LA GRAVITÉ SELON NEWTON

PGC-06

UN PEU D'HISTOIRE

Video: L'histoire de l'astronomie

~ 400 — Grèce antique

BC

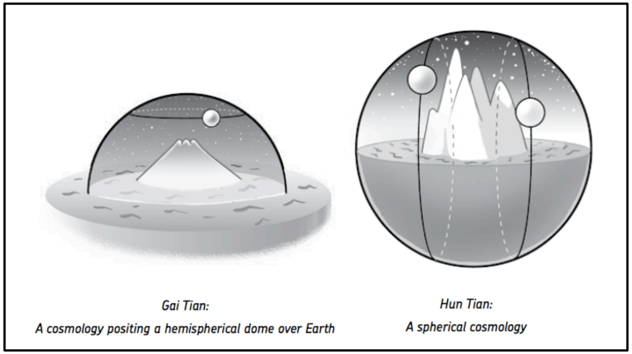
AC

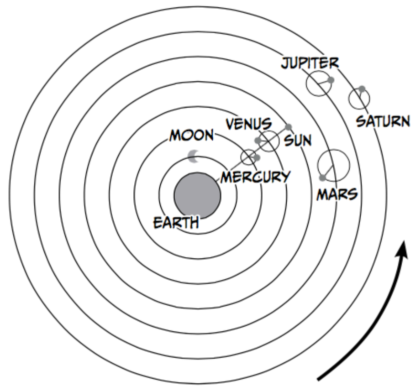
~150 — Ptolémée

1543 — Copernic
'De Revolutionibus'

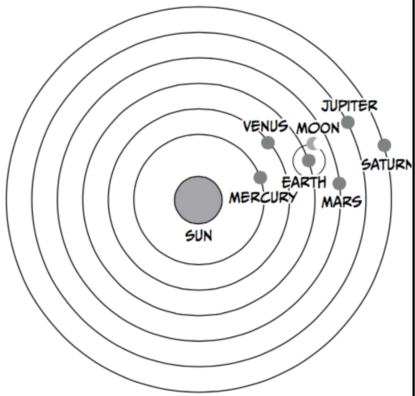
Kepler
1609 — 'Astronomia nova'
1619 — 'Harmonices Mundi'

Galilée
1609 — Téléscope
1632 — 'Dialogo sopra i due massimi sistemi del mondo'





Modèle géocentique

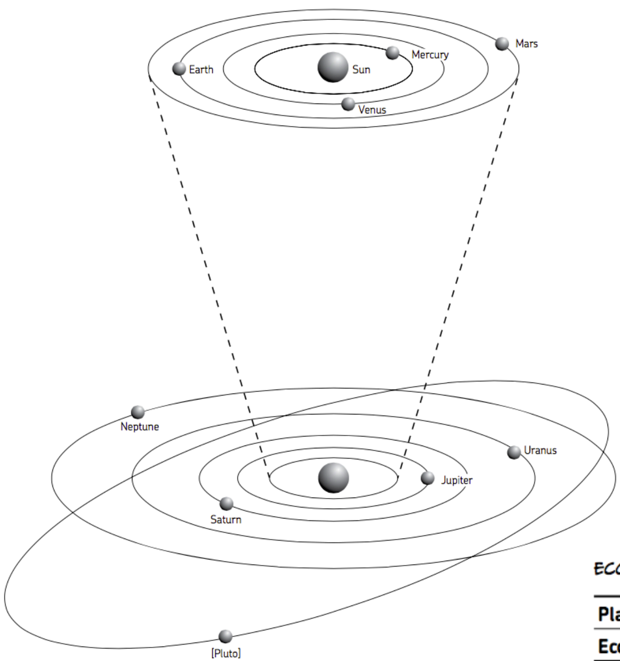


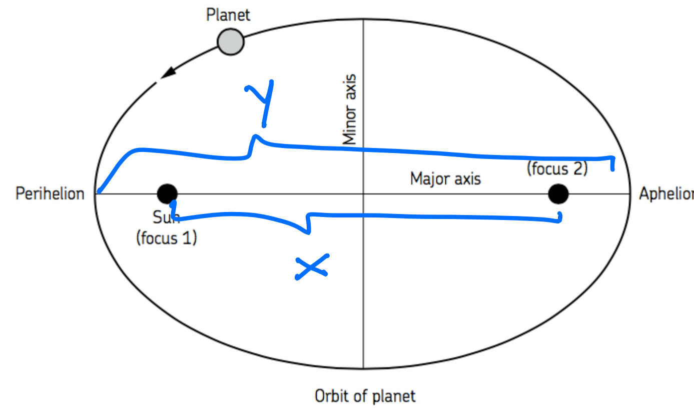
Modèle heliocentrique

Dessins: The Manga Guide To Astronomy

LES TROIS LOIS DE KEPLER

1.





