ÉLASTICITÉ ET OSCILLATIONS

PGC-08

L'ÉLASTICITÉ

L'élasticité étudie le comportement de matériaux et de structures sous contraintes. Un solide soumis à une force extérieure (contrainte) peut être comprimé, étiré ou cisaillé.

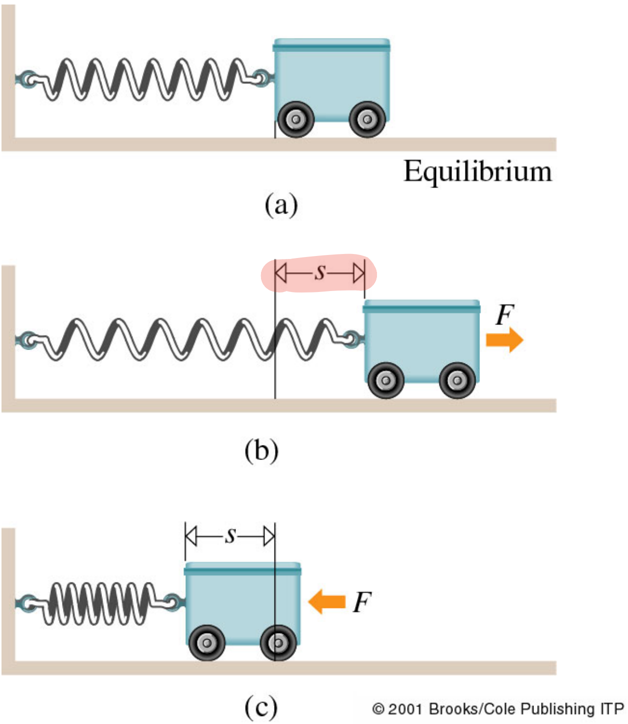
deformation.

$$\underbrace{F = k\,s}$$

k : const. d'elasticité

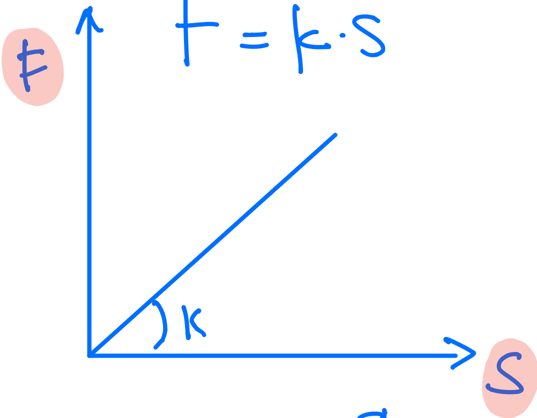
$$[k] = \frac{[F]}{[s]} = \frac{N}{m}$$

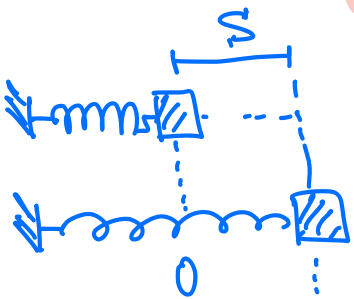
Loi de Hooke



ÉNERGIE POTENTIELLE ÉLASTIQUE

$F = k \cdot S$





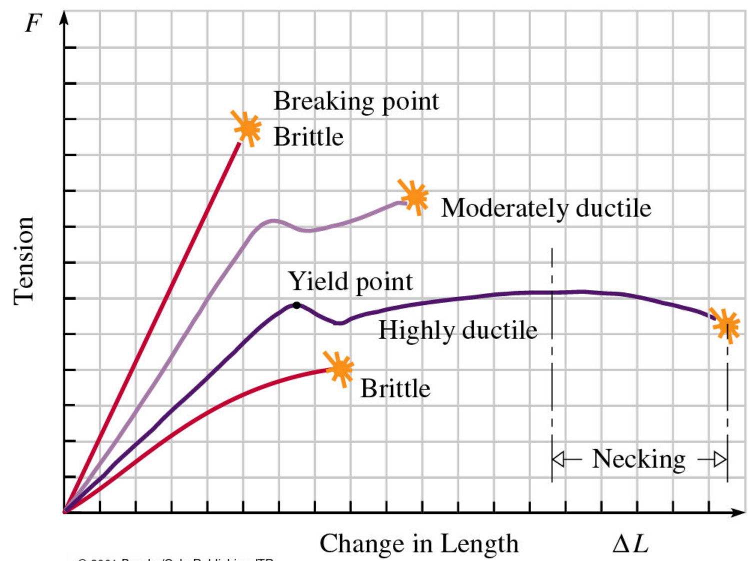
$$W = \int_0^s F\,ds = \int_0^s ks\,ds = \frac{1}{2}ks^2$$

