Simulation and image reconstruction tools for AX-PET

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on behalf on the AX-PET collaboration

Monte Carlo in PET imaging

- AX-PET is a novel concept: high energy and spatial resolution, 3D interaction position recovery, high sensitivity.

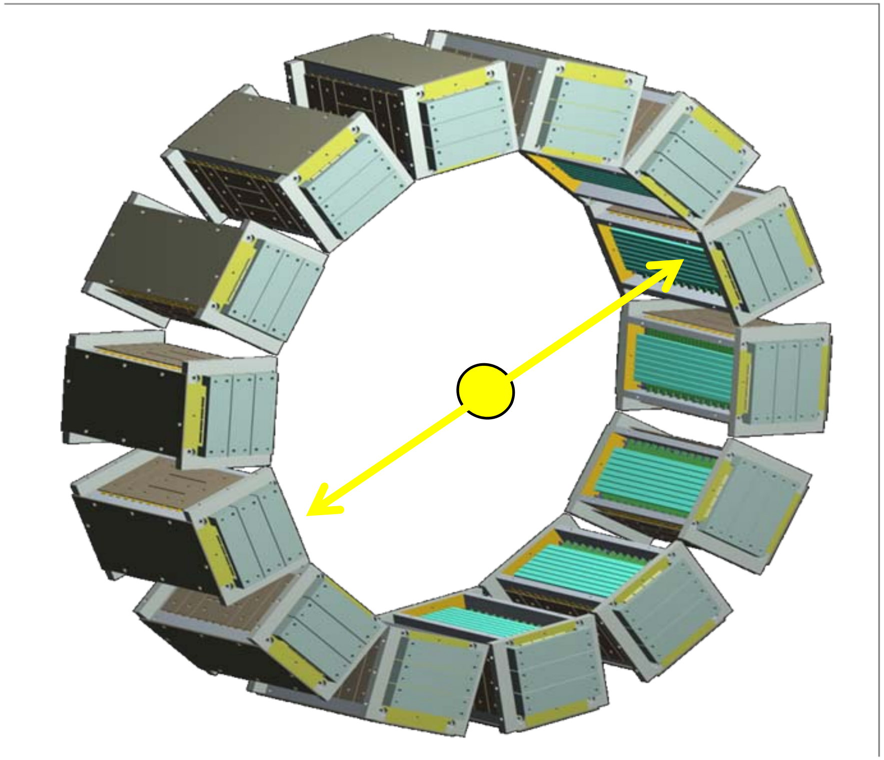
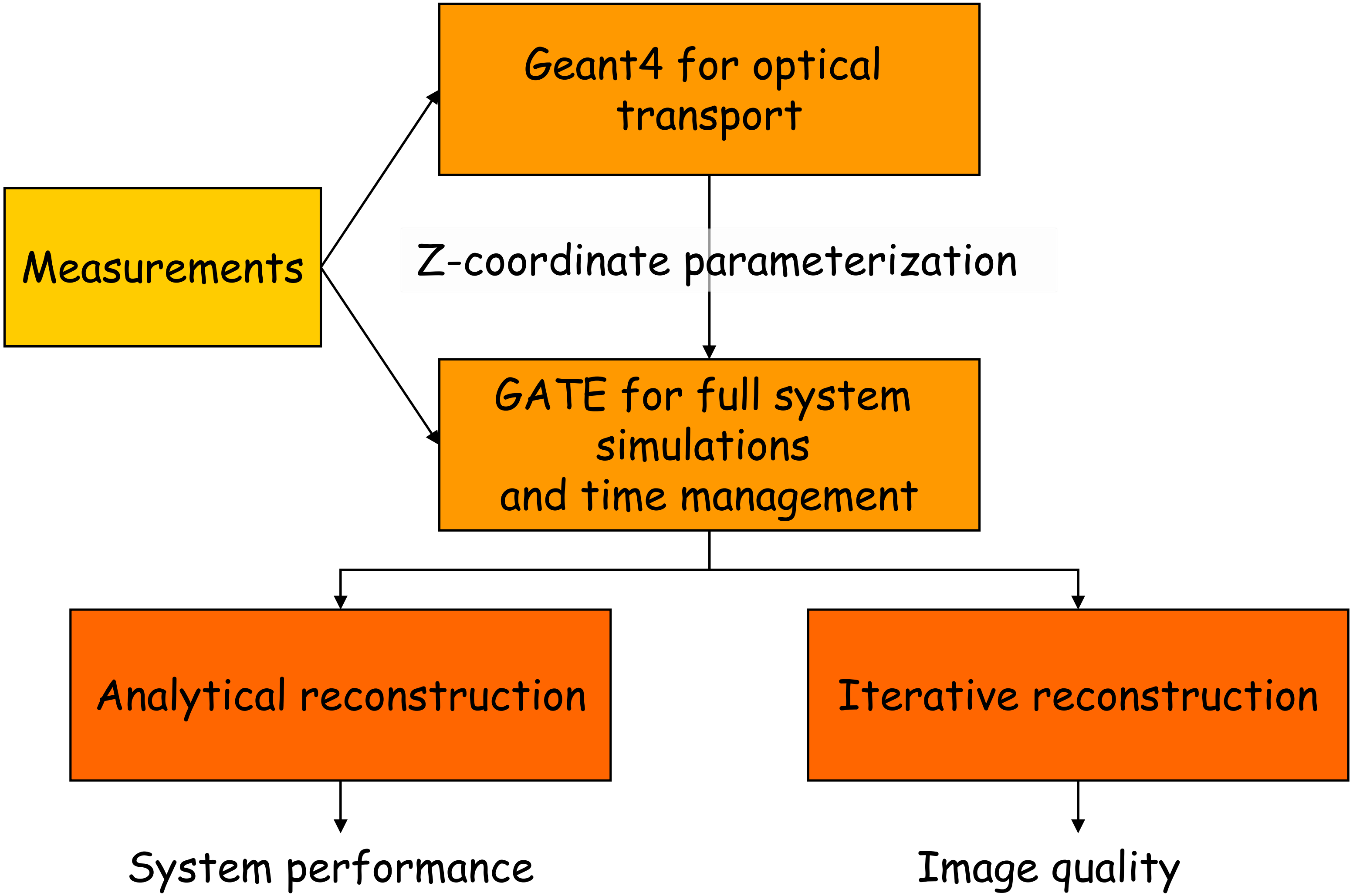


Image quality is a delicate balance of all the system properties! No recipe available...

Scheme of the simulation and software



Measurements

Geant4 for optical transport

Z-coordinate parameterization

GATE for full system simulations and time management

Analytical reconstruction

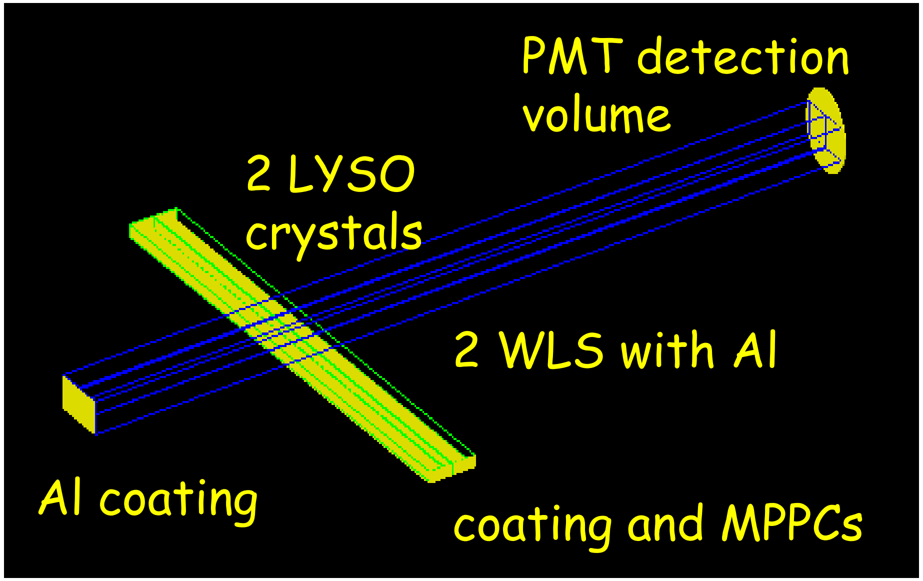
Iterative reconstruction

System performance

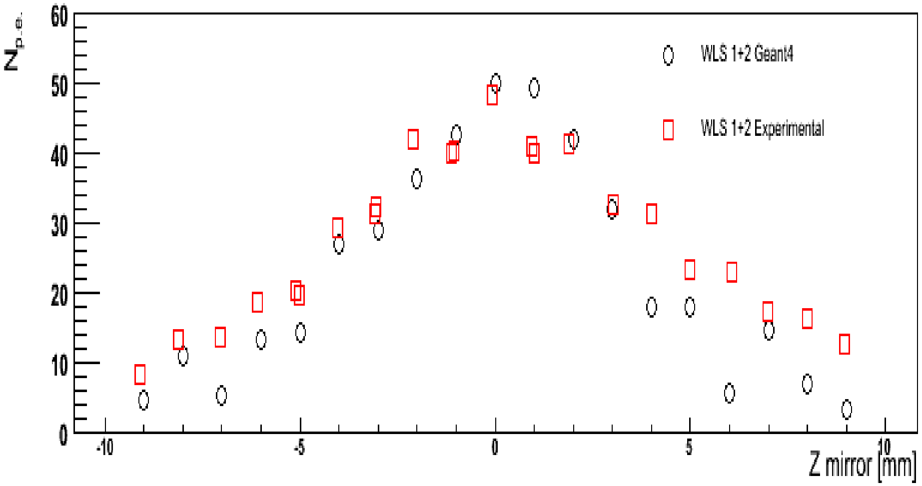
Image quality

Geant4 simulation validation

- Optical material properties modeled according to experimental data.
- Bunch of <9.7> incoming 25 keV e-, gaussian distributed (σ ~1 mm)