Orbit of a planet according to Kepler's First Law

$\frac{x}{y}$

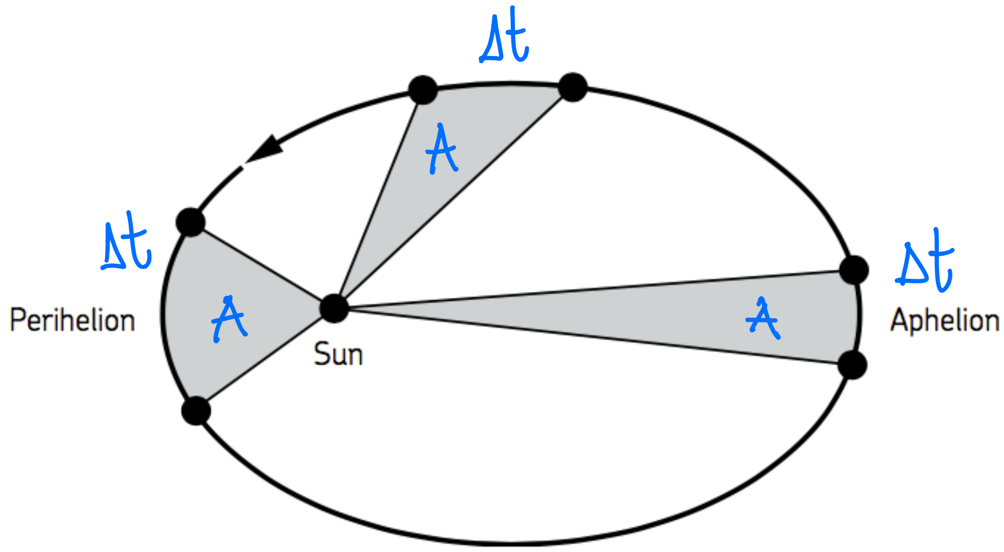
0 : pour un circle

ECCENTRICITY OF EACH PLANET IN THE SOLAR SYSTEM

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Eccentricity	0.2056	0.0068	0.0167	0.0934	0.0485	0.0555	0.0463	0.0090

LES TROIS LOIS DE KEPLER

2.



Orbit of a planet according to Kepler's Second Law

LES TROIS LOIS DE KEPLER

3.

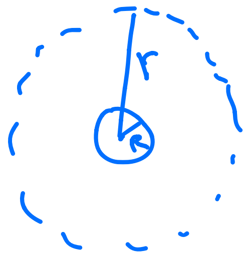
$T^2 \propto r^3$

SEMIMAJOR AXIS OF A PLANET'S ORBIT AND ORBITAL PERIOD

Planet	Semimajor axis of orbit a (AUs)	a^3	Orbital period relative to the fixed star's P (solar years)	P^2	a^3/P^2
Mercury	0.3871	0.05800555	0.2409	0.05803281	0.9995
Venus	0.7233	0.37840372	0.6152	0.37847104	0.9998
Earth	1.0000	1	1.0000	1	1.0000
Mars	1.5237	3.53751592	1.8809	3.53778481	0.9999
Jupiter	5.2026	140.819017	11.8620	150.707044	1.0008
Saturn	9.5549	872.32524	29.4580	867.773764	1.0052
Uranus	19.2184	7098.25644	84.0220	7049.69648	1.0055
Neptune	30.1104	27299.1783	164.7740	27150.4711	1.0055

