à la quelle la voiture peut tourner sans l'assistance du frottement.



$$\Sigma F_y = 0 \Rightarrow F_{N_y} = mg \Rightarrow$$

$$\Rightarrow F_N \cos\theta = mg \quad (1)$$

$$\Sigma F_r = F_c = \frac{mv^2}{r} = F_{N_r} \Rightarrow$$

$$\Rightarrow F_N \sin\theta = \frac{mv^2}{r} \Rightarrow$$

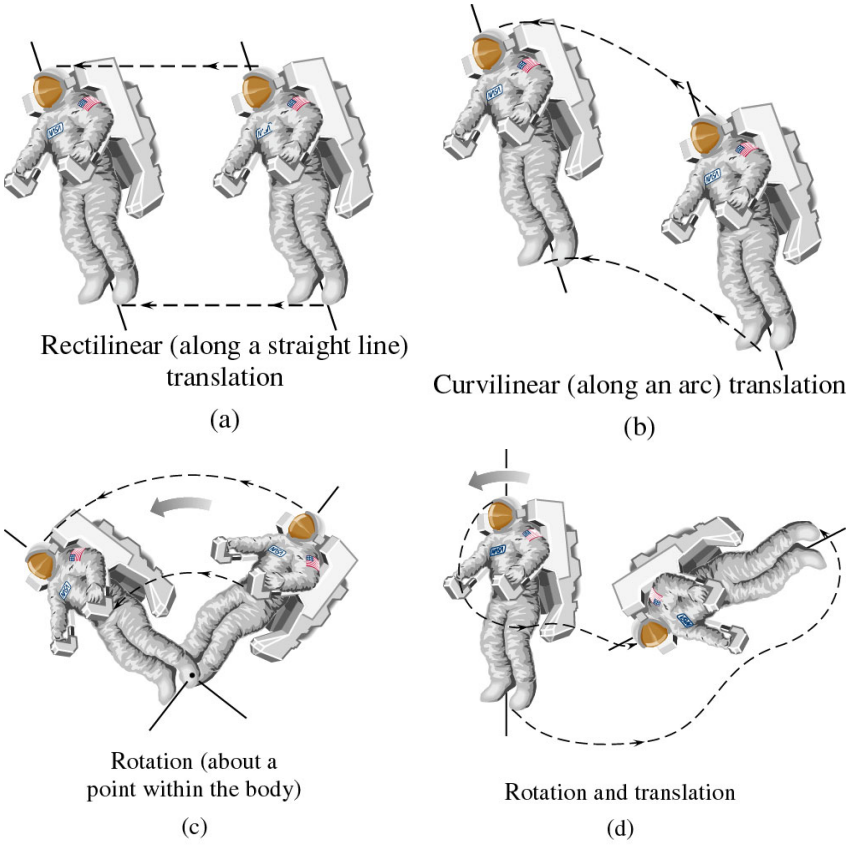
$$(1) \Rightarrow \frac{mg}{\cos\theta} \cdot \sin\theta = \frac{mv^2}{r} \Rightarrow$$

$$\Rightarrow v_0 = \sqrt{gr \tan\theta}$$

MOUVEMENT DE ROTATION

Translation

Rotation.



Rectilinear (along a straight line) translation

(a)

Curvilinear (along an arc) translation

(b)

Rotation (about a point within the body)

(c)

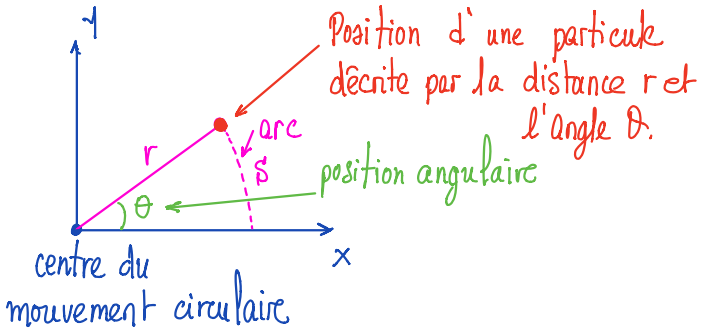
Rotation and translation

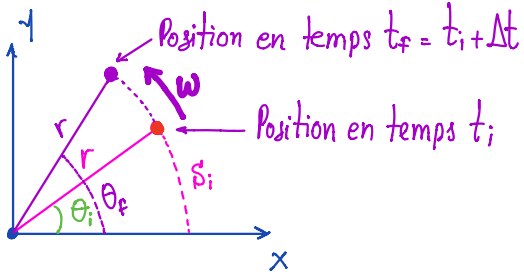
(d)