(NIM A 586 (2008) 300)



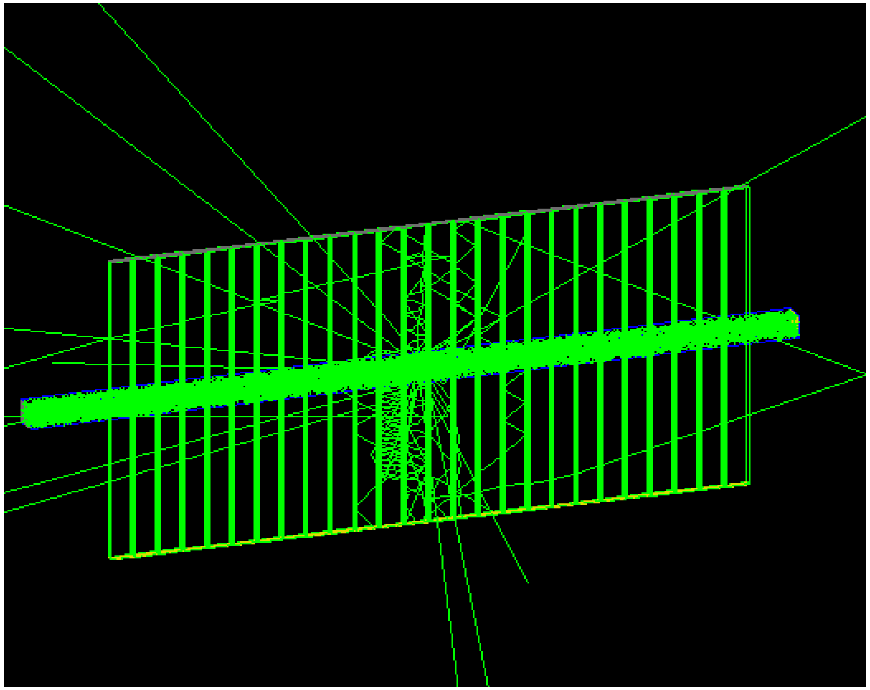
- The large width of the signal on the WLS is due to the small DOI of e- in LYSO.
- The light yield is fine tuned to achieve an optimum agreement with the signal amplitude on WLS strips and the p.e. detected in LYSO (1200 @511 keV).

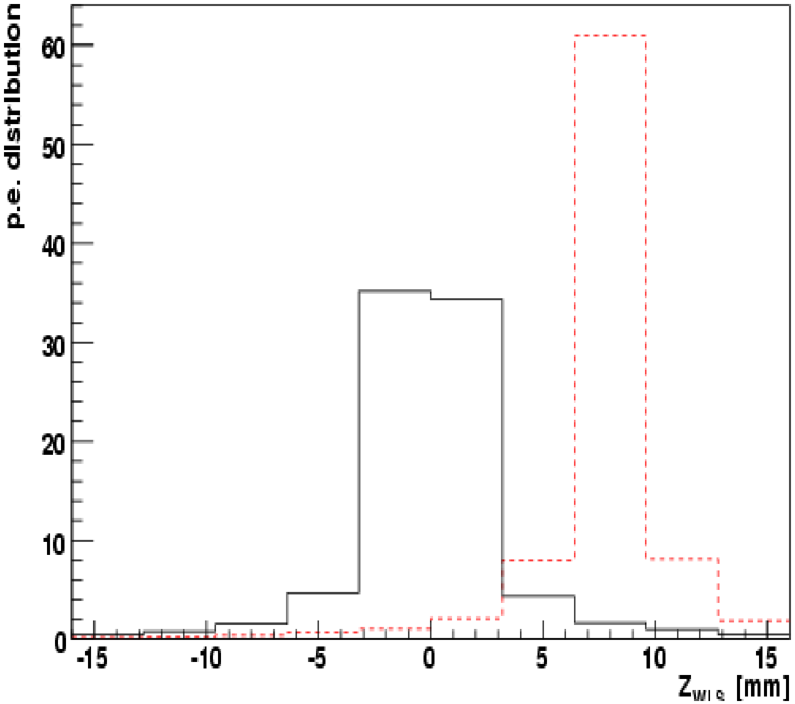
Toward the parameterization of the z coordinate

Simulation set-up: $3 \times 3 \times 100$ mm^3 LYSO crystals with 26 $1 \times 40 \times 3$ mm^3 WLS strips (0.2 mm wls-wls, 0.1 mm wls-lyso).

The resolution is sampled with 1 mm step in the axial direction, with uniform random distribution in the transveral plane.

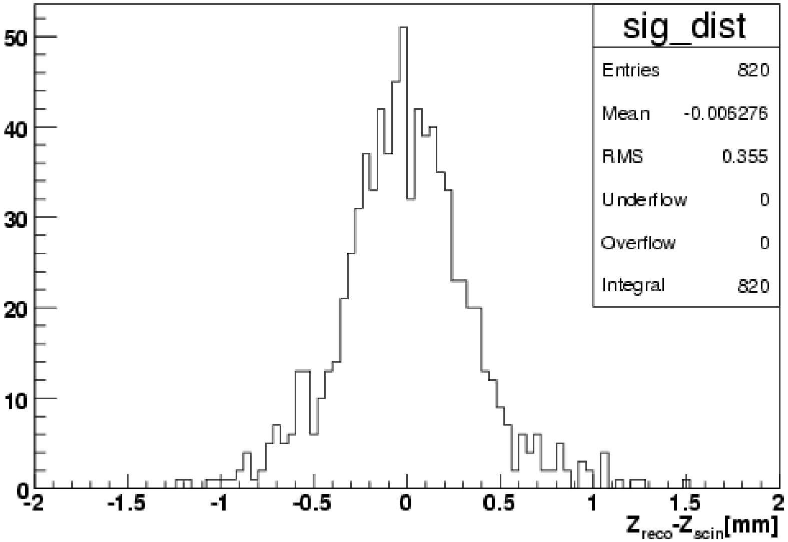
Optical photons are isotropically emitted: <N> ~10,000 with 3.5% (R_{intr}) (assumption of a point-like 511 keV energy deposition)





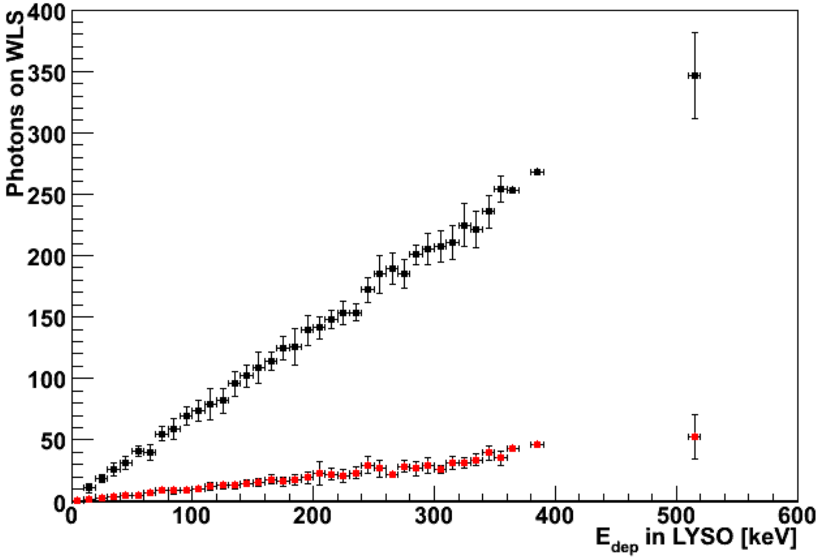
Z coordinate is estimated by means of the Centre of Gravity taking into account the maximum signal WLS ± 2 WLSs:

$$z_{reco} = \frac{1}{S_{tot}} \sum_{i=max-2}^{i=max+2} S_i \cdot z_{WLS,i}$$



The z resolution (RMS) is 0.36 mm (-41.5 < z < 41.5 mm), obtained by assuming a point-like full energy deposition: recoil electron weighted range < 0.05 mm.