NEWTON 1642-1727

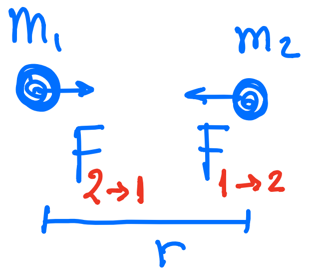
$$\frac{g_T}{g_{T_L}} = \left(\frac{(1/R)}{(1/r)} \right)^2$$



$$F \propto \frac{1}{r^2}$$

LOI DE GRAVITÉ DE NEWTON

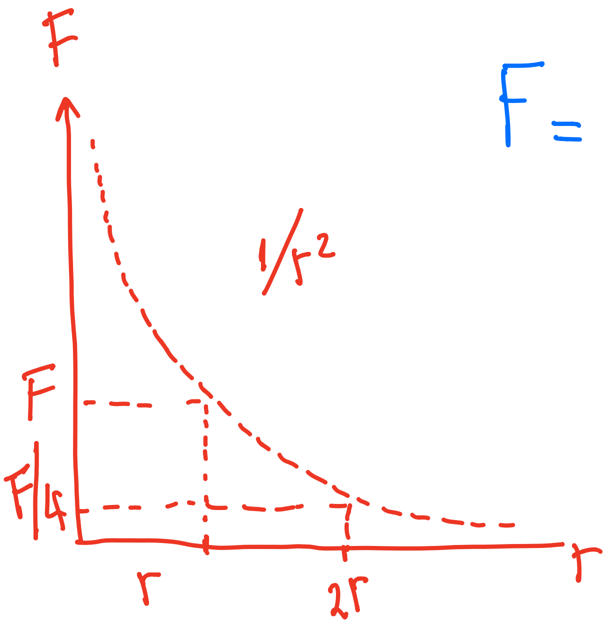
$$F \propto \frac{m_1 m_2}{r^2}$$



$$F = G\,\frac{m_1 m_2}{r^2}$$

$$G = 6.67\times10^{-11}\ Nm^2/kg^2$$

$$\vec{F}_{1\to2} = -\vec{F}_{2\to1}$$



GRANDEUR DE LA FORCE GRAVITATIONNELLE

$$\left.\begin{array}{l} m_1 = 1\,kg \\ m_2 = 1\,kg \\ r = 1\,m \end{array}\right\} \Rightarrow F = G\,\frac{m_1 m_2}{r^2} = 6.67\times10^{-11}\,\frac{N\cancel{m^2}}{\cancel{kg^2}}\cdot\frac{\cancel{kg^2}}{\cancel{m^2}} = 6.67\times10^{-11}\,N$$

$$\left.\begin{array}{l} m_1 = M_T = 6\times10^{24}\,kg \\ m_2 = 1\,kg \\ r = R_T = 6.4\times10^{6}\,m \end{array}\right\} \Rightarrow \underline{F = 9.8\,N} \rightarrow \text{le } \underline{\underline{\text{poids}}}\,!$$

PRINCIPE DE L'ÉQUIVALENCE

$m_{grav} = \frac{r^2 F}{G M_T}$ $\rightsquigarrow$ pas d'acceleration

$m_{inertie} = \frac{F}{a}$ $\rightsquigarrow$ pas de gravité

$m_{inertie} = m_{gravitationnelle}$.