$$= \Delta E_p$$

MATÉRIAUX ÉLASTIQUES

- zone d'élasticité $F = k s$
- limite d'élasticité
- zone plastique -- non réversible!
- limite de rupture

Courbes de traction

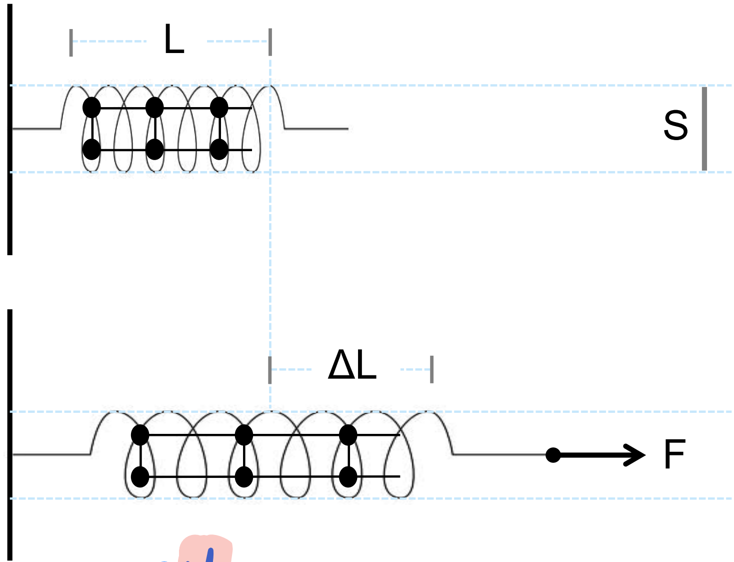


CONTRAINTE ET DÉFORMATION

$$\boxed{\sigma = \frac{F}{S}}$$

$$[\sigma] = \frac{N}{m^2}$$

"P"



$$\boxed{\varepsilon = \frac{\Delta L}{L}}$$

$$\boxed{Y} = \frac{\sigma}{\varepsilon} = \frac{F}{S} \cdot \frac{L}{\Delta L} = \frac{L}{S \cdot \Delta L} \cdot F \Rightarrow F = \frac{S\boxed{Y}}{L} \cdot \Delta L$$

$$\boxed{F = \boxed{k}\ \Delta L}$$

LES OSCILLATIONS

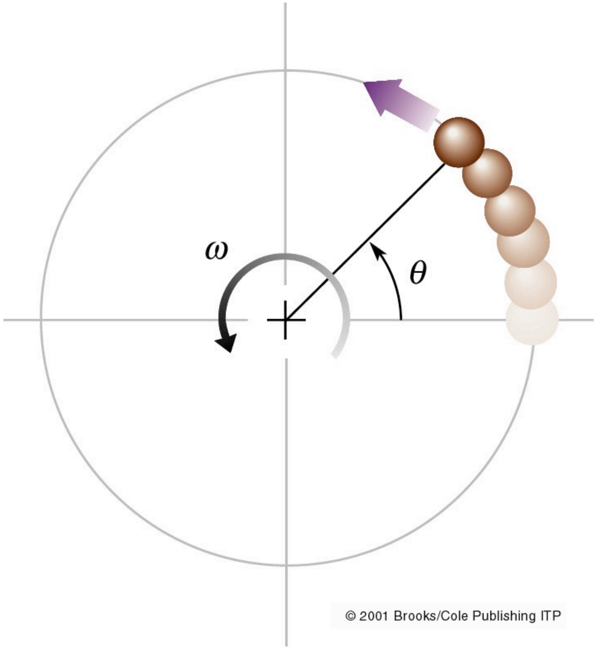
LE MOUVEMENT PÉRIODIQUE

- La periode T (s)
- La frequence f (Hz)

$$f = 1/T$$

- vitesse angulaire ω

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \left(\frac{rad}{s}\right)$$