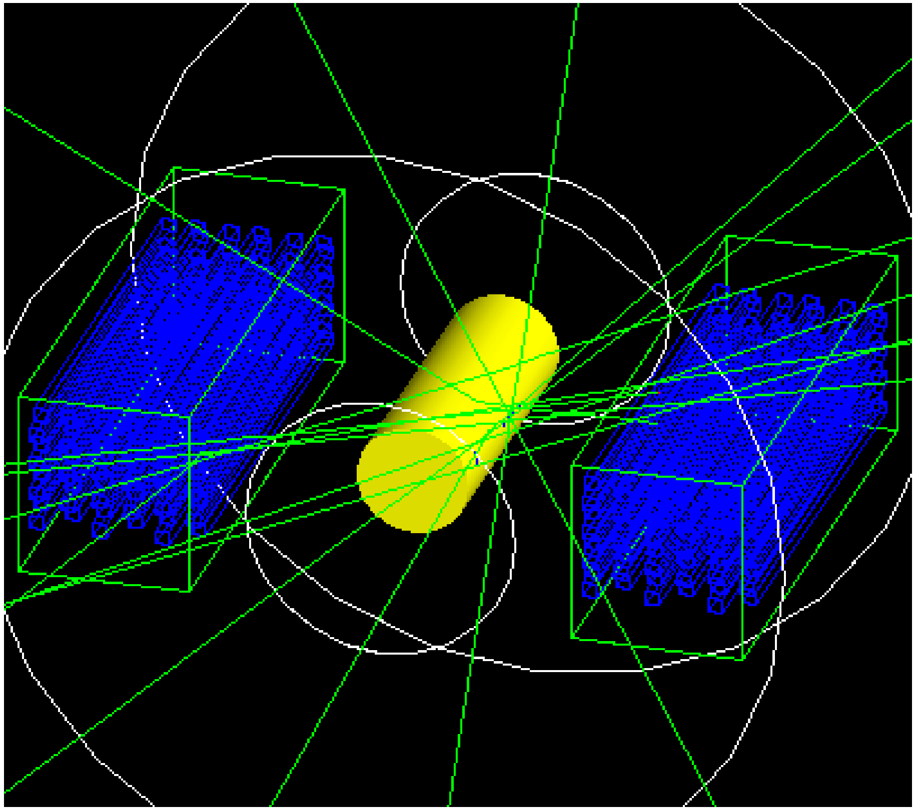
Study of the dependence of the WLS's signal on the energy deposited in the crystal.
Ratio of light in neighboring strips ~7.



System modelling by GATE

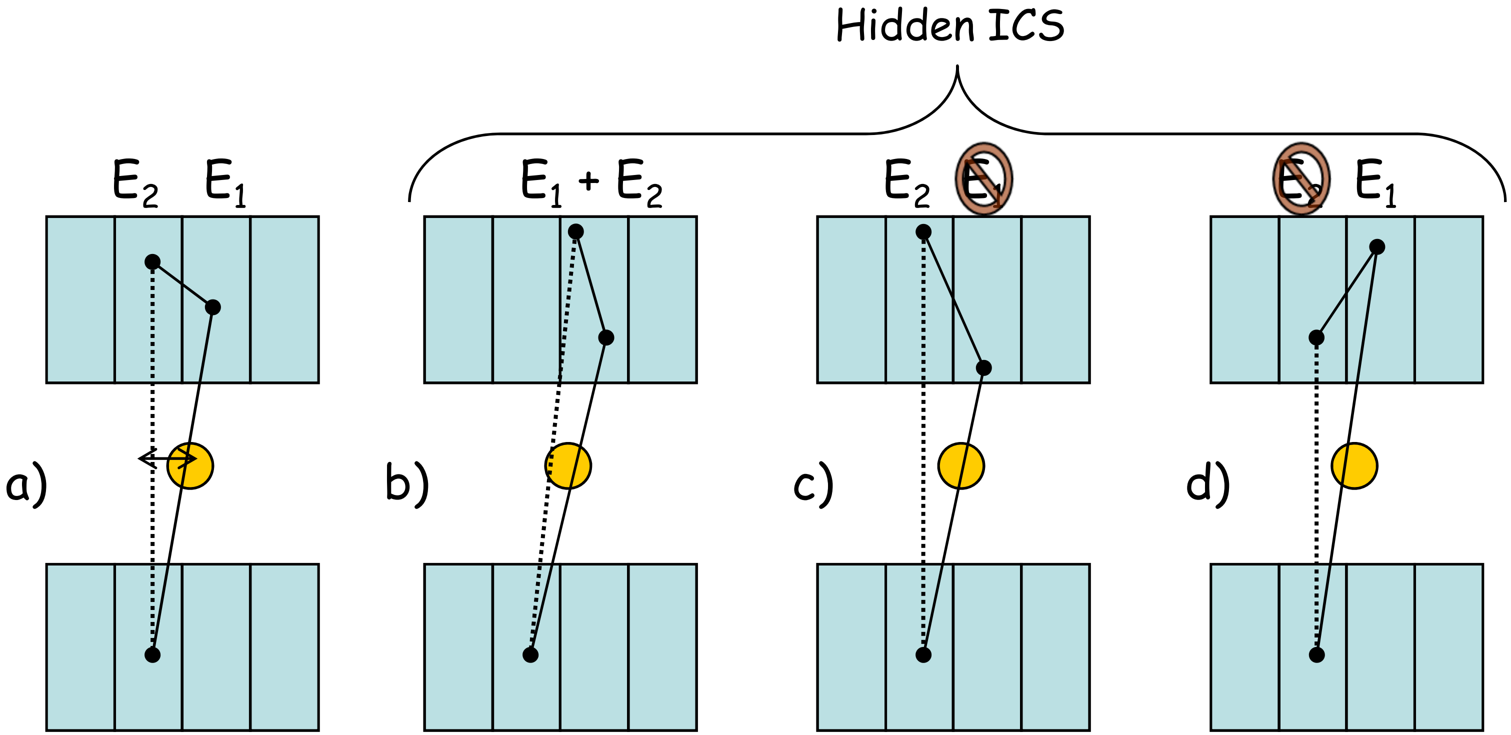
GATE: Geant4 application to Positron Emission Tomgraph.

The AX-PET geometry and related features are accurately modelled in GATE.



Simulation's parameters:

- Crystal matrix size, offsets
- Distance between detectors, rotation steps
- Phantom and source distribution
- Digitizer chain: LET-UET, δE, δt, τ, dead time.
- Acquisition scheme