THÉORIE DE GRAVITÉ DE NEWTON

Theorie de Gravité

1. $F = G\dfrac{m_1 m_2}{r^2}$

2. principe d'equivalence

3. 3 lois de Newton

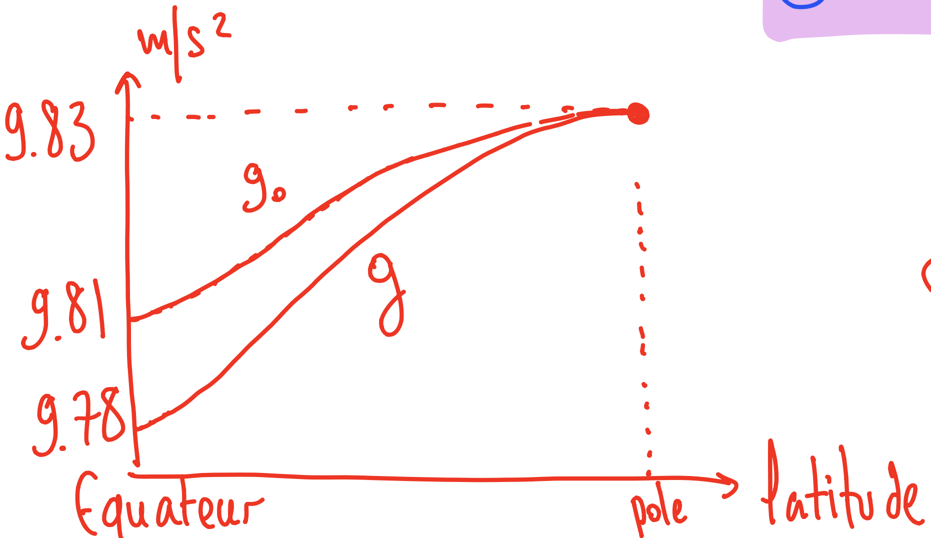
GRAND G ET PETIT g

$$\left.\begin{array}{l} F_G = \dfrac{GMm}{r^2} \\[2ex] F_D = mg \end{array}\right\} \Rightarrow mg = \frac{GMm}{r^2} \Rightarrow$$

$$\Rightarrow \boxed{g_0 = \frac{GM}{r^2}}$$

Mars:

$g_M = 3.8\ m/s^2$



$$g = \frac{GM}{r^2} - a_c$$

LA MASSE DE LA TERRE?

$$g = \frac{GM}{r^2} \Rightarrow M = \frac{g r^2}{G}$$

g : cinematique

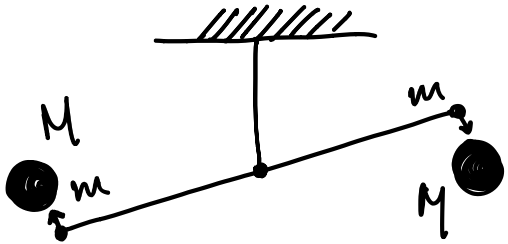
r : technique
surveillance

G : Exp. Cavendish

$$\Downarrow$$

$$M = \frac{g r^2}{G} \checkmark$$

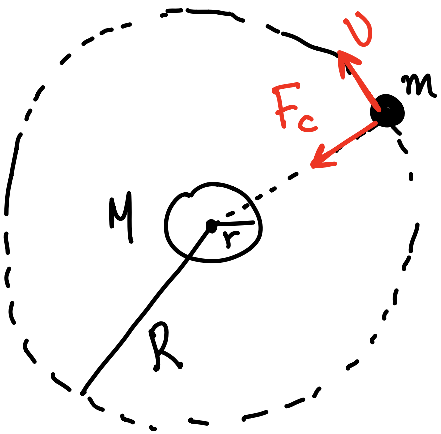
$$G = \frac{F_{mM} \cdot r^2}{Mm}$$



Henry Cavendish

$\underline{\underline{1\%}}$

ORBITE DE SATELLITE



$$F = \frac{GmM}{R^2} = m a_c = \frac{mv^2}{R}$$

$$\frac{G\cancel{m}M}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

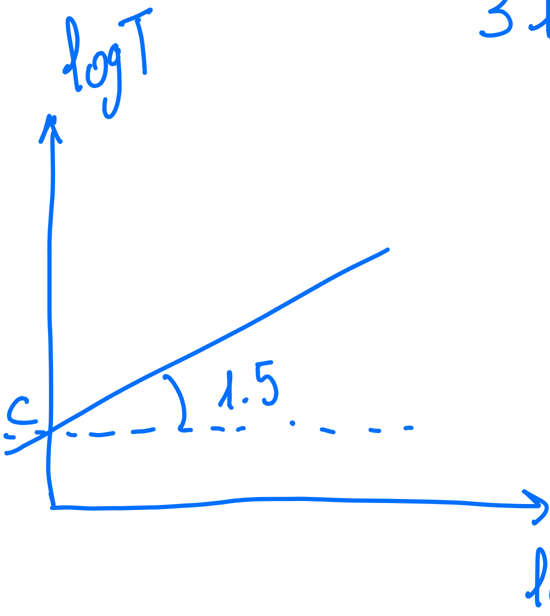
$$v = \frac{\text{circ}}{\text{per}} = \frac{2\pi R}{T}$$

$$\Rightarrow \frac{2\pi R}{T} = \sqrt{\frac{GM}{R}} \Rightarrow T^2 = \frac{4\pi^2}{GM}R^3 \Rightarrow \boxed{\frac{R^3}{T^2}} = \boxed{\frac{GM}{4\pi^2}}$$

