ÉQUILIBRE & DYNAMIQUE EN ROTATION

PGC-04

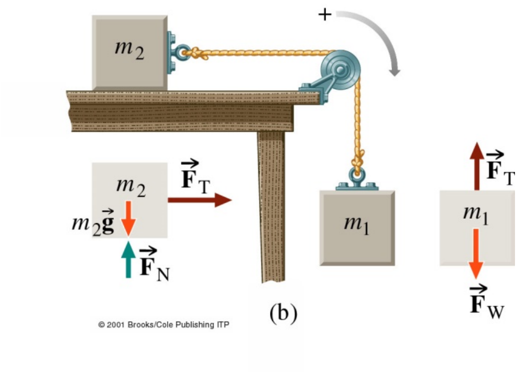
(PGC-02)

ÉQUILIBRE

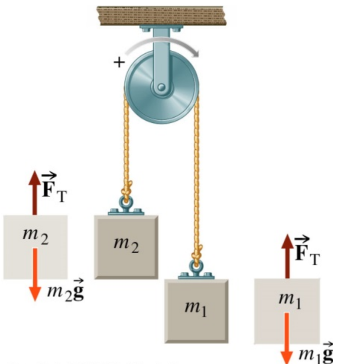
PGC-04

(PGC-02)

RAPPEL – MOUVEMENT COUPLÉ



(b)



(a)