LE MOUVEMENT SINUSOÏDAL

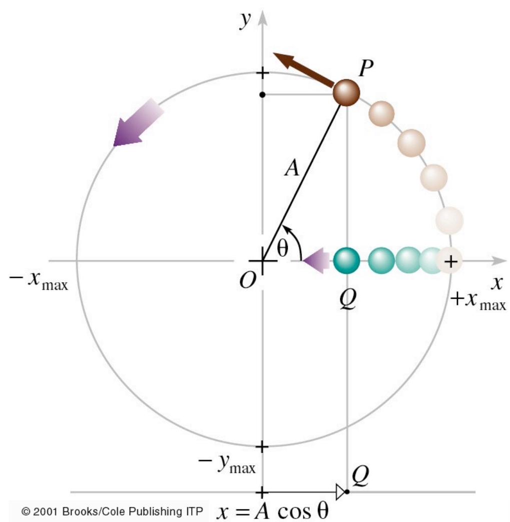
$$\left.\begin{array}{l} x = A \cdot \cos\theta \\ A = x_{max} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = x_{max} \cdot \cos\theta \\ \omega = \theta / t \Rightarrow \theta = \omega t \end{array}\right\} \Rightarrow x = x_{max} \cos \omega t$$

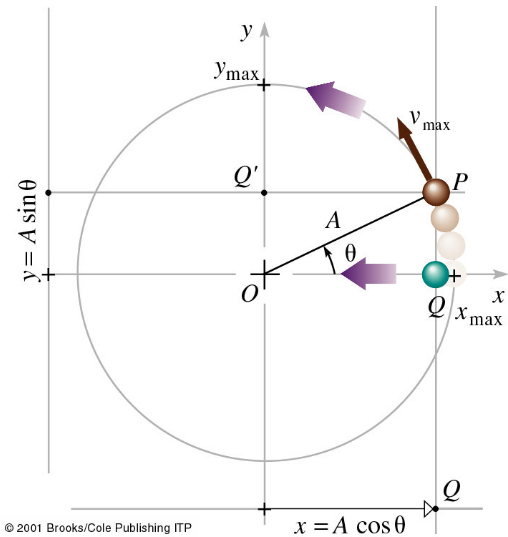
$$\downarrow f(t)$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$x = x_{max} \cos \omega t = x_{max} \cos \frac{2\pi}{T} t = x_{max} \cos 2\pi f t$$

