DYNAMIQUE DE ROTATION

PGC-04

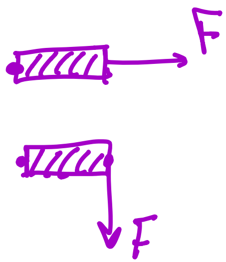
LE MOMENT DE FORCE

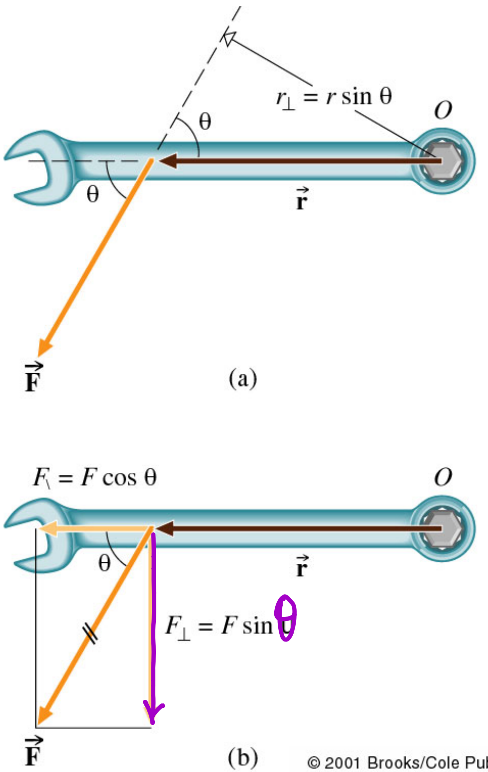
$$\vec{\tau}_o = \vec{r} \times \vec{F}$$

$$\tau_o = r_\perp \cdot F = r \cdot \sin\theta \cdot F = r \cdot F \cdot \sin\theta = r \cdot F_\perp$$

bras de levier

$$[\tau] = [F] \cdot [r] = N.m$$





DYNAMIQUE DE ROTATION - INTRO

$a \Leftrightarrow F$ m : inertie

$F = ma$

$a_{ang} \Leftrightarrow \tau$ I : inertie

$\tau = I \, a_{ang}$ moment d'inertie

MOMENT D'INERTIE

$$\left.\begin{array}{l}\tau_o = r\cdot F\\ F = m_o\cdot a_t\end{array}\right\} \Rightarrow \left.\begin{array}{l}\tau_o = r\, m_o\, a_t\\ a_t = r\cdot a_{ang}\end{array}\right\} \Rightarrow \tau_o = \boxed{m_o\cdot r^2}\cdot a_{ang}$$

$$\tau_o = I_o\cdot a_{ang}$$

$$\Updownarrow$$

$$(F = m\, a)$$

$$\tau_o = I_o\cdot a_{ang}$$

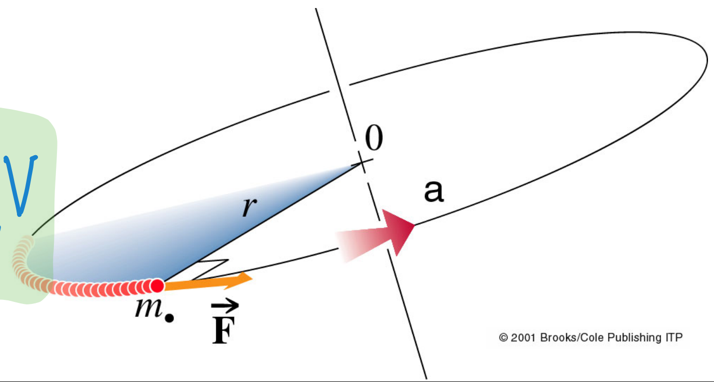
$$\tau_o^{TOT} = \sum \tau_o = \boxed{\sum_i (m_{oi}\, r_i^2)}\, a_{ang}$$

$$I_o = \sum_i (m_{oi}\, r_i^2)$$

$$\underline{(dm)}$$

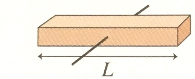
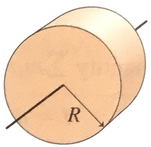
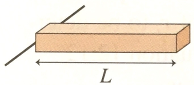
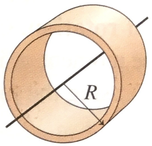
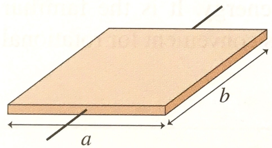
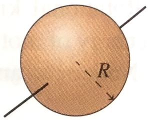
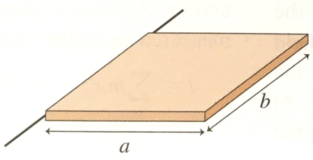
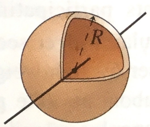
$$=$$

$$\left.\begin{array}{l}I_o = \int r^2\, dm\\ dm = \rho\, dV\end{array}\right\} \Rightarrow \boxed{I_o = \int r^2 \rho\, dV}$$