Au repos

$\vec{F}_T = \vec{F}_W$

$m_1 > m_2$

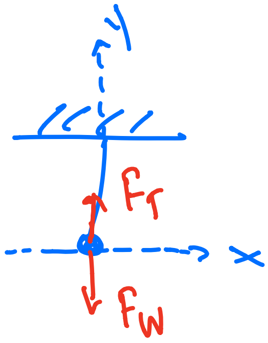
$a_1 = a_2$

ÉQUILIBRE STATIQUE

$\boxed{\sum \vec{F} = 0} \Leftrightarrow \vec{U} = \text{const}$

$\sum F_x = 0 \qquad \sum F_y = 0$

$F_T = F_W$

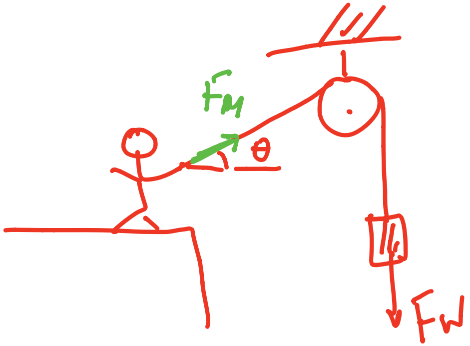


SYSTÈMES DE FORCES PARALLELES

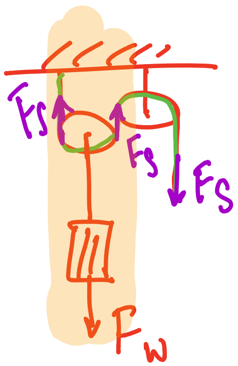
Poulies

- Reorienter Forces

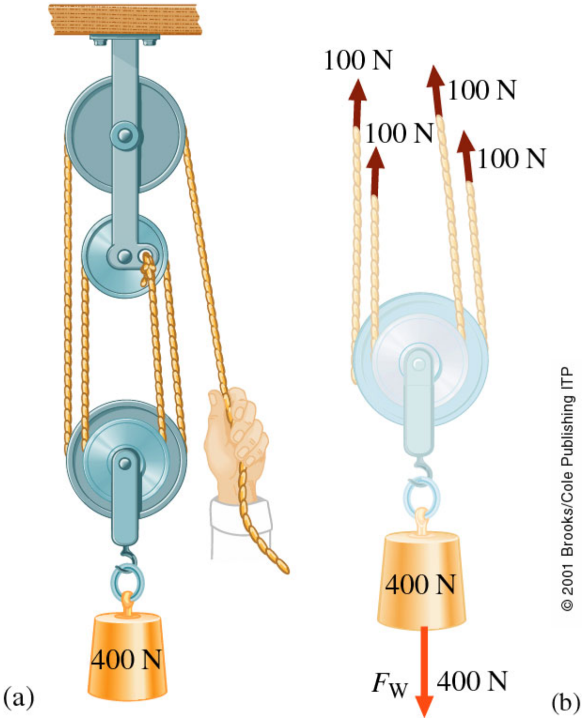
$F_M = F_w$

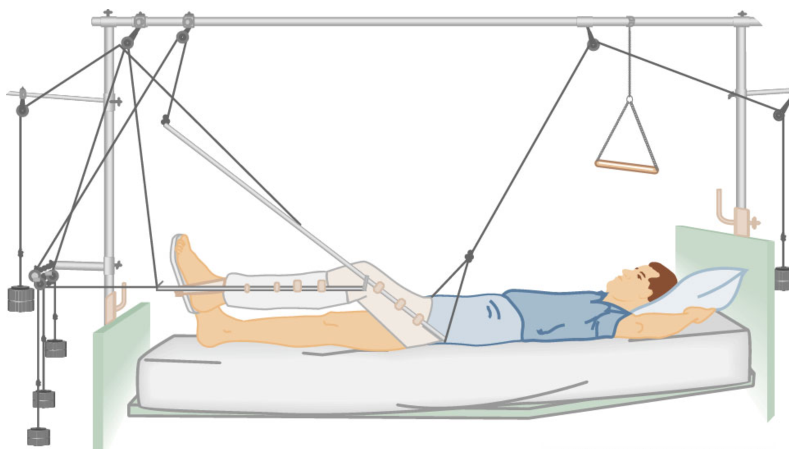


- Demultiplier Forces

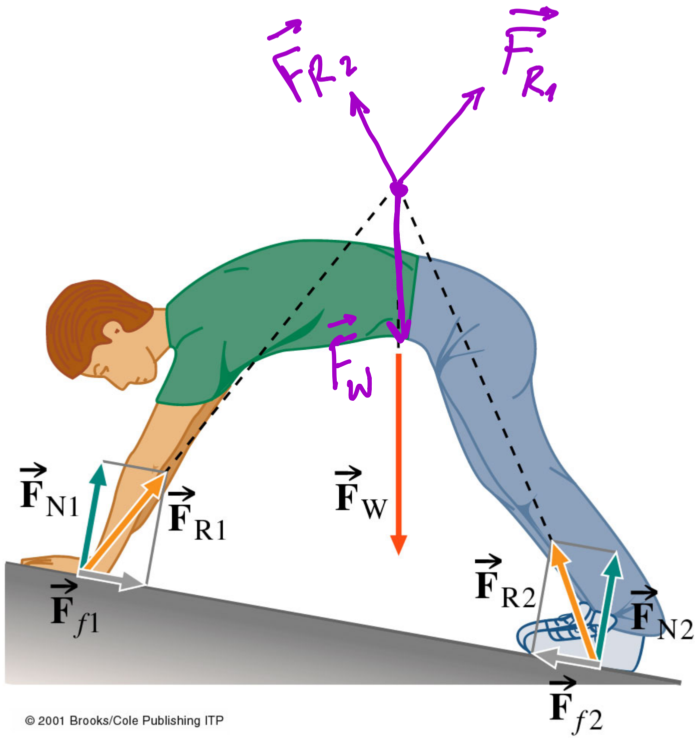


$F_S = \frac{1}{2} F_W$


100 N
100 N
100 N
100 N
400 N
400 N
F_W 400 N
(a)
(b)



FORCES CONCOURANTES

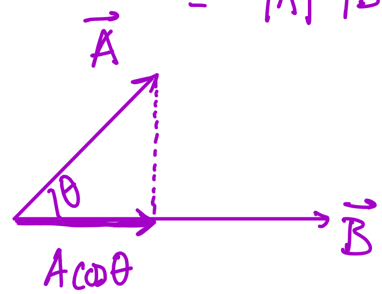


Produit scalaire

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y =$$
$$= |A| \cdot |B| \cdot \cos\theta$$



min $\theta = 90°$

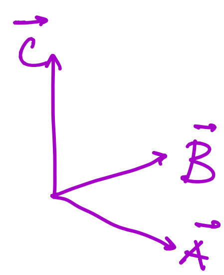
max $\theta = 0°$

Produit vectoriel

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} A_y B_z - A_z B_y \\ -(A_x B_z - A_z B_x) \\ A_x B_y - A_y B_x \end{pmatrix}$$

$$|\vec{C}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin\theta$$



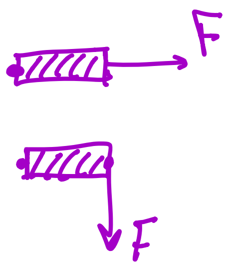
Notez: si $A_z = 0$ et $B_z = 0$, i.e. si vecteurs $\vec{A}$ et $\vec{B}$ se trouvent sur surface x-y, le vecteur $\vec{C}$ a une seule composante: z, puisque C_x et C_y resultent à 0 !