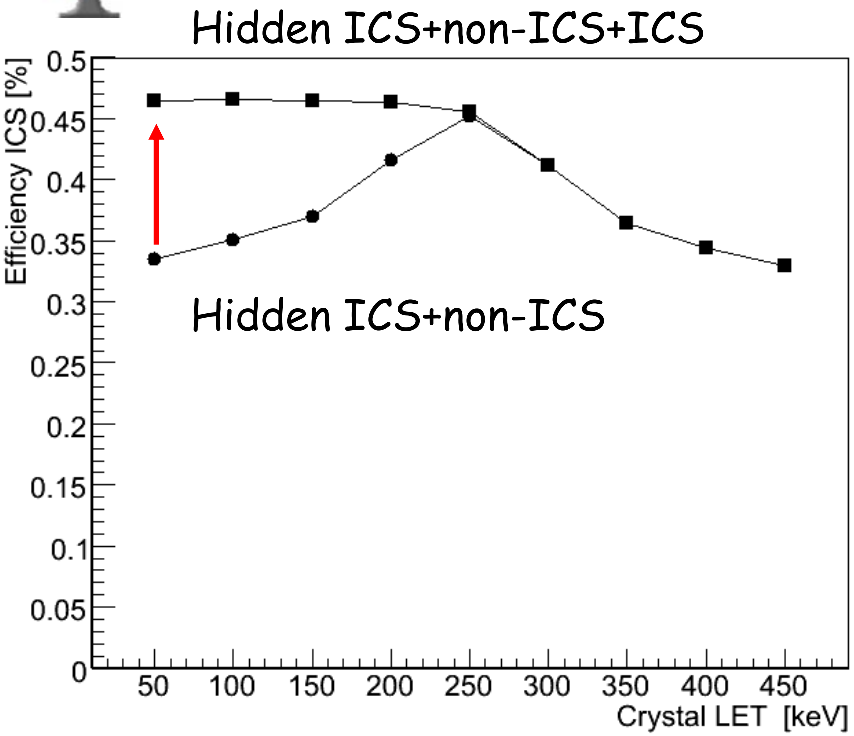


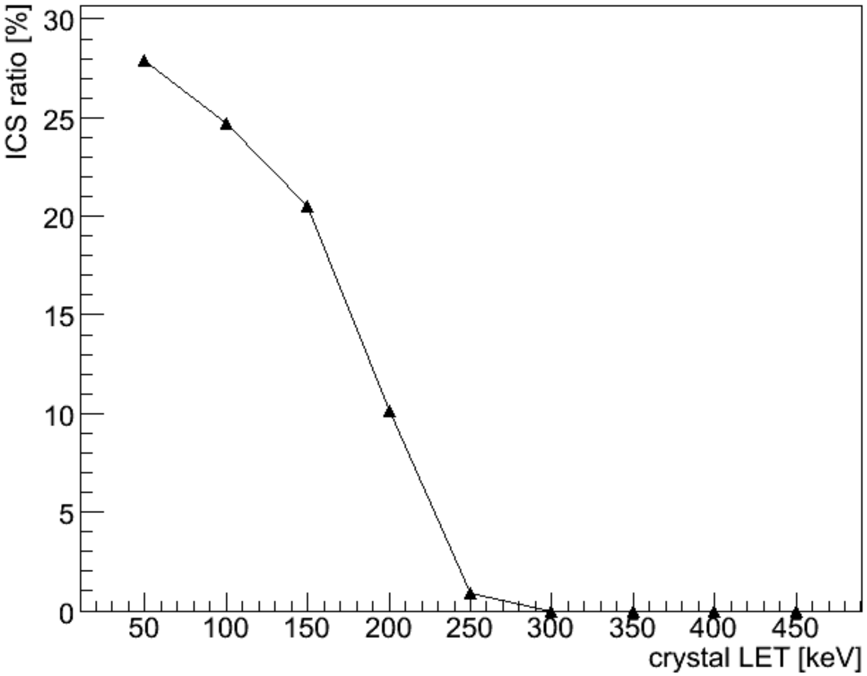
An uncorrect identification of ICS results in a misplacement of the true LOR and a consequent image degradation.

Misplacement derives from:

- Wrong reconstruction
- Hidden ICS: hits within the same pixel or one hit lieing below LET.

AX-PET applies a low LET (30-50 keV) and its layered geometry permits to resolve geometrically multiple hits, without jeopardizing the spatial resolution.





One module efficiency: singles are selected for E_{module} > 450 keV.

There is ~30% sensitivity gain (<u>on singles</u>) by considering the ICS events for 50 keV LET at crystal level.

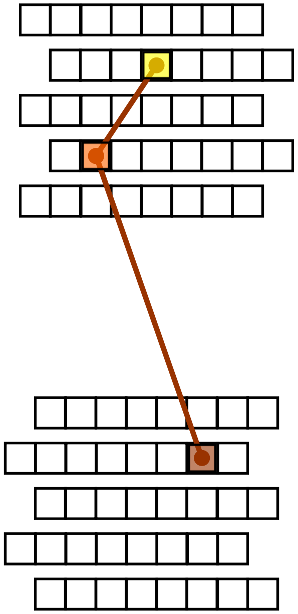
ICS reconstruction by Neural Network

NN is the first approach to ICS identification.

<u>TMultiLayerPerceptron</u> by ROOT is fast and can be easily incorporated into the reconstruction algorithm.

First trial: 1 Compton->double hit->full energy released.

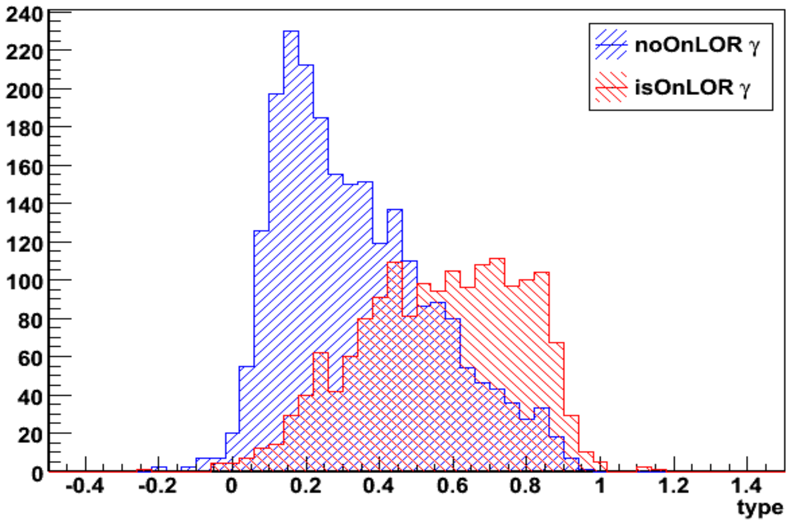
The network is fed with 11 variables: $E_1, x_1, y_1, z_1, E_2, x_2, y_2, z_2, x_0, y_0, z_0$ (E_1+E_2 = 511 keV).



🟨 E2, x2,y2,z2

🟧 E1, x1,y1,z1

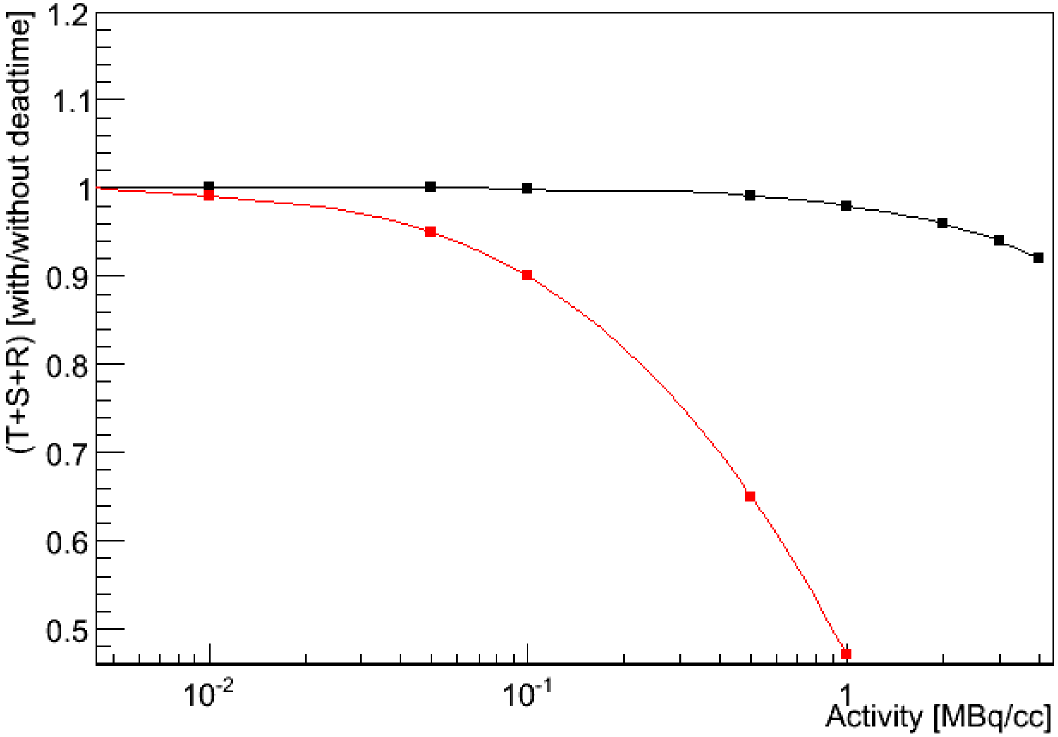
🟫 x0,y0,z0 (511 keV)



Maximum achieved effciency: 78% , but energy and z coordinate blurring not included yet.

Constraints on the electronics

The high sensitivity offered by the layered stucture of AX-PET as well as the capability to identify multiples needs to be supported by a fast electronics.