PARAMÉTRISATION DU MOUVEMENT DE ROTATION

θ : radiants

$1\ \text{rad} = \frac{360°}{2\pi} = 57.3°$



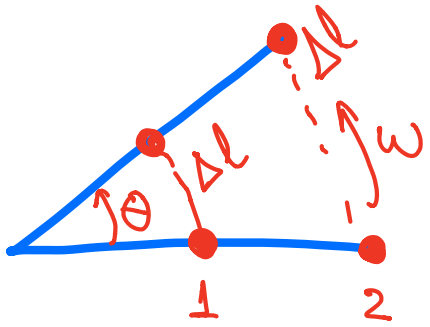


LA VITESSE ANGULAIRE

$$\Delta l = r\,\Delta\theta$$

$$v_m = \frac{\Delta l}{\Delta t} = r\,\frac{\Delta\theta}{\Delta t}$$

$$v = r \cdot \omega$$



ω : const

$\omega \neq$ const

$$\left.\begin{aligned} v_1 &= r_1 \omega_1 \\ v_2 &= r_2 \omega_2 \end{aligned}\right\}$$

ACCÉLÉRATION ANGULAIRE

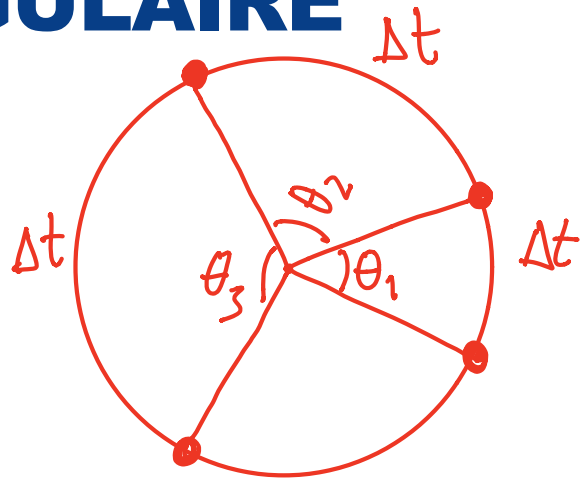
$$\omega = \frac{\Delta\theta}{\Delta t} \neq \text{const}$$

$$a_{ang}^{m} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

$$[a_{ang}] = \frac{[\omega]}{[t]} = \frac{rad}{s^2}$$

$\Delta t \to 0$

$$a_{ang} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad a_{ang} \neq a$$



$$v = \omega r \Rightarrow \frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow$$

r: const!

$$\Rightarrow a_T = r\, a_{ang}$$

ACCÉLÉRATION ANGULAIRE ET ACCÉLÉRATION CENTRIPÈTE

$$\ell = r \cdot \theta \qquad [\ell] : m \qquad [\theta] : rad$$

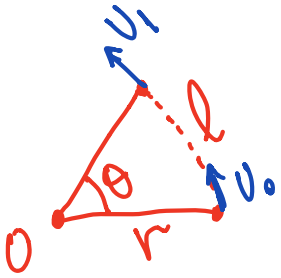
$$\frac{d\ell}{dt} = r\frac{d\theta}{dt} \Rightarrow v = r \cdot \omega \qquad [v] : m/s \qquad [\omega] : rad/s$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow a_T = r\,a_{ang} \qquad [a_T] : m/s^2 \qquad [a_{ang}] = rad/s^2$$

a_T → Δ(module v)

$$a_c = \frac{v^2}{r} \qquad [a_c] = m/s^2$$

a_c → Δ(direction v)



Données $r = 50\,m$

$\omega = 0.60\ rad/s$

$a_{ang} = 0.20\ rad/s^2$

EXEMPLE

Une voiture de Formule 1 prend un virage de 50m de rayon avec une vitesse angulaire de 0.60 rad/s et une accélération angulaire de 0.20 rad/s². Calculez sa vitesse linéaire au début du virage, son accélération centripète, ses accélérations tangentielle et totale.

Trouver: $v,\ a_c,\ a_T,\ a_{TOT}$

$$v = r \cdot \omega = 50m \cdot 0.60\ rad/s = 30\ m/s$$

$$a_c = \frac{v^2}{r} = \frac{30^2}{50}\ m/s^2 = 18\ m/s^2$$

$$a_T = r \cdot a_{ang} = 50 \cdot 0.20\ m/s^2 = 10\ m/s^2$$

$$a_{TOT} = \sqrt{a_c^2 + a_T^2} = 21\ m/s^2$$

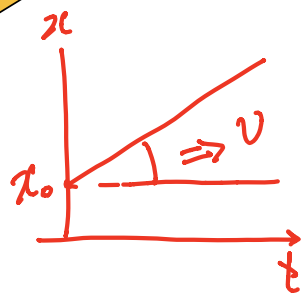
$$\tan\theta = \frac{a_c}{a_T} = \frac{18}{10} \Rightarrow \theta = \dots$$

RESUMÉ

MRU $a=0$

$v=\frac{dx}{dt}=\text{const}$

$x=v_0 t+x_0$



MRUA

$a=\text{const}$

$v=at+v_0$ ① $v=f(t)$

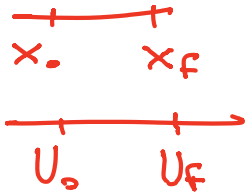
$x=\frac{1}{2}at^2+v_0 t+x_0$ ② $x=f(t)$