ORBITES GEOSTATIONNAIRES

$$\frac{R^3}{T^2} = const \Rightarrow \log \frac{R^3}{T^2} = const \Rightarrow \log R^3 - \log T^2 = const \Rightarrow$$

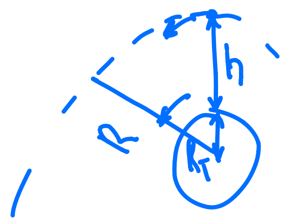
$$3 \log R - 2 \log T = const \Rightarrow \log T = 1.5 \log R + c$$



pour $T = 24h$

sat: periode terre !

$R = r_T + h$



$h \simeq 5.6\, r_T$

ÉNERGIE POTENTIELLE GRAVITATIONNELLE TERRESTRE

$$\Delta E_c = W = F \cdot h = mgh$$

$$\Delta E_m = 0 \Rightarrow -\Delta E_c = \Delta E_p$$

ÉNERGIE POTENTIELLE GRAVITATIONNELLE TERRESTRE

$$\Delta E_p = E_{pf} - E_{pi} = W_F = \int_{r_i}^{r_f} \vec{F}_G \, d\vec{r}$$

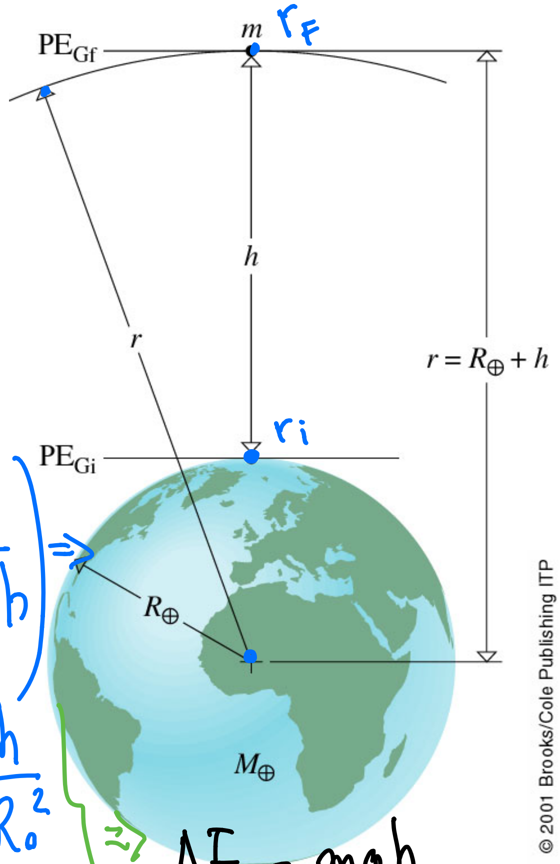
$$F_G = G\frac{Mm}{r^2}$$

$$\Delta E_p = \int_{r_i}^{r_f} \frac{GMm}{r^2}\,dr \qquad \left(\int \frac{1}{r^2}\,dr = -\frac{1}{r}\right)$$

$$\Delta E_p = G\frac{Mm}{r_i} - G\frac{Mm}{r_f} = GMm\left(\frac{1}{R_0} - \frac{1}{R_0+h}\right) \Rightarrow$$

$$\left.\begin{array}{l}\Delta E_p = GmM\cdot\dfrac{h}{R_0(R_0+h)} \\ h \ll R_0 \Rightarrow R_0 + h \simeq R_0\end{array}\right\} \Rightarrow \Delta E_p = GmM\frac{h}{R_0^2}$$

$$\left.\begin{array}{l}\Delta E_p = GmM\dfrac{h}{R_0^2} \\ g = GM/R_0^2\end{array}\right\} \Rightarrow \Delta E_p = mgh$$



