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Nonlinear theory of cosmic rays acceleration by steady stellar wind

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Abstract. We consider the model of particles acceleration to reletevistic energies by spherical symmetric strong shock wave of stellar wind. It's supposed, that the particles are accelerated by the Fermi mechanism. The effect of the accelerated particles on the hydrodynamic parametres of stellar wind is also taken into account.

Our method allows to find the self-consistent steady solution for a strong shock, significantly modified by accelerated particles. On the hydrodynamic stage we give the pressure of accelerated particles and find the profile of hydrodynamical stream as a solution of hydrodynamic equations. On the kinetic stage we calculate the spectrum of accelerated particles and determine its pressure by self-consistent method. In this report we also discuss possibility of nonthermal X-ray radiation and give recommendations for observations.

1 Statement of problem

. Interaction of strong stellar wind with surrounding medium creates shock fronts (Weaver et al.,1977), which is capable of generating accelerated particles. The linear theory of acceleration process by spherical steady shock wave in stellar wind was developed in papers (Webb et al.,1983),(Klepach et al.,1997),(Toptygin,1999). If acceleration is efficient enough, nonlinear effect of accelerated particles on the stellar wind dynamics is significant (see solution of analogous problem for supernova explosion in Toptygin(2000)). The consideration of nonlinear effects is important for correct calculation of the energetic spectrum of accelerated particles and energy distribution between relativistic particles and hot surrounding gas.

In the given paper we consider nonlinear problem formation of the energetic spectrum of accelerated particles by stellar wind shock wave. We take into account the back effect of particles on hydrodynamics of the stellar wind. In stellar wind and outside it, in interstellar medium, several discontinuities are formed (Weaver et al.,1977). The supersonic region of stellar wind and the first schock wave, nearest to star, play the main role in particle acceleration. Our evaluations give, that particles receive about 90 % of energy in this region. We consider isotropic diffusion of particles in acceleration region and two models of diffusion coefficient dependence on particle energy. The first is stepped dependence: $\kappa(r,p) = \kappa_0(r)$, if particle momentum $p \le p_m$, and $\kappa \to \infty$, if $p > p_m$. Here p_m is momentum of escaped particles. The second is dependence $\kappa(r,p) = \kappa_0(r)(p/p_0)^a$, were $0 < a \le 2$. We suppose, that particles are injected into acceleration regime at the shock front and the rate of injection Q_0 is free parameter in our consideration.

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2 Equations

. For solving of the formulated problem we must combine macroscopic and kinetic approaches. Gasdynamical parameters of stellar wind and shock front must be determined from conservation laws in integral and differential forms. Nevetheless the computation of the basic macroscopic parameters of the accelerated particles — their pressure and energy density, and also energy spectrum — requires a kinetic approach. In static case and for spherical simmetry of system conservation laws have the form

$$4\pi r^2 \rho u = J = const \qquad (1)$$

$$\frac{J}{4\pi r^2}\frac{du}{dr} + \frac{dPg}{dr} + \frac{dPc}{dr} = 0 \qquad (2)$$

$$\frac{Ju^2}{2} + 4\pi r^2 \left\{ \frac{\gamma_g}{\gamma_g - 1} u P_g + \frac{\gamma_c}{\gamma_c - 1} u P_c - \frac{\overline{\kappa}(r)}{\gamma_c - 1}\frac{dP_c}{dr} \right\} + S(r) = const \qquad (3)$$

ρ - density of an incindent flow, u - its velocity, P_g - pressure of thermal plasma, P_c - pressure of accelerated particles, $\overline{\kappa}(r)$ - diffusion coefficient, averaged on particles momentum, γ_g and γ_c - parameters of thermal plasma and accelerated particles polytrope, $S(r) = 4\pi \int_0^r Q_m \theta(\xi) \xi^2 d\xi$ - flow of escaped particles energy (Q_m and $\theta(r)$ - flow intensity and spatial distribution of escaped particles).

The kinetic equation for distribution function of accelerated particles $N(r,p)$, in case of spherical symmetry are $u(r)$ - hydrodinamical velocity of plasma, $\kappa(r,p)$ - diffusion coeficient, $Q(r,p)$ - injection rate)

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\kappa(r,p)\frac{\partial N}{\partial r}-u(r)\frac{\partial N}{\partial r}+\frac{p}{3}\frac{\partial N}{\partial p}\frac{1}{r^2}\frac{d(r^2u)}{dr}=-Q(r,p) \quad (4)$$

This equation should be solved in internal and external areas of shock front. If we designate the appropriate function distributions through N_1 and N_2, then boundary conditions and conditions at the front are:

$$N_1(r,p)\overset{r\to 0}{\longrightarrow}0 \qquad , \qquad N_2(r,p)\overset{r\to\infty}{\longrightarrow}0 \qquad (5a)$$

$$\left.\begin{array}{c} N_1(r,p)=N_2(r,p) \\ \kappa_1\dfrac{\partial N_1}{\partial r}+\dfrac{u_*}{3}p\dfrac{\partial N_1}{\partial p}=\kappa_2\dfrac{\partial N_2}{\partial r}+\dfrac{u_2}{3}p\dfrac{\partial N_2}{\partial p}+Q_0 \end{array}\right\} r=r_0 \qquad (5b)$$

Here κ_1, κ_2, u_* and u_2 - corresponding values on internal and external surfaces of shock transition.