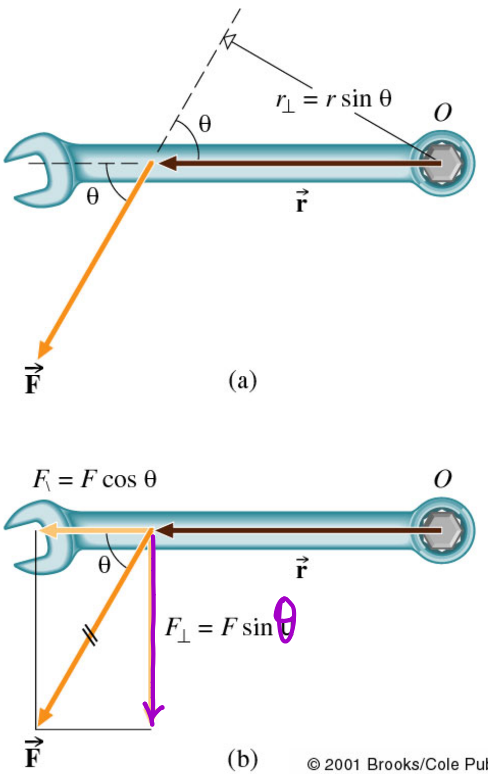
LE MOMENT DE FORCE

$$\tau_O = r_\perp \cdot F = r \cdot \sin\theta \cdot F =$$
$$= r \cdot F \cdot \sin\theta =$$
$$= r \cdot F_\perp$$

bras de levier

$$[\tau] = [F] \cdot [r] = N \cdot m$$



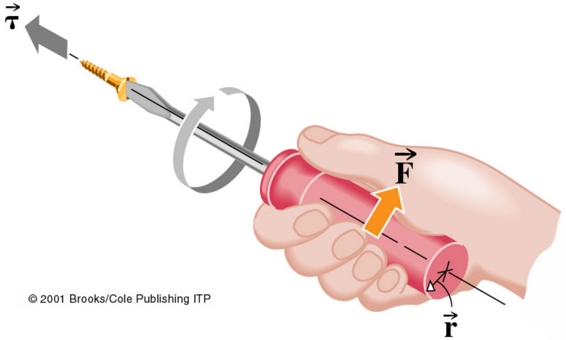


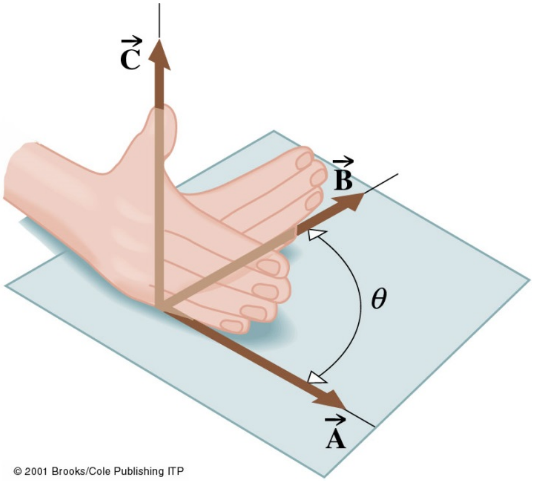
LE MOMENT DE FORCE

$\vec{\tau}$: vecteur !