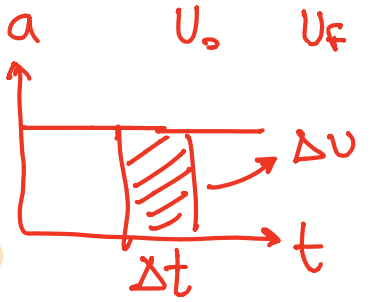
① $\Rightarrow t=\frac{v-v_0}{a}$

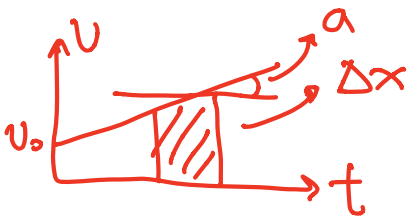
② $\Rightarrow v^2=2ax+v_0^2$ ③ $v=f(x)$

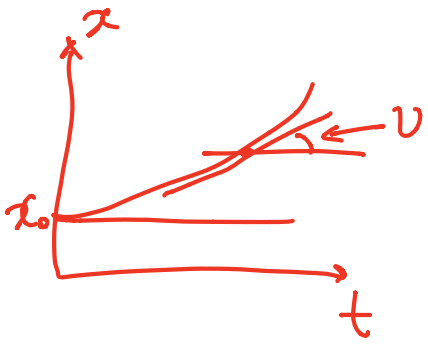
$x_0=0$

(maisoy!)









Rappel!

MOUVEMENT CURVILIGNE UNIFORMÉMENT ACCÉLÉRÉ

MRUA

$$v = v_0 + at$$

$$v_m = \frac{1}{2}(v + v_0)$$

$$x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 - v_0^2 = 2ax$$

MCUA

$$v = v_0 + a_T t$$

$$v_m = \frac{1}{2}(v_0 + v)$$

$$l = v_0 t + \frac{1}{2} a_T t^2$$

$$v^2 - v_0^2 = 2 a_T l$$

$$\omega = \omega_0 + a_{ang} t$$

$$\omega_m = \frac{1}{2}(\omega + \omega_0)$$

$$\theta = \omega_0 t + \frac{1}{2} a_{ang} t^2$$

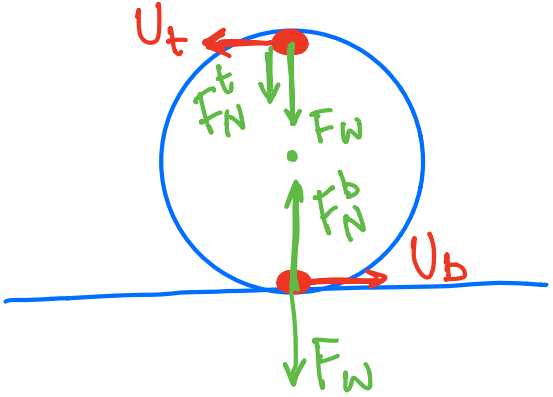
$$\omega^2 - \omega_0^2 = 2 a_{ang} \theta$$

$$l = r\theta$$

$$v = r\omega$$

$$a_T = r a_{ang}$$

EXEMPLE – LOOP VERTICAL

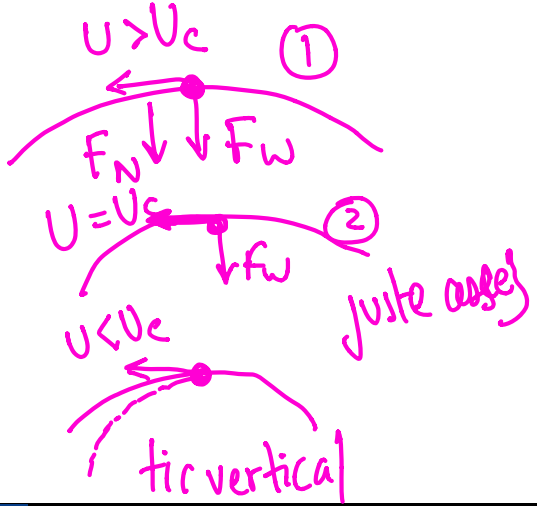


$$\Sigma F_r^b = F_N^b - F_W = \frac{mU_b^2}{r} \Rightarrow F_N^b = \frac{mU_b^2}{r} + mg > mg !$$

$$\Sigma F_r^t = F_W + F_N^t = \frac{mU_t^2}{r} \Rightarrow$$

$$\Rightarrow F_N^t = \frac{mU_t^2}{r} - mg \Rightarrow$$

$$\frac{mU_t^2}{r} = F_N^t + mg$$



Si $F_N^t = 0$ ⇝ No contact $F_W = F_c$

⇝ $\frac{mU_t^c}{r} = mg \Rightarrow U_t^c = \sqrt{gr}$

ROULEMENT SANS GLISSEMENT