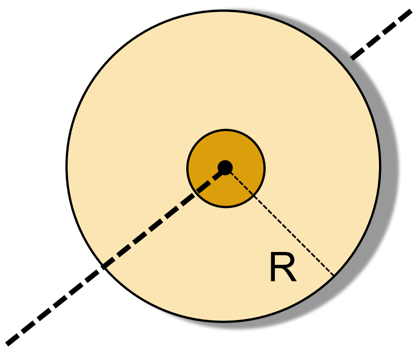
MOMENT D'INERTIE DES CORPS SIMPLES

TABLE 12.2 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

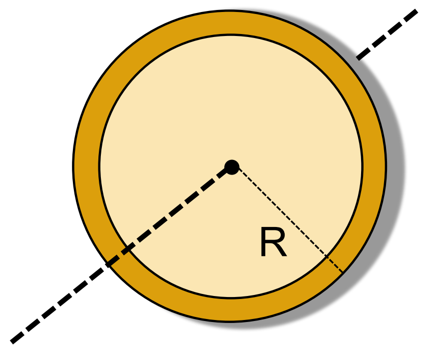
MOMENT D'INERTIE

Masse M



Objet 1

Masse M



Objet 2

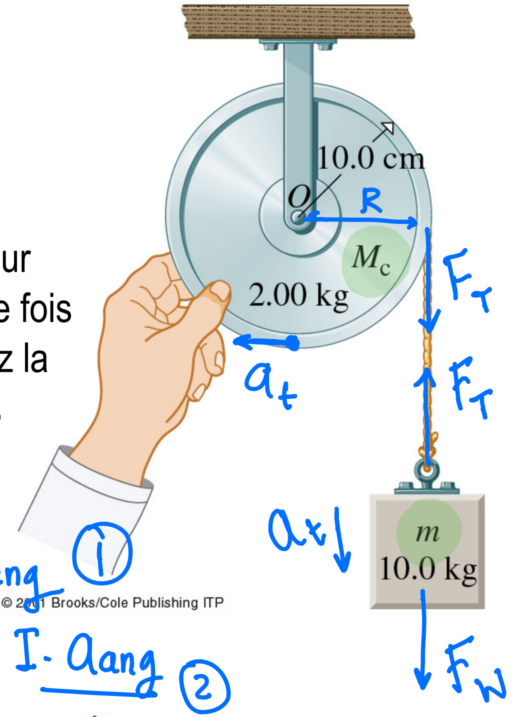
(a) $I_1 > I_2$

(b) $I_1 = I_2$

(c) $I_1 < I_2$

EXEMPLE

Une masse $m = 10.0$ kg est suspendue à une corde enroulée autour d'un cylindre de rayon $R = 10.0$ cm et de masse $M_c = 2.00$ kg. Une fois lâché, le cylindre est libre de tourner autour de son axe. Déterminez la tension de la corde, et les accélérations du cylindre et de la masse.



$F_T = ?$ $\quad a_t = ?$

$\sum F = m a_t = m R a_{ang} \Rightarrow F_W - F_T = m R a_{ang}$ ①

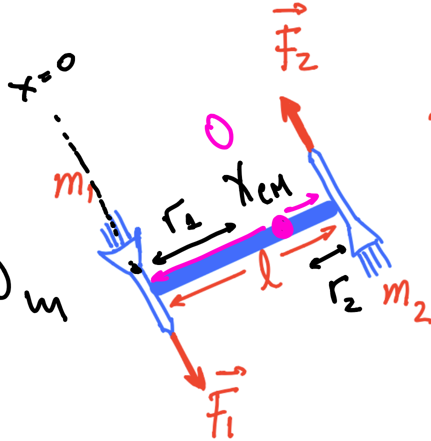
$\sum \tau_o = I \cdot a_{ang} \Rightarrow R \cdot F_T = I \cdot a_{ang} \Rightarrow F_T = \frac{I \cdot a_{ang}}{R}$ ②

① ② $\Rightarrow$ $mg - \frac{I \cdot a_{ang}}{R} = m R a_{ang} \Rightarrow a_{ang} = \frac{mg}{mR + I/R}$

Pour cylindre $\quad I = \frac{1}{2} M_c R^2$

$a_{ang} = \frac{mg}{mR + \frac{1}{2} M_c R} = \dots = 89.2\ rad/s^2 \Rightarrow a_t = 8.92\ m/s^2 < g$!