$$y = y_{max} \sin \omega t$$





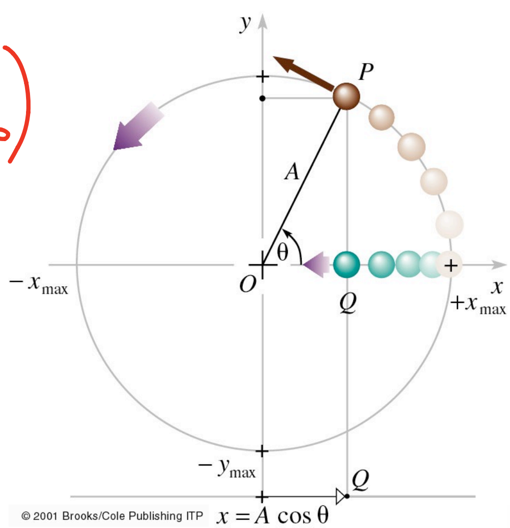
LE MOUVEMENT SINUSOÏDAL

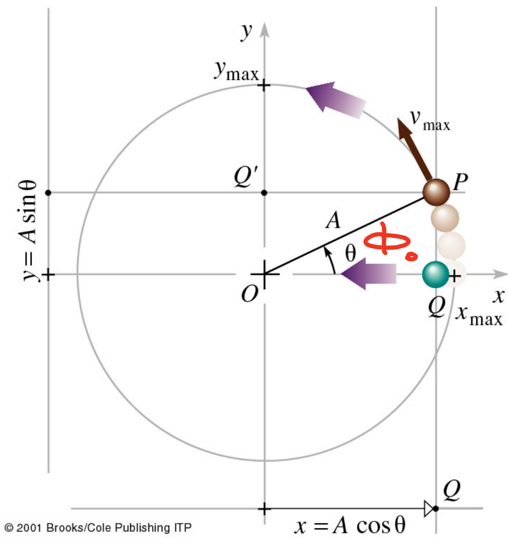
Et si t=0 quand θ > 0?

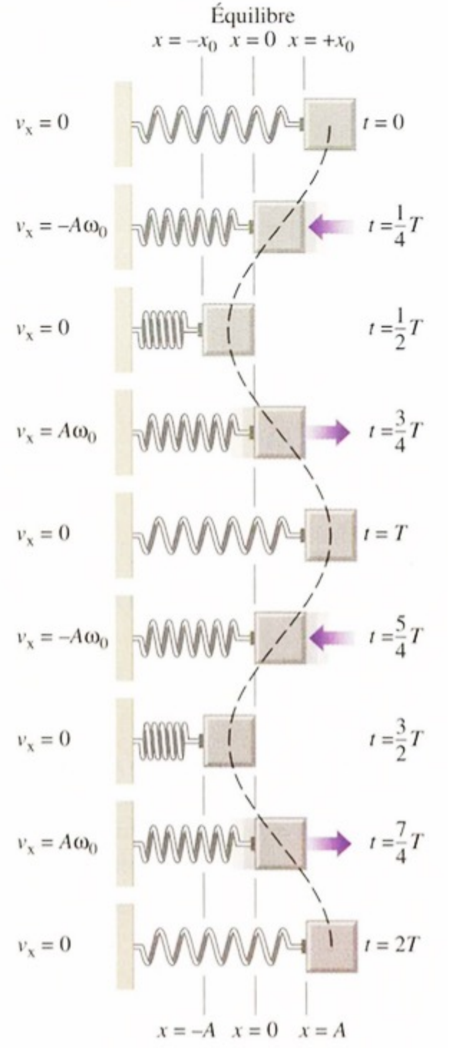
$$\theta = \phi_0$$

$$x(t=0) = x_{max} \cos \phi_0$$

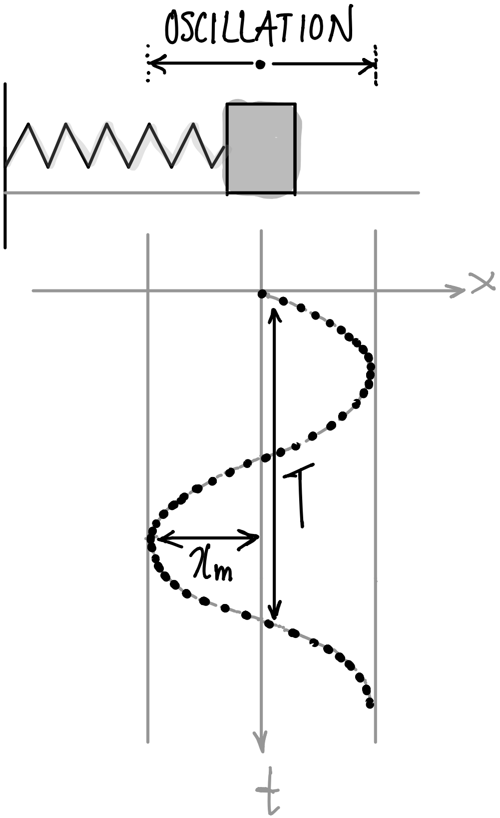
$$x(t) = x_{max} \cos(\omega t + \phi_0)$$