3 The solution of equation system.

The equations (1) - (3) were solved numerically in internal and external areas. The measure of accelerated particles pressure influence was the value of its pressure at the front (P_{c0}). Depending on different values of this parameter, character of incindental flow velocity changes considerably (see fig. 1). There are characteristic inflected profiles of flow velocity at the large pressure of the accelerated particles (see fig. 1b). When the pressure is not too high it is possible to approaximate the numerical solutions by power-type functions with enough accuracy: $u(r)=1+(u_*-1)(r/r_0)^\mu$ (u_* - value of velocity at the front). Parameter μ is defined by the condition, so that approach of velocity, is in accordance with the given value of accelerated particles pressure at the front. As detailed calculations show, power expressions approximate the numerical solution for velocity well enough. So the error does not exceed 2-3 % in most cases.

As for external area, it is possible to note insufficient (¡ 10 %) difference of function $z(x)=u(r)r^2/u_2r_0$ from 1 at $r<5\div 10r_0$. Thus, with acceleration, the particles take energy basically from internal area,without rendering strong influence on external.

Let's consider, that the particles are injected into the process of acceleration in the narrow area around the front. It can be motivated by the greatest injection being expected in the area with the greatest acceleration of particles. The Fermi mechanism asserts that the more gradient of velocity is, the more effectivity the particles are accelerated. It takes place at the front. So we shall consider, that injection occurs with constant of injection rate Q_0 from background of particles with momentum p_0. The particles also escape the system with constant rate Q_m from narrow area around the front as soon as they have achived momentum p_m. This model corresponds to step dependence diffusion coefficient from momentum: $k(r,p)=k(r)$ at $p<p_m$, and $k(r,p)=\infty$ at $p\geq p_m$. Thus the function of injection rate $Q(r,p)$ should be chosen as $Q(r,p)=Q_0p_0^{-2}\delta(r-r_0)\delta(p-p_0)-Q_mp_0^{-2}\delta(r-r_0)\delta(p-p_m)$

In such model the equation (4) can be solved using Mellin transformation from p to s:

$$\overline{N}(r,s)=\int_0^\infty N(r,p)p^{s-1}dp \qquad (6)$$

. Thus the equation becomes the one of second order on r, which is transformed into hypergeometric equation. Boundary conditions (5a) and (5b) are transformed into the system of linear equations which allows to find free constants.

The Mellin retransformation is given by integral, which is equal to the sum of residues in all special points. At power-law approximation of velocity there is one pole. As a result power-type spectrum of the accelerated particles on momenum is obtained. The degree of particles momentum spectrum , varying depending on parameters of a task in limits $3<\alpha<5$.

The step dependence of diffusion coefficient on particles momentum can be considered only as model. According to the standard estimations this dependence approximately is power-type $\kappa(p)\sim p^a$, where the parameter a ranges from 0 to 2.

Expressing function of distribution in external area through function in internal using (5a) and the solution for external area, it is possible to write down a condition of particles flows equality at the front (5b) as

$$\kappa_1(p)\frac{\partial N_1}{\partial r}+\frac{\Delta}{3}p\frac{\partial N_1}{\partial p}+F(p)N_1(p)=Q_0 \qquad \text{at } r=r_0 \qquad (7)$$

Here Δ - jump of velocity at the front, $\kappa_1(p)$ - diffusion coefficient in the interior of front,

$$F(p)=\exp\left(\frac{1}{k_2p^\alpha}\right)/\left[p^\alpha\left(1-\exp\left(\frac{1}{k_2p^\alpha}\right)\right)\right]$$

Thus, the finding of the distribution function is reduced to solution of equation (4) with the conditions (7) and the function is finit at $r\to 0$.

Here we give linear dependence on momentum, though all calculations can be generalized for any degree. Let's search for the solution as a double sum

$$N(r,p)=\sum_i c_{0i}x^{\nu_i}p^{\frac{3\nu_i}{2}}\exp\left(-\frac{3k_1\nu_i(\nu_i+2)}{2}p\right)(1+ \varepsilon a_1(p,c_{1i})x^\mu+\varepsilon^2a_2(p,c_{2i})x^{2\mu}+...)\,. \quad (8)$$

Where $\varepsilon=1-u_*/u_1$ (u_1 - meaning of plasma velocity in the diturbance area far from front), $x=r/r_0$. We can obtain the differential equations of the first order for functions $a_k(p,c_{ki})$, by equating the items at identical degrees ε.The unknown constants ν_i and c_{ki} can be defined from a boundary condition (7). At final number of items in a sum on i this condition can be satisfied only approximately. We can minimize relative error of condition (7), for reception of numerical values ν_i and c_{ki}, concerning error as function of all unknown constants. The more items are took into account, the fewer momentums are provided equality of the particle flows at the front with proper accuracy. So in case $\varepsilon = 0$ (non-distorted profile at linear acceleration) for maintenance of equality flows at the large pulses ($p \to \infty$) it is enough to take into account one item. In this case it appears $\nu = \frac{2}{3}(\sigma - 3)$. For performance of equality of flows with 5 % accuracy for all pulses $p \geq 10p_0$ (p_0 - pulse of injection) it is necessary to take into account not less than 8 - 10 items. Thus minimizing difference in flows, it is possible to achieve performance of condition (7) with any accuracy for any range of momentum.