EXEMPLE



$m_1 = 100{,}000$ kg

$m_2 = 200{,}000$ kg

$\ell = 90$ m

$F_1 = F_2 = 50{,}000$ N

ω at $\Delta t = 30$ s?

$\Downarrow$

a?

$$x_{CM} = \frac{\cancel{m_1 x_1} + m_2 \cdot x_2}{m_1 + m_2} = \frac{m_2 \cdot \ell}{m_1 + m_2} = 60 \text{ m}$$

$x_1 = 0 \quad x_2 = \ell$

$$\Sigma\tau = a_{ang} \cdot I$$

$r_1 = 60$ m

$r_2 = 30$ m

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = 540.000.000 \text{ kg/m}^2$$

$$\tau = F_1 r_1 + F_2 r_2 = \cdots = 4500000 \text{ Nm}$$

$$\tau = a_{ang} \cdot I \Rightarrow a_{ang} = \frac{\tau}{I} = 0.00833 \text{ rad/s}^2$$

$$\omega = a_{ang} \cdot \Delta t \Rightarrow \omega = 0.25 \text{ rad/s}$$

EXEMPLE

$$\tau = I \cdot a_{ang}$$

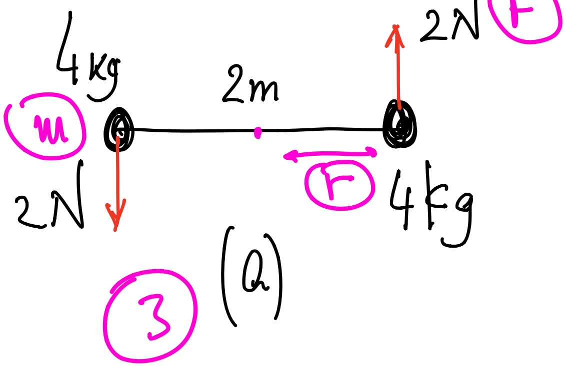
$$a_{ang} = \frac{\tau}{I}$$

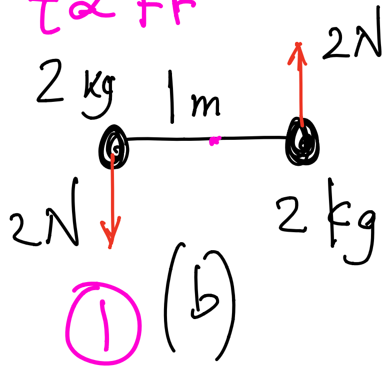
$$I = 2mr^2 \Rightarrow I \propto mr^2$$

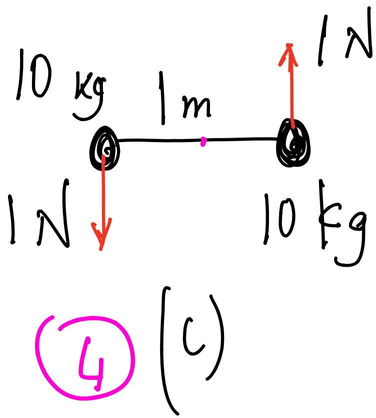
$$\tau \propto Fr$$

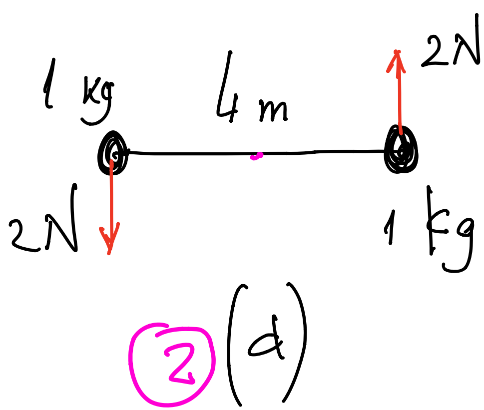
Où a_{ang} max ?