LET = 50 keV

δE = 12% (FWHM)

δT = 3 ns

τ = 10 ns

Dead time at crystal level:

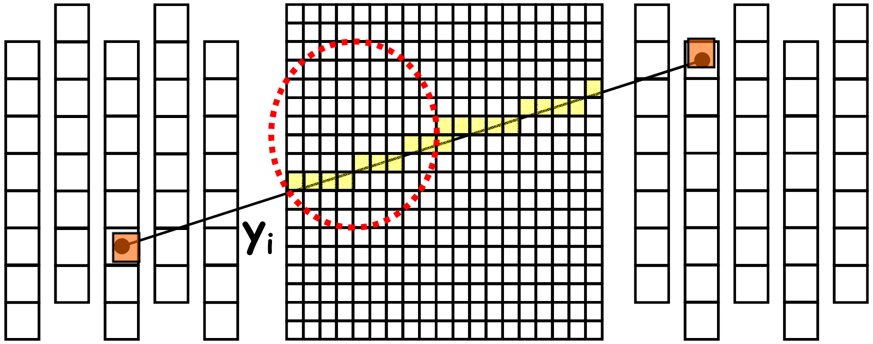
500 μs and **10 μs**

Image reconstruction

- Two big families of algorithms: analytical and statistical iterative.
- The second ones offer higher image qualities but an accurate model of the system is fundamental!
- In iterative image reconstruction it is crucial to compute a system matrix to accurately model the relationship between projection and image space:

$$\overline{y} = \mathrm{A} \cdot f$$

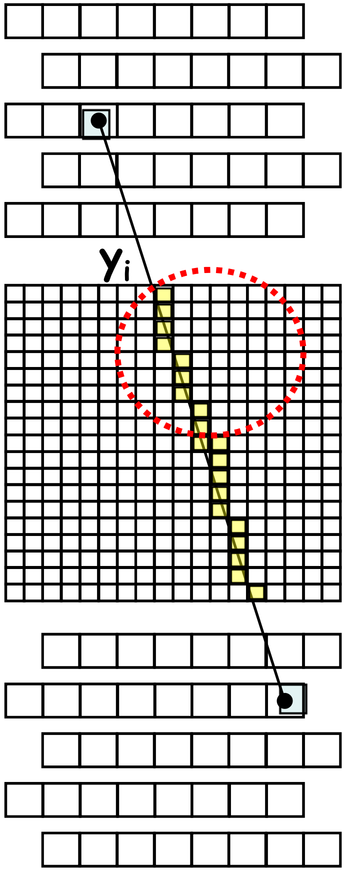
where y is the measured data, f the voxelized image and A is the system matrix.

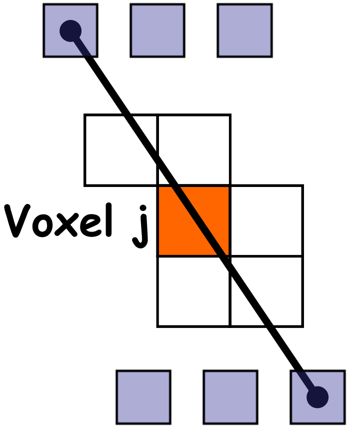


System matrix element **A**ij: probability to detect the activity in voxel j in y_i.

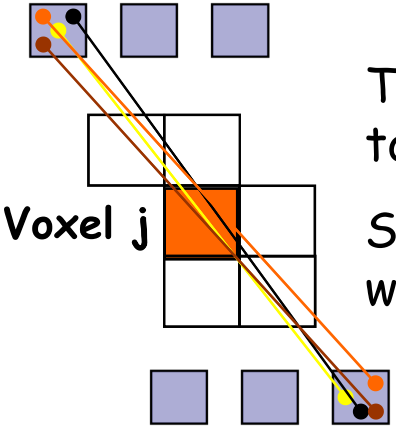
System matrix

Computation of A can be performed by analytical and Monte Carlo approaches. The first approach has been chosen for the moment.





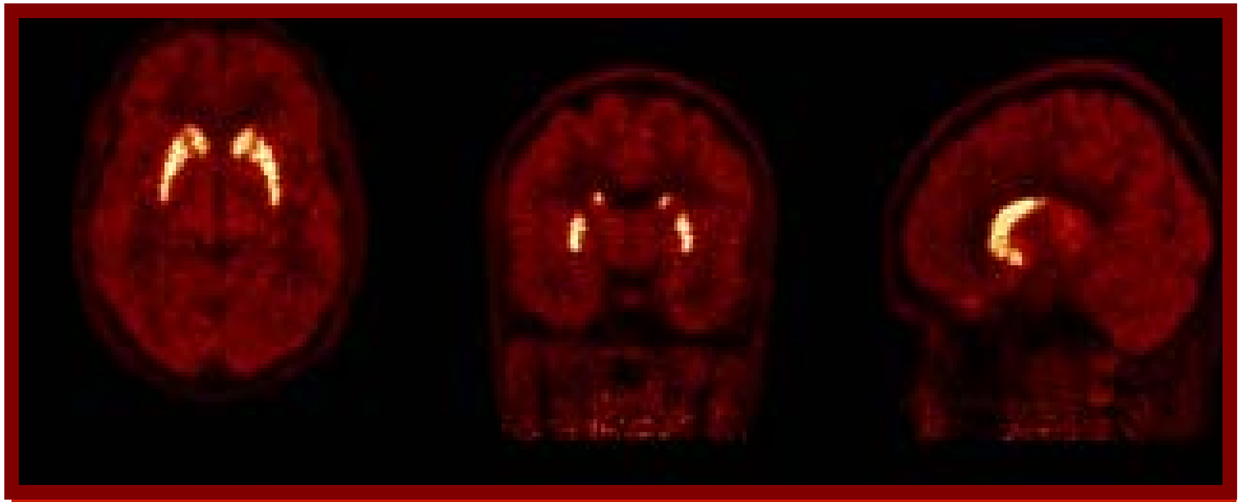
Siddon's ray approach: p is proportional to the length of the LOR's segment traversing j.



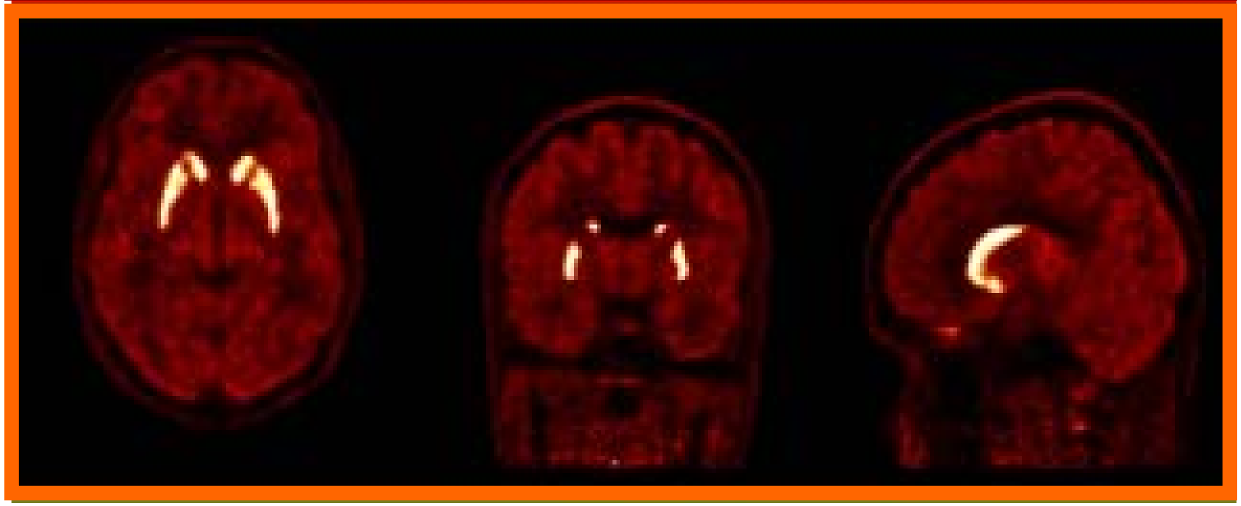
TOR replaces LOR: LOR is no proper due to the discrete nature of crystal matrix.

Sub-sampling: to compensate for TOR's width.

"Screening" effect due to attenuation in neighboring crystals is considerered.