$\tau_o = r \sin\theta F$

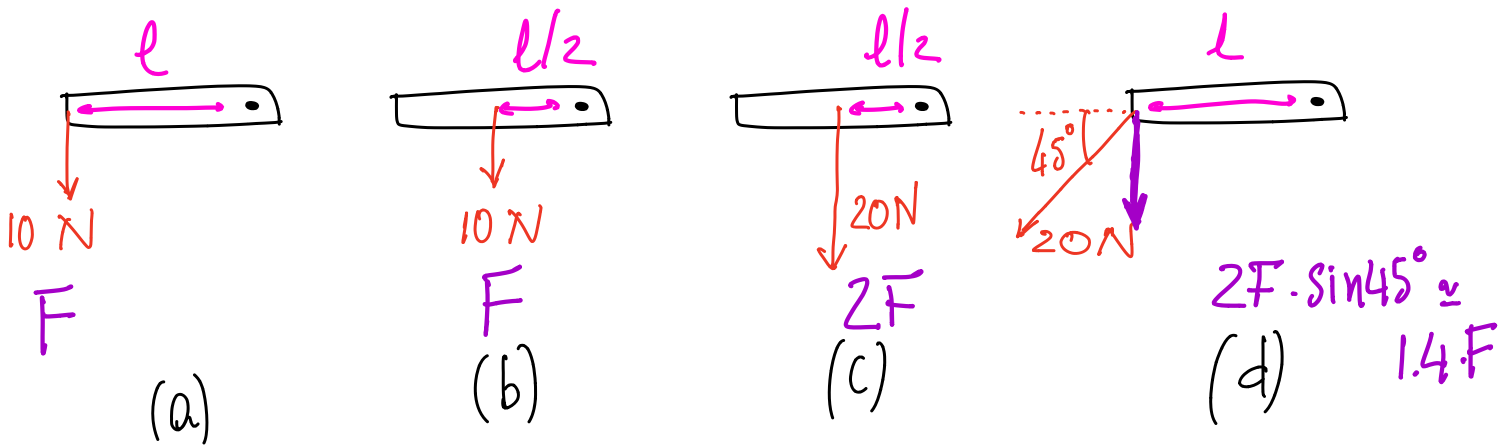
$\vec{\tau}_o = \vec{r} \times \vec{F}$





QUESTION

Dans quelle situation le moment de force est-il plus grand?

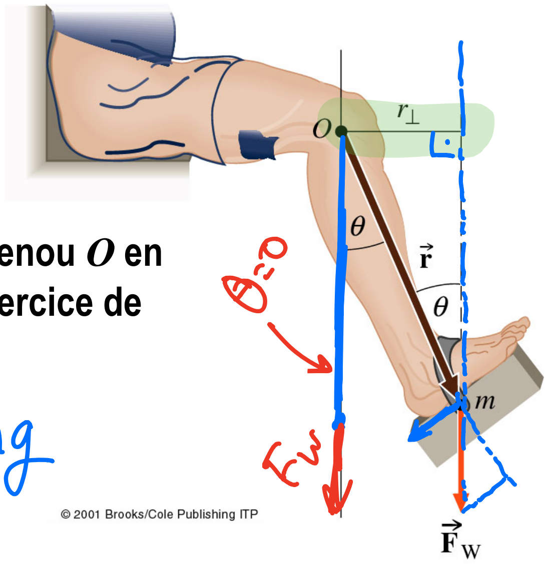


$$\tau = F \cdot \ell \qquad F \cdot \frac{\ell}{2} \qquad 2F \cdot \frac{\ell}{2} \qquad 1.4F \cdot \ell$$

$$F\ell \qquad F\ell/2 \qquad F\ell \qquad 1.4\,F\ell$$

EXEMPLE

QUESTION: Exprimez le moment des forces par rapport au genou *O* en fonction de θ, *m*, et la distance *r* du genou au talon dans l'exercice de musculation ci-contre. Négligez la masse de la jambe.