$$a_{ang} = \frac{\tau}{I} \propto \frac{F}{mr}$$









MOMENT CINÉTIQUE

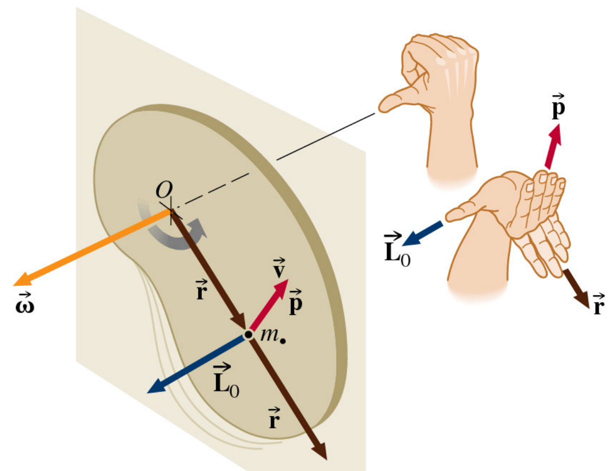
Equiv. Rotat $\vec{F} \Rightarrow \vec{\tau}$

— // — $\vec{p} = m\vec{v} \Rightarrow \vec{L}$ moment cinetique

$$\left.\begin{aligned} \vec{\tau} &= \vec{r} \times \vec{F} \\ \vec{L} &= \vec{r} \times \vec{p} \\ \vec{p} &= m\vec{v} \end{aligned}\right\} \Rightarrow \vec{L} = m\,\vec{r} \times \vec{v}$$

$$\vec{L} = I \cdot \vec{\omega}$$

$$(\vec{p} = m \cdot \vec{v})$$



$$\left(\vec{F} = \frac{\Delta \vec{p}}{\Delta t}\right)$$

CONSERVATION DU MOMENT CINÉTIQUE

$$\tau = I \cdot a_{ang} = I \cdot \frac{\Delta \omega}{\Delta t}$$

$$L = I\omega \Rightarrow \Delta L = I \Delta \omega$$

$$\Rightarrow \tau = \frac{\Delta L}{\Delta t}$$

et pour $\Delta t \to 0$: $\tau = \frac{dL}{dt}$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \left(\text{eq. } \vec{F} = \frac{d\vec{p}}{dt}\right)$$

Si $\vec{\tau} = 0 \Leftrightarrow \vec{L}$: conservé !

(si $\vec{F} = 0 \Leftrightarrow \vec{p}$ ou $\vec{v}$ est conservé)

RECAP

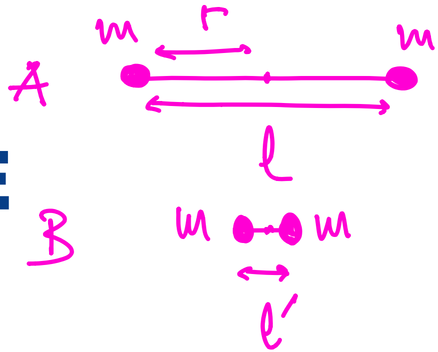
Symétrie entre les lois de translation et celles de la rotation

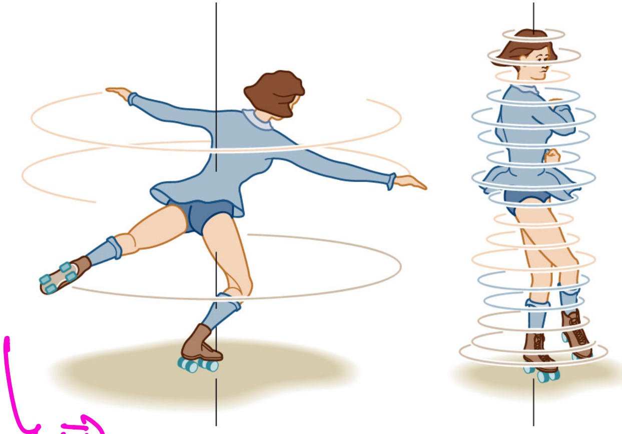
$$\vec{F} = m\vec{a} \qquad \vec{\tau} = I\alpha_{ang}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I\vec{\omega}$$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \qquad \vec{\tau} = \frac{\Delta\vec{L}}{\Delta t}$$

EXEMPLE





(a) (b)

A: $L_A = I_A \omega_A = 2mr^2\omega_A$

B: $L_B = I_B \cdot \omega_B = 2mr'^2\omega_B$

$\Rightarrow$

$\left(\text{conser } L\right)$ $L_A = L_B \Rightarrow r^2\omega_A = r'^2\omega_B \Rightarrow$

$$\Rightarrow \frac{\omega_A}{\omega_B} = \frac{r'^2}{r^2}$$

The most non-intuitive subject of 8.01

(perhaps of all physics)