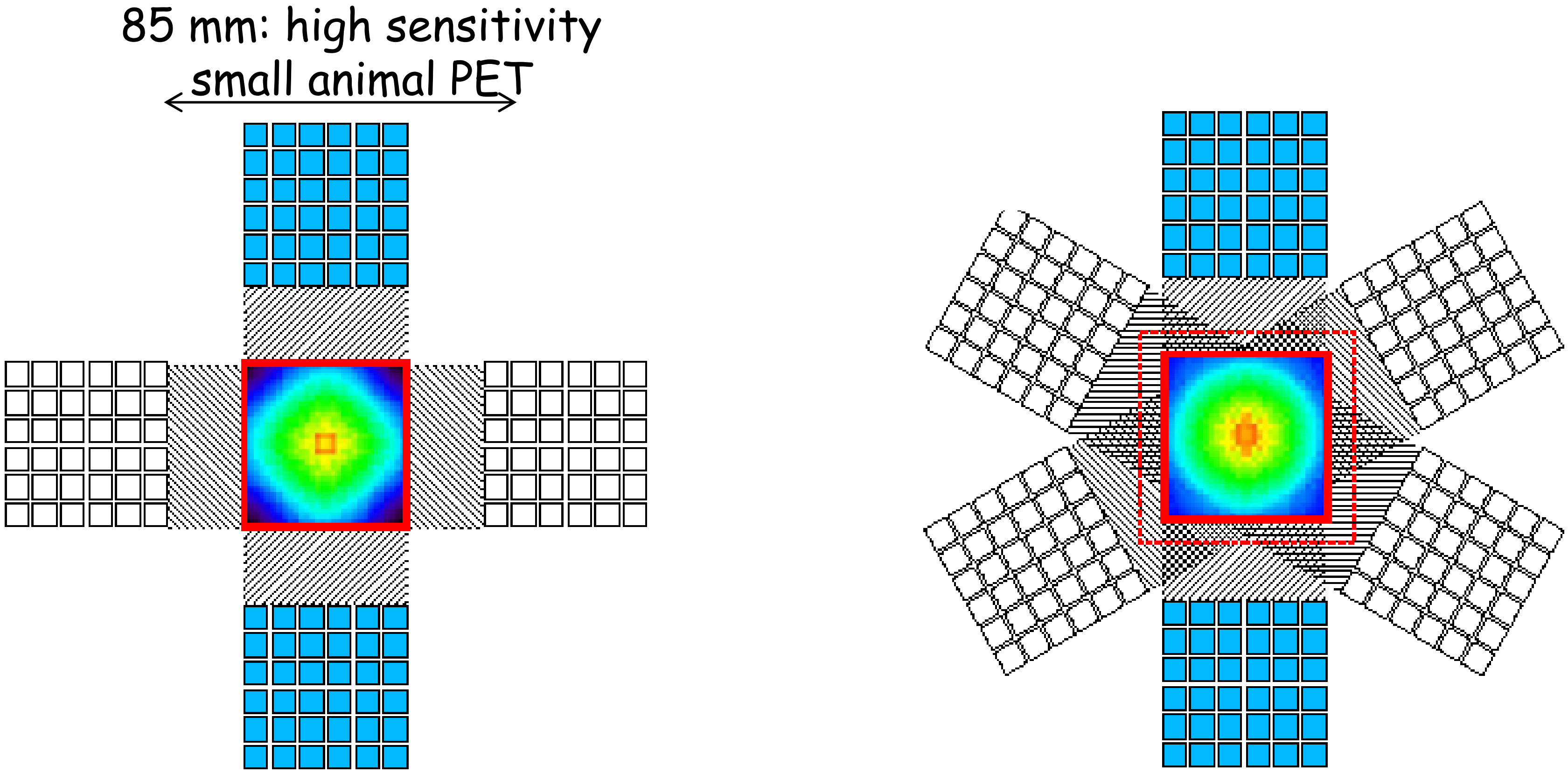
Siddon's ray approach



Siddon's ray approach plus detector point spread function model

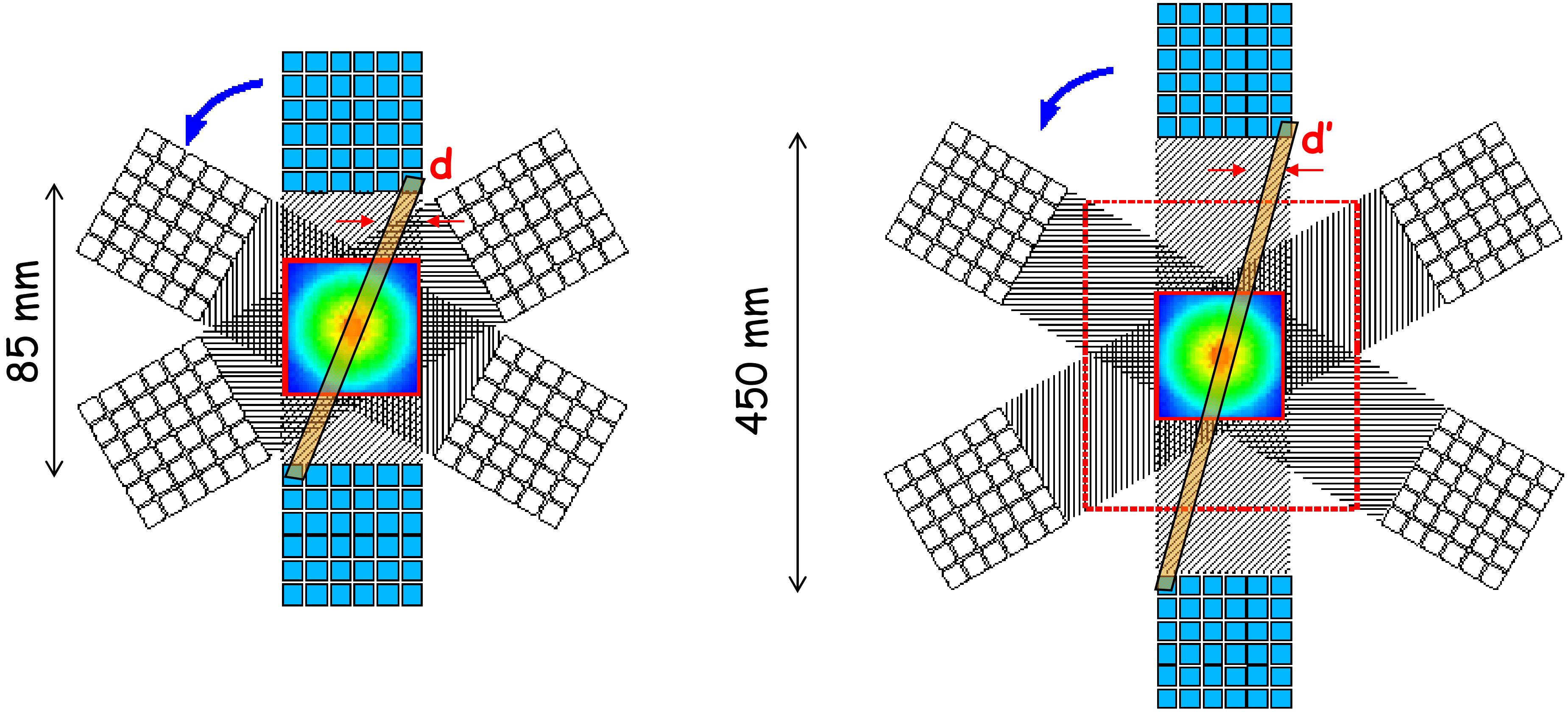
Courtesy of C. Comtat

Effect of the rotation steps on the sensitivity matrix



- Within 30x30x86 mm^3 FOV: no significant gain in the sampling for more than 3 steps.
- Larger number of steps would allow larger FOV.

Effect of the distance between detectors on sensitivity matrix



☐ Potential higher FOV

☐ More steps needed to compensate for the missing data

☐ Smaller parallax error (small gain: d=3.73 mm -> d'=3.17 mm)