$$\tau_o = r_\perp \cdot F = r_\perp \cdot mg = r \sin\theta \, mg$$

τ_o min $\theta : 0$

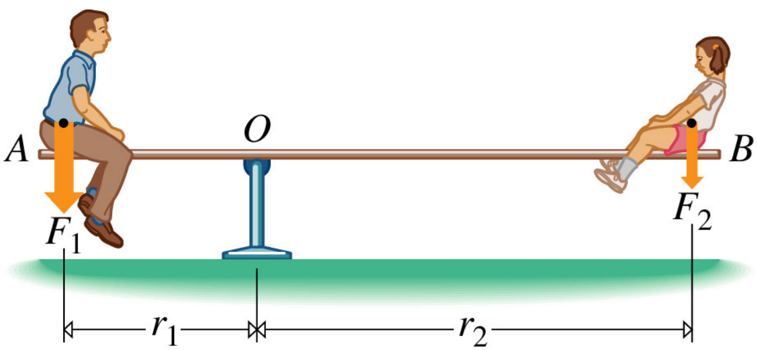
τ_o max $\theta = 90°$

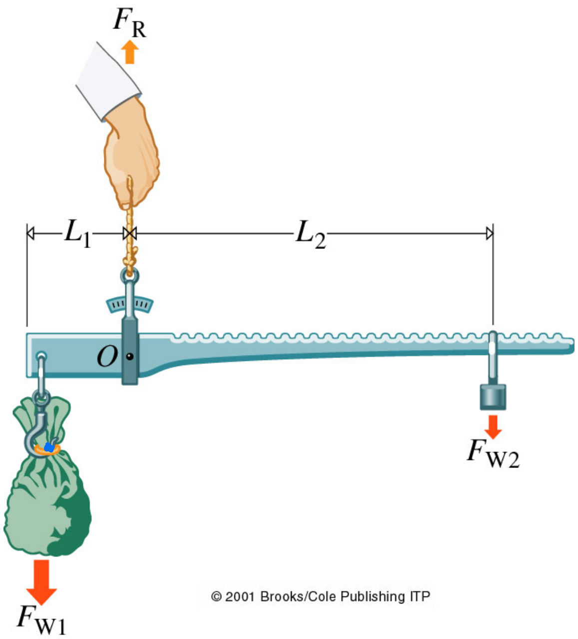
SECONDE CONDITION D'ÉQUILIBRE

$$\sum \vec{F} = 0 \Leftrightarrow \Delta \vec{v} = 0$$

$$\sum \vec{\tau} = 0 \Leftrightarrow \Delta \omega = 0$$

$$\sum \vec{\tau}_O = 0 \Rightarrow F_1 \cdot r_1 = F_2 \cdot r_2$$





QUESTION

Quelle est l'indication de la balance?