Équilibre
$x = -x_0$ $x = 0$ $x = +x_0$
$v_x = 0$
$t = 0$
$v_x = -A\omega_0$
$t = \frac{1}{4}T$
$v_x = 0$
$t = \frac{1}{2}T$
$v_x = A\omega_0$
$t = \frac{3}{4}T$
$v_x = 0$
$t = T$
$v_x = -A\omega_0$
$t = \frac{5}{4}T$
$v_x = 0$
$t = \frac{3}{2}T$
$v_x = A\omega_0$
$t = \frac{7}{4}T$
$v_x = 0$
$t = 2T$
$x = -A$ $x = 0$ $x = A$

LE MOUVEMENT HARMONIQUE SIMPLE



$$x(t) = x_m \cos(\omega t + \phi)$$

x_m : amplitude

$\omega t + \phi$: phase du mouvement

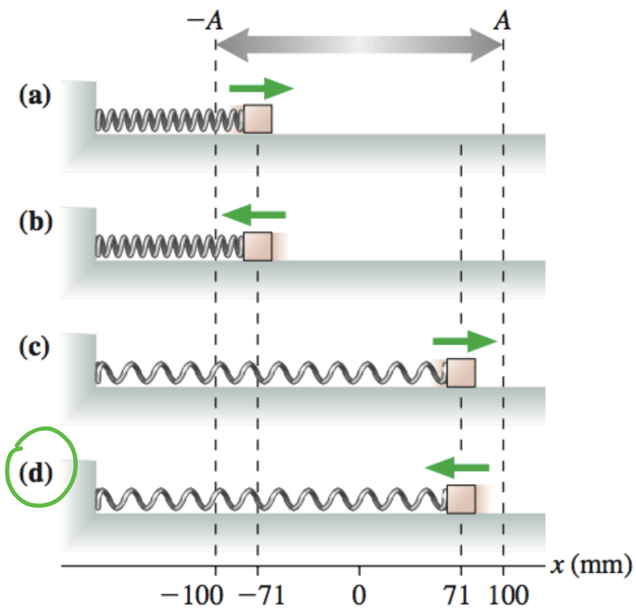
ϕ : phase initiale

ω : frequence angulaire

$$\omega = \frac{2\pi}{T} = 2\pi f \quad (rad/s)$$

QUESTION

La figure montre quatre oscillations à t = 0 s. La quelle a une phase initiale de π/4 rad?



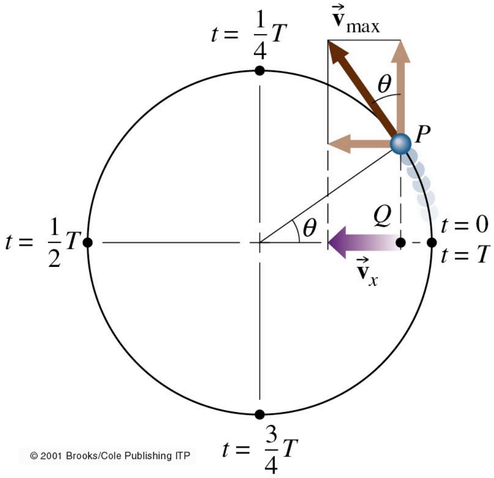
MHS – LA VITESSE

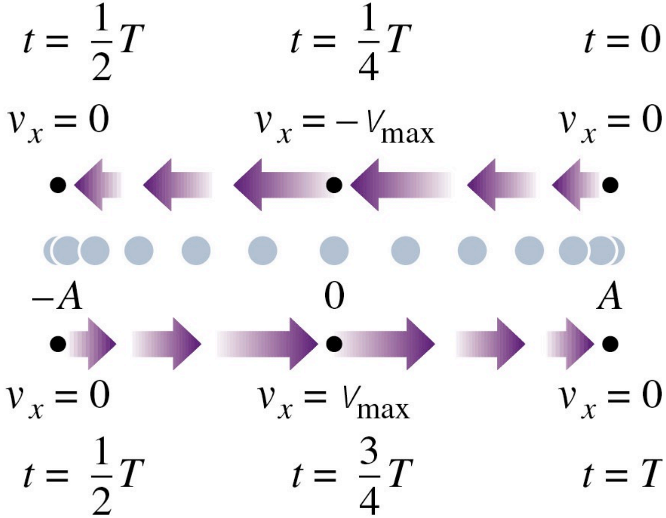
$$v = \frac{dx}{dt} = \frac{d}{dt}\left(X_{max}\cos(\omega t + \phi)\right) = -X_{max}\,\omega \sin(\omega t + \phi)$$