Conclusions

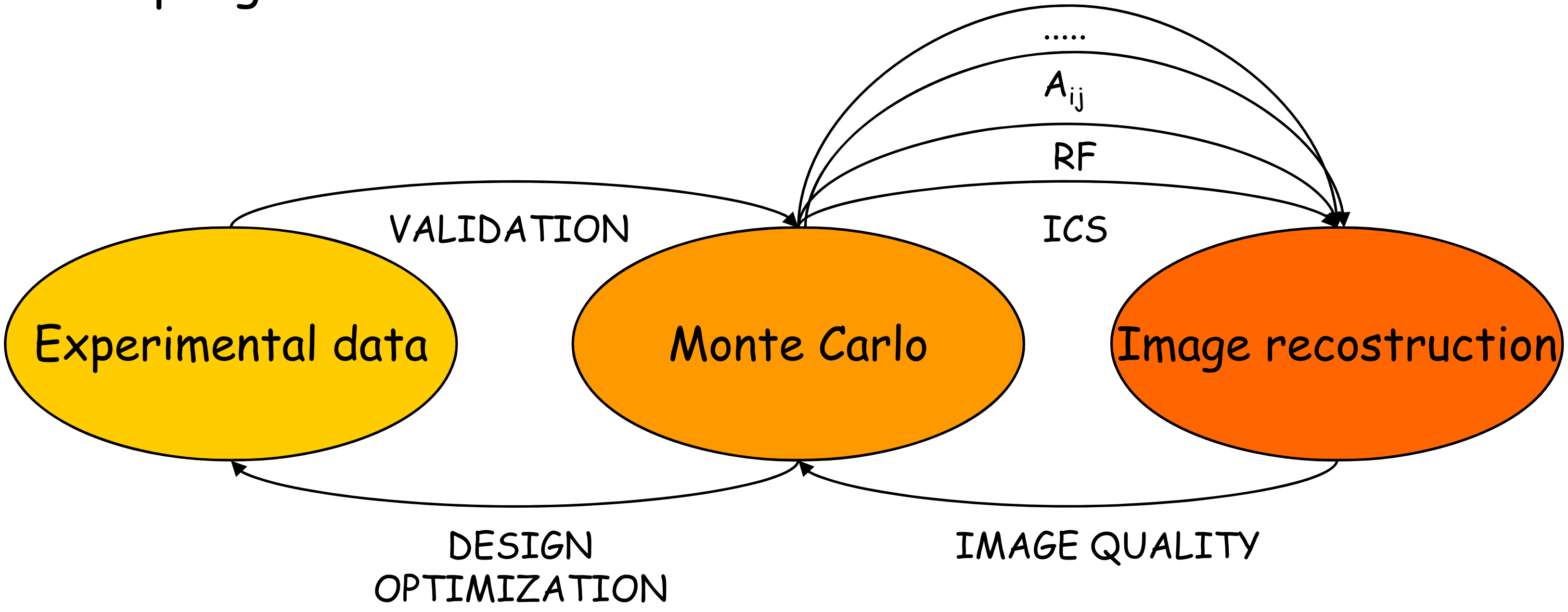
- AX-PET is a novel PET concept, however to exploit at best its potential, a huge effort in terms of dedicated software developement is required.
- The production of synthetic data is mandatory to foresee the detector performance for a specific application, train reconstruction algorithms, and to improve the system design.
- A first accurate description of the system, by means of GATE and Geant4, is almost completed, and first performance studies are ongoing.
- MLEM recontruction has been selected because it is capable to provide the maximum achievable image quality, but a proper system response matrix is required.
- Currently under study: Different modelling approaches and system sampling capability.

Back-up slides

Monte Carlo in PET imaging

- AX-PET is a novel concept: high energy and spatial resolution, 3D interaction position recovery, high sensitivity, good time resolution.

- <u>Image quality</u>:optimum acquisition sheme, optimum electronics, dedicated image reconstruction algorithms, homogeneous sampling of the FOV.



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A_{ij}

RF

VALIDATION

ICS

Experimental data

Monte Carlo

Image recostruction

DESIGN
OPTIMIZATION

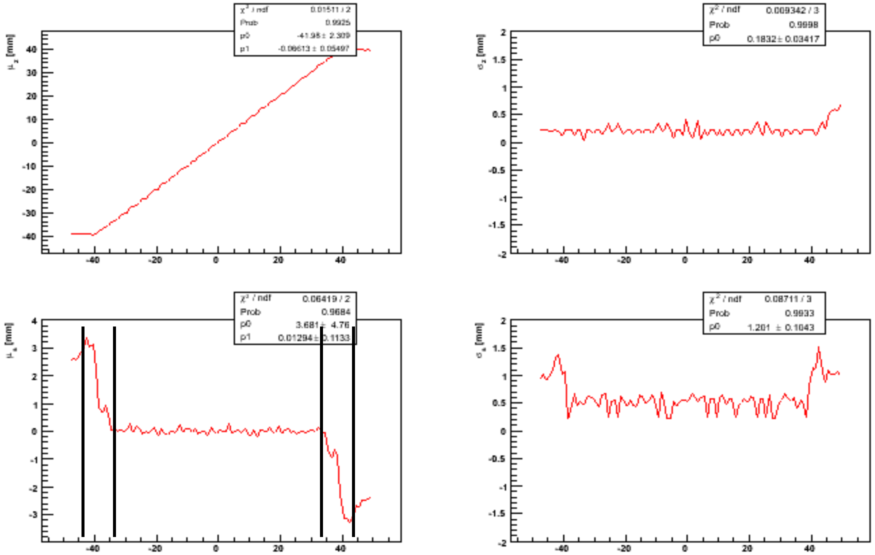
IMAGE QUALITY

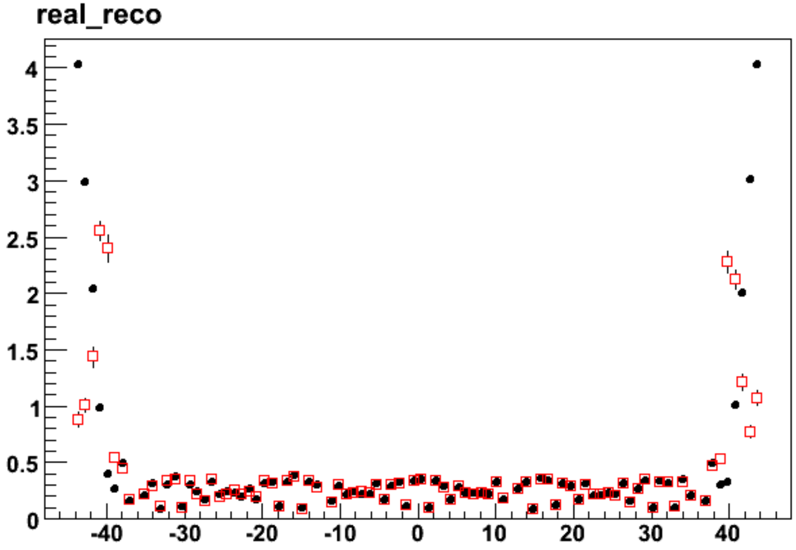
Skewness technique close to the crystal's edges

$$z_b = \frac{1}{Q}\sum_i q_i z_i$$

$$s_z = \frac{\sum_i \frac{q_i}{Q}(z_i - z_b)^3}{[\sum_i \frac{q_i}{Q}(z_i - z_b)^2]^{3/2}}$$

$$\frac{\partial[\log(f_b(x_b)\cdot f_s(s_z))]}{\partial z_h} = 0$$





GATE simulation

GATE: Geant4 application to Positron Emission Tomgraph.

- User friendly (script interface to the user).
- Not as flexible as Geant4 in the geometrical description.
- Easy modeling of the phantom and source distribution.
- Simulation of the system gantry rotation.
- Output stored in ROOT format: hits, singles, coincidences.

GATE digitizer

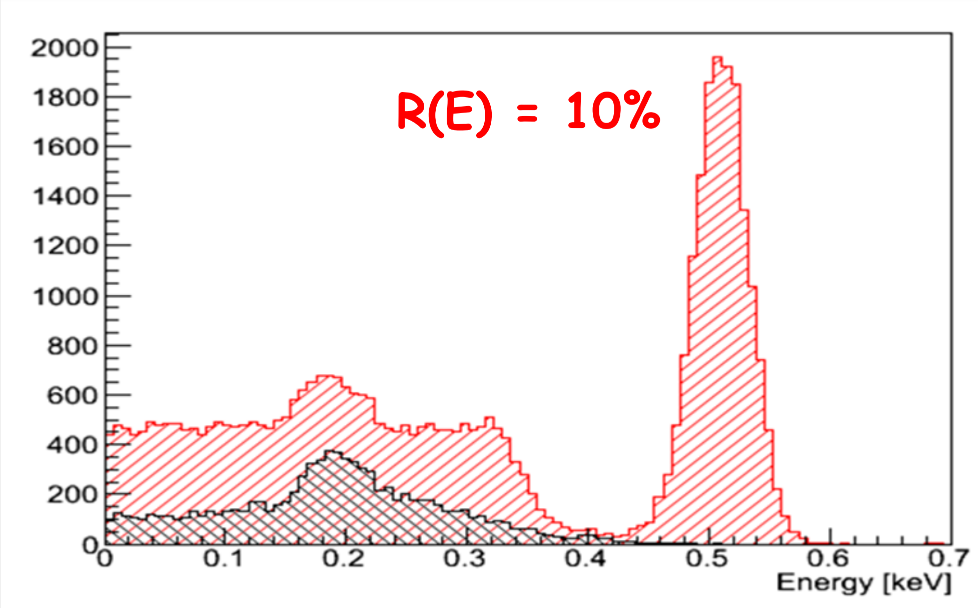
- Energy blurring, LET and HET
- Noise: LYSO radioactivity background and crystal optical cross-talk (expected to be negligible) -> single and matrix measurements planned
- Time resolution, dead-time, pile-up, coincidence window
- Coincidence policy

Aluminum housing background

Simplified geometry: 36 crystals, 4 mm thick Aluminum housing.

A point-like source is placed in the middle of the FOV (1 kBq for 3000 s long acquisition).