(a) 200 N

(b) 400 N

(c) 100 N

(d) 0 N

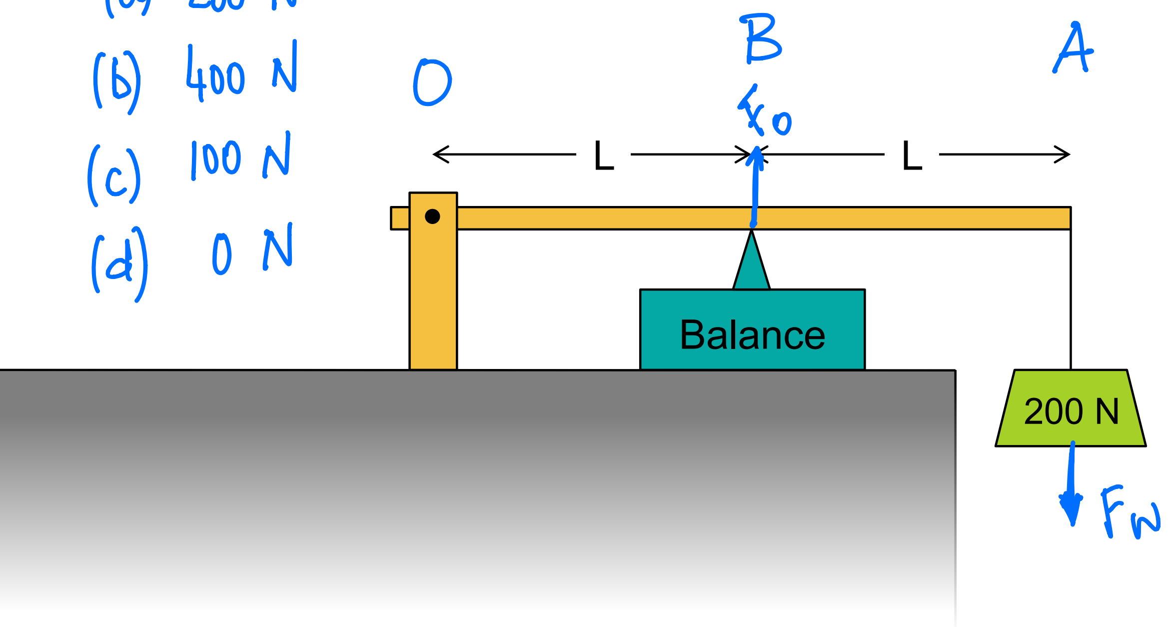
$$\sum \vec{\tau} = 0$$

$$-\vec{\tau}_{AO} = \vec{\tau}_{BO}$$

$$\tau_{AO} = \tau_{BO}$$

$$F_W \cdot 2\cancel{L} = F_o \cdot \cancel{L} \Rightarrow$$

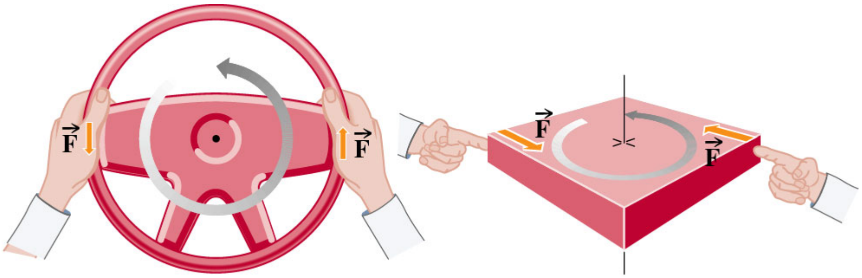
$$F_o = 2F_W$$



FORCES NON-CONCOURANTES

Un couple des forces

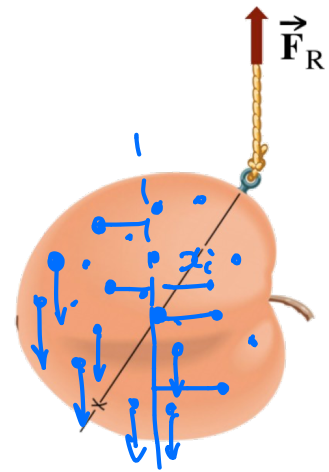
$\Sigma \vec{F} = 0$ <u>mais</u> $\Sigma \vec{\tau} \neq 0$



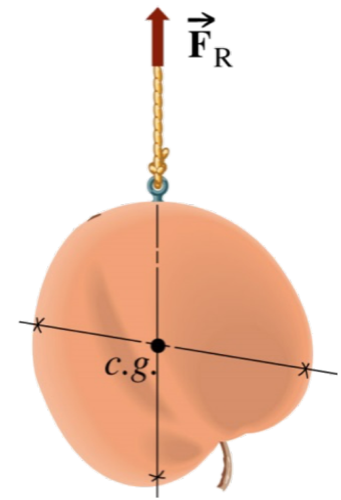
(a) (b)