ÉNERGIE POTENTIELLE GRAVITATIONNELLE TERRESTRE

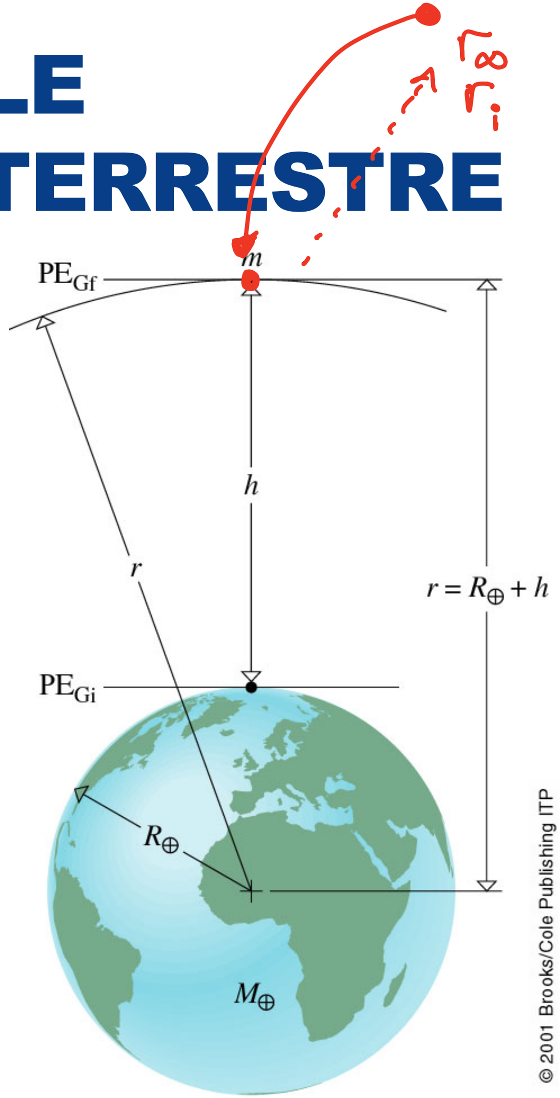
$$\Delta E_p = GmM\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$E_p = 0$ @ $r_\infty \gg R_o$ $\quad r_f = R_o + h$

$$\Delta E_p = GmM\left(\cancel{\frac{1}{r_\infty}}^{\,0} - \frac{1}{R_o + h}\right) \Rightarrow$$

$$\Delta E_p = -\frac{GmM}{R_o + h} = -\frac{GmM}{r}$$

$$\boxed{\Delta E_p = -\frac{GmM}{r}}$$

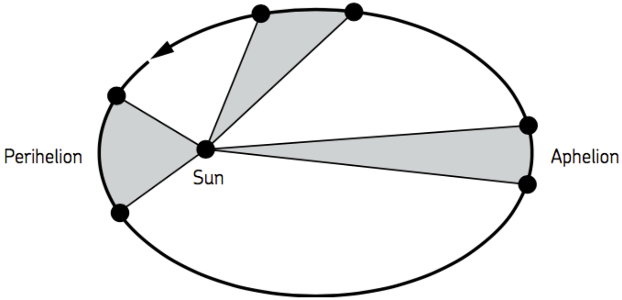


EXEMPLE – ÉNERGIE GRAVITATIONNELLE

Pourquoi une planète accélère-t-elle sur son orbite en s'approchant du Soleil ?

$E_p + E_{cin} =$ constante

Puisque $E_p \propto -\frac{1}{r}$



Orbit of a planet according to Kepler's Second Law

quand r devient plus petit, ie en s' approchant au soleil

$E_p \Downarrow$ donc $E_c \Uparrow$

VITESSE DE LIBERATION

Quelle est la vitesse suffisante pour partir de la terre sans y retourner? i.e. pour "vaincre" la gravité

E_c : suffisante pour vaincre gravité

$$E_i = E_{ci} + E_{pi} = \frac{1}{2} m v_{LIB}^2 + \left(-\frac{GmM}{R}\right)$$

$E_f = 0$ à une distance ∞

$$E_i = E_F \Rightarrow \frac{1}{2} m v_{LIB}^2 = \frac{GmM}{R} \Rightarrow v_{LIB} = \sqrt{\frac{2GM}{R}}$$

et puisque $g = \dfrac{GM}{R^2}$: $\boxed{v_{LIB} = \sqrt{2gR}}$