4 Discussion and conclusions.

The ways of the equation solution mentioned above allow to find spectra of particles at step and power-law dependences of diffusion coefficient from particle momentum. With the help of the found distributions it is possible to define pressure of the accelerated particles for each meaning of particles injection rate. It allows to match injection rate with a set hydrodynamical parameters of the front, determining a concrete mode acceleration and, thus, to coordinate a task.

The figure 2b, shows, that, as well as in flat case, the solution appears multiple-valued for step dependence of diffusion coefficient. The question wich of possible solutions could be realized in nature, is being solved by the analysis of stability of the offered solutions in relation to small influences. Similar research in case of the flat front shows, those conditions, which have smaller compression at equal capacities of injection are more stable.

To the number of features of the solution in the flat case, which also take place in the symmetric case it is possible to relate a final interval of injection rates in the process of acceleration at which the thermal jump can exists.

At power-type dependence of diffusion coefficient the injection rate grows monotonously with the growth of shock compression in considered interval of compressions. Besides there is no necessity to enter momemtum of escaping, because distribution function can be normed at any compression. Due to the presence of exhibitors the spectrum appears to be cut off. The very important difference between solutions mentioned above is exponential asymptotic of solution with power-type diffusion coefficient.

It is well known (Kaplan and Pikel'ner,1963), in plasma behind the front of adiabatic strong shock waves, temperature kept behind the front is defined expression $T_2 = 3\mu m_H v_1^2/16k$, where m_H - weight of atom of hydrogen, molecular weight $\mu \approx 1.3$ in areas HI and $\mu \approx 0.7$ in zones HII. It has been note above, that a branch of the decision after a maximum of a curve in a fig. 2, is unstable, probably. Even considering,

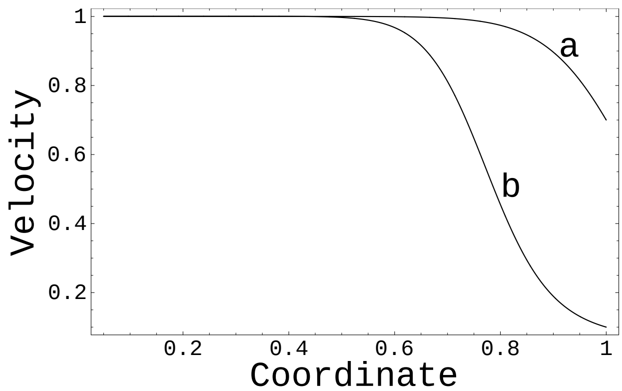

Fig. 1. The dependence of plasma velocity ($u(r)/u_1$) on coordinate (r/r_0).

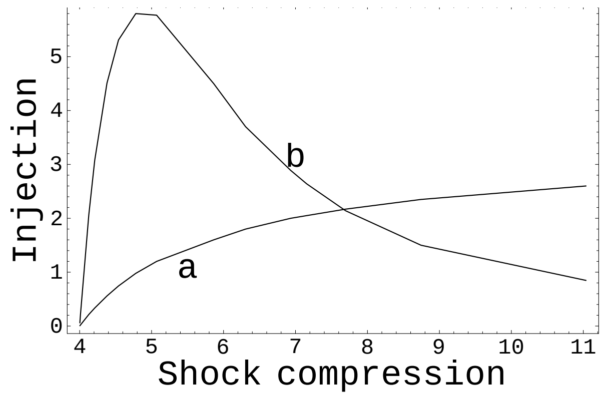

Fig. 2. The dependence of injection rate ($Q_0/(10^{-4}n_g u_1)$, n_g-concentration of thermal particles) on shock compression ($\sigma = u_1/u_2$).

that the greatest compression with the account the accelerated particles, can not exceed the meaning, which is achieved in a maximum of a curve in a fig. 2, velocity under action of accelerated components can be reduced by $\approx 15\%$. It means, that even at rather low of injection rates $\sim 10^{-5} \div 10^{-4}$, the decrease of temperature behind the front by $\approx 30\%$ is possible. It in turn can result in change of a spectrum of thermal plasma radiation. Except thermal radiation, the process of particles acceleration results the mechanism of radiation, in x-ray range. The question about the radiation in similar objects deserves close attention, because of it will most likely be possible to judge about the acknowledgement or refutation of the given theory according to the observating data of radiation.

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