CENTRE DE MASSE

$x_{c.g.}$: point qui concentre toute la masse d'un solide

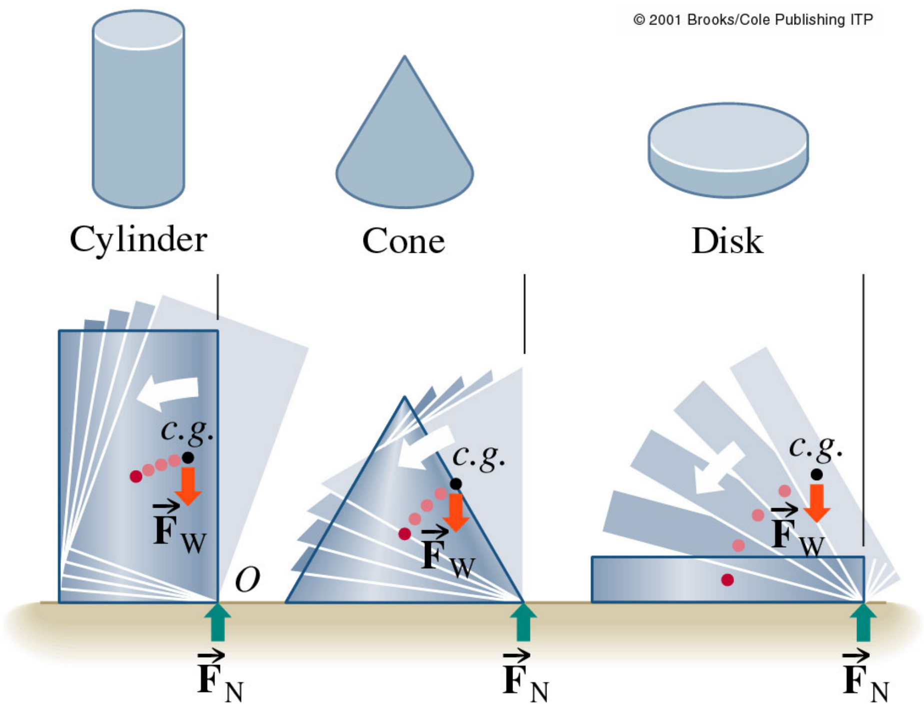


$$x_{cg} = \frac{\sum_{i=1}^{N} F_{W_i}\, x_i}{\sum_{i=1}^{N} F_{W_i}}$$



$$\sum F_{W_i} \cdot x_{cg} = \sum F_{W_i}\, x_i$$

STABILITÉ ET ÉQUILIBRE



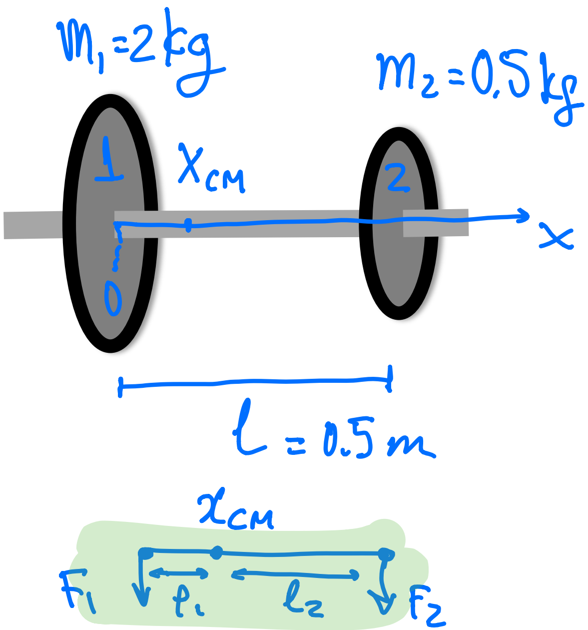
LE CENTRE DE MASSE

Un haltère se compose d'un disque de 500 gr d'une coté et d'un disque de 2 kg de l'autre coté. On considère la barre qui les connecte sans masse et le longueur 50 cm. Calculer le centre de masse.

$$x_{CM} = \frac{\sum_i F_i x_i}{\sum F_i} \qquad F_i = m_i g$$

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 \cdot x_1 + m_2 x_2}{m_1 + m_2}$$

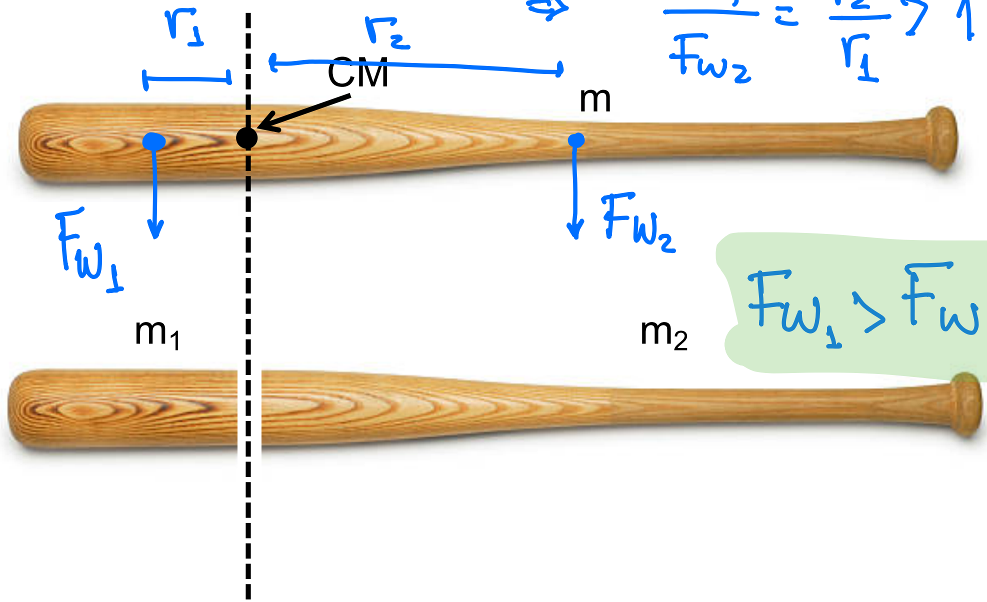
$$= \frac{2 \cdot 0 + 0.5 \cdot 0.5}{2 + 0.5}\, m = 0.1\, m$$



LE CENTRE DE MASSE - QUESTION

$\sum \vec{\tau} = 0 \Rightarrow F_{W_1} \cdot r_1 = F_{W_2} \cdot r_2$

$\Rightarrow \frac{F_{W_1}}{F_{W_2}} = \frac{r_2}{r_1} > 1 \Rightarrow$



$F_{W_1} > F_{W_2}$

(a) $m_1 > m_2$

(b) $m_1 = m_2$

(c) $m_1 < m_2$