= En avance de T/4

$$v = -X_{max}\,\omega \sin(\omega t + \phi)$$

= $v = 0$ pour X_{max} et max pour $x = 0$

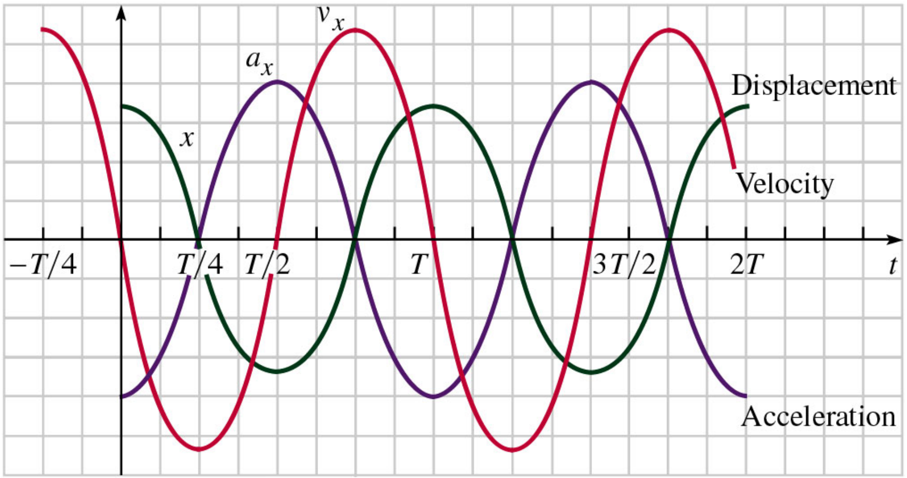




MHS – L'ACCÉLÉRATION

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(-X_{max}\,\omega \sin(\omega t+\phi)\right) = -\underbrace{X_{max}\,\omega^2 \cos(\omega t+\phi)}_{x}$$

$$\Rightarrow \underline{\underline{a = -\omega^2 x}}$$



MHS – FORCE ASSOCIÉE

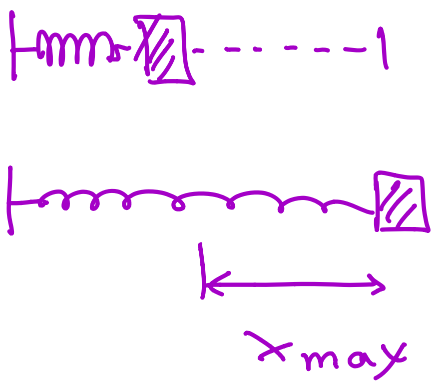
Une particule de masse *m* soumise à une force de rappel proportionnelle à son déplacement suit un mouvement harmonique simple.

$$\left.\begin{aligned} F &= ma \\ a &= -\omega^2 x \end{aligned}\right\} \Rightarrow F = -m\omega^2 x$$

$$F = -Kx$$

"Force de rappel"

MHS EXEMPLE – LE RESSORT



$$F = -kx$$
$$F = ma$$
$$\Rightarrow ma = -kx \Rightarrow$$
$$m\frac{d^2x}{dt^2} = -kx \Rightarrow$$

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x} \;*$$

$$\Rightarrow x = A\cos(\omega t + \phi) \quad \text{avec} \quad \omega = \sqrt{\frac{k}{m}}$$

Pour $t = 0$

$$\left.\begin{array}{l} x(t=0) = x_{max} = A\cos\phi \\ v(t=0) = 0 = -A\omega\sin\phi \Rightarrow \phi = 0 \end{array}\right\} A = x_{max}$$