	CF_{Al} (LET = 0 keV)	CF_{Al} (LET = 350 keV)
4 mm – 4 mm module level read-out	21 %	2 %

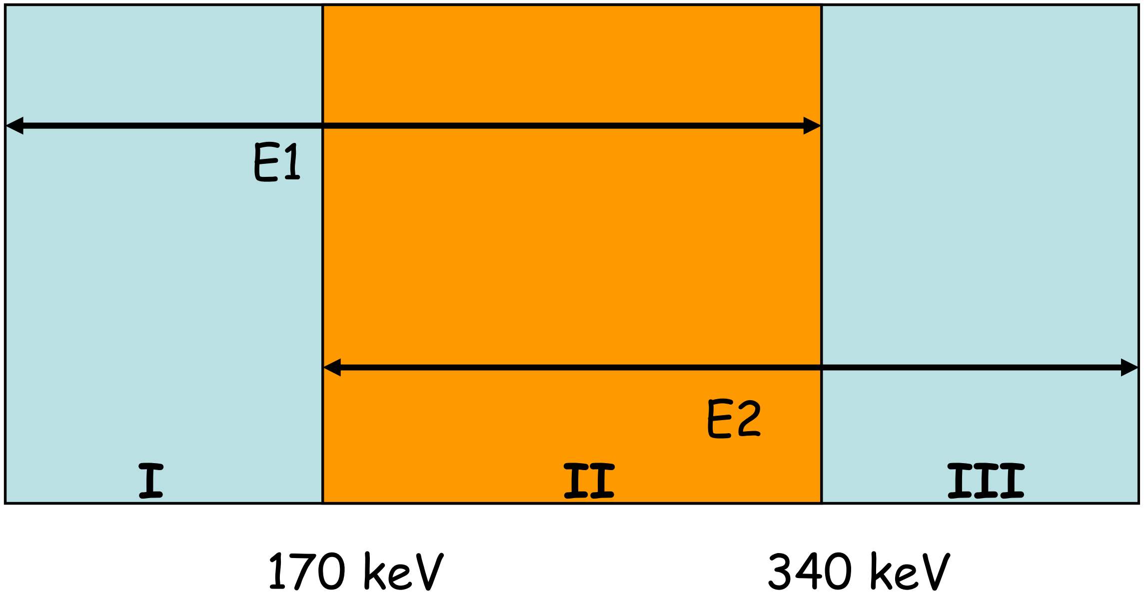


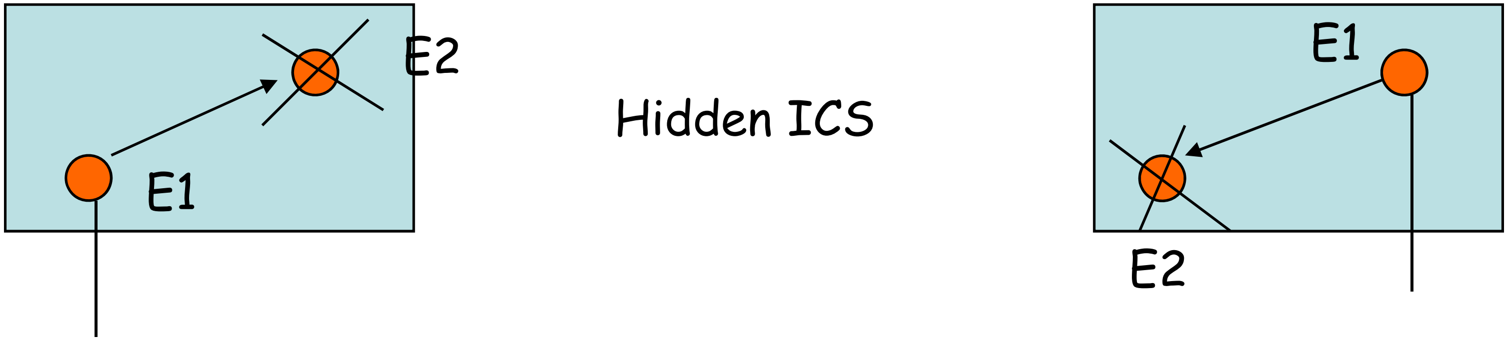
Compton fraction in the Al housing is reduced to 2 % by applying a 350 keV LET, but it still remains a problem concerning ICS accidentals.

The effect might be negligible compared to object scatter.

Compton ICS

E1+E2 = 511 keV.



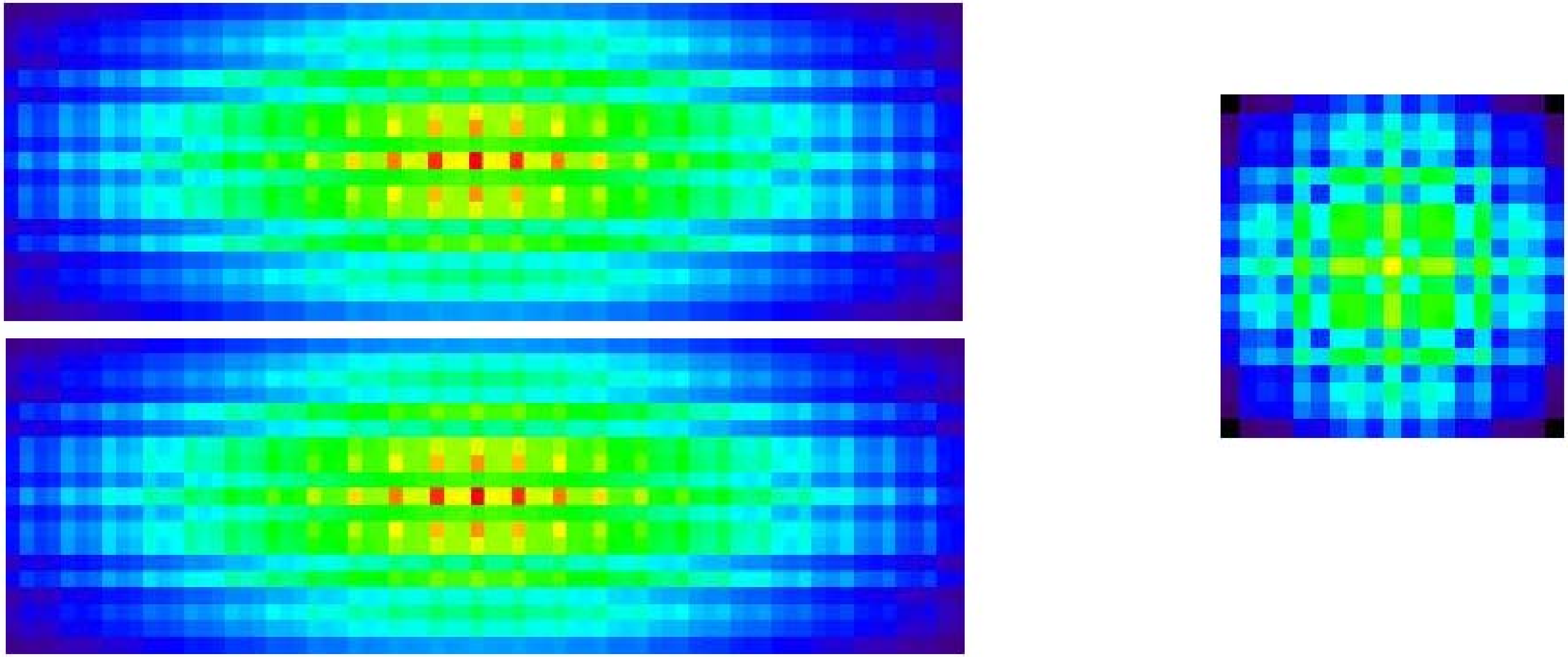


Inter-crystal scattering

Most popular available techiques:

- Depth of interaction algorithm: the crystal with interaction depth closest to the patient is selected (theoretical limit $\theta<90°$ is 2/3).
- Minimum/maximum-energy algorithm: crystal with lower/higher deposited energy is selected.
- Compton kinematics: selection applied to the energy depositions or minimization of $K = |\cos\theta_E - \cos\theta_G|$ for each possible LOR.
- Klein-Nishina scheme: selected LOR the one with the highest probable sequence.

Effect of geometry in FOV sampling



Symmetric crystal matrix: effect of gaps still visible after 2 steps rotation.

MLEM

$$\vec{y} = A\vec{f} + \vec{x}$$

$$\Phi \rightarrow p(\vec{y} \mid \vec{f})$$

$$\Phi : \prod \left[\frac{e^{-(Af)_m} (Af)_m^{y_m}}{(y_m)_i} \right]$$

Cost function: probability to measure y if you have f. It's based on Poisson distribution, noise can be incorporated in the cost function.

The best way to maximize ML is to use EM.

$$f_j^{k+1} = \frac{f_j^k}{\sum_i A_{ij}} \sum_i \frac{y_i}{\sum_{ji} A_{ij} f_j^k} A_{ij}$$

Last image

System matrix

Control convergence and smoothing

Forward BP: algorithm converges when $y_i = y_i^k$