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Monte Carlo calculations of particle trajectories at superluminal shock waves

A. Meli and J. J. Quenby

Imperial College, Blackett Laboratory, Astrophysics Group, London SW7 2BW

Abstract. We calculate the spectrum shape and the energy gain per shock crossing at oblique super-luminal shock waves using the picture of shock drift mechanism. We use high gamma flows with Lorentz factors ranging 5-40 which are relevant to Active Galactic Nuclei ultra-relativistic shock configurations. We closely follow the particle's trajectory along the magnetic field lines, constantly measuring its phase space co-ordinates in the fluid frames where $\boldsymbol{E}$=0 and through its motion across the shock. We calculate the energy gain for a number of different inclinations and a calculation of the spectrum of the accelerated particles is given.

1 Introduction

Detection of $10^6 - 10^{20}$eV nuclei through much of the observable universe has stimulated the development of the diffusive shock acceleration model whereby particles are repeatedly accelerated in multiple crossings of a shock interface due to collisions with upstream and downstream magnetic scattering centres. The pionnering works of the late 70s (Bell, 1978a,b; Blandford & Ostriker, 1978; Krismky 1977; etc) boosted the knowledge of particle diffusive acceleration. Since then controversies exist not only refering to parallel shock configurations (where much of the work has been addressed), but mostly to oblique shocks and various parameters in relation to those. At oblique shock fronts the motion of particles is more complicated as they may either transmitted or reflected by the shock surface. Previous works have shown that in the non-relativistic case, the inclination of the field to the shock normal does not affect the spectrum if the diffusion approximation is used in conjuction with first adiabatic invariant conservation at the shock interface in the $\boldsymbol{E}$=0 frame. On the other hand Kirk & Heavens (1989) find spectral flattening at relativistic speeds in inclined shocks and an increased anisotropy both in fluid frames and the $\boldsymbol{E}$=0 frame. At oblique shocks particle acceleration occurs by means of two different mechanisms. The *diffusive acceleration* (first order Fermi acceleration) which is a mechanism relying on repeated crossings of the front by a particle (eg Bell 1978a,b) and the *shock-drift* acceleration, in which energy is gained by a particle as its gyrocenter makes a single crossing of the shock front from upstream to downstream (eg Webb et al, 1983) and by doing so it drifts in a direction parallel or antiparallel to the electric field $\boldsymbol{E}$. There is a debate regarding on which of these two mechanisms are at work in various astrophysical situations and under what circumnstances. De Hofmann and Teller (1950) showed that hydrodynamic shocks may be classified as 'subluminal' or 'superluminal' ones, being depedent on the upstream flow velocity and the orientation of the magnetic field with respect to the shock normal. The difference between these two categories is that in sub-luminal case it is possible to find a transformation to a frame of reference in which the shock front is stationary and the electric field is zero $\boldsymbol{E}$=0 in both upstream and downstream regions, the so called de Hoffmann-Teller frame. On the other hand though superluminal shocks fronts do not admit a transformation to such a frame of reference as they correspond to shock fronts in which the point of intersection of the front with a magnetic field line moves with a speed greater than c. Super-luminal shocks admit a Lorentz transformation into a frame of reference in which the magnetic field is perpendicular to the shock normal and the shock is at rest. It is most likely that the diffusive shock acceleration in super-luminal shock configurations cannot take place (eg Axford, 1981). On the other hand though there are indications showing that strong scattering across the field lines, or non-negligible fluctuations of the direction of the magnetic field at the shock, could mean that difussive accelration is still at work (eg Jokipii,1982). As the upstream flow velocity increases, the range of the field angles for which the shock is subluminal decreases. In our investigation the velocity of the join point of the magnetic field with the shock surface is greater than the c for angles greater than $45°$ in the shock frame: $V_1 tan\psi > c$, taking under consideration that in highly relativistic situations we have always

Correspondence to: a.meli@ic.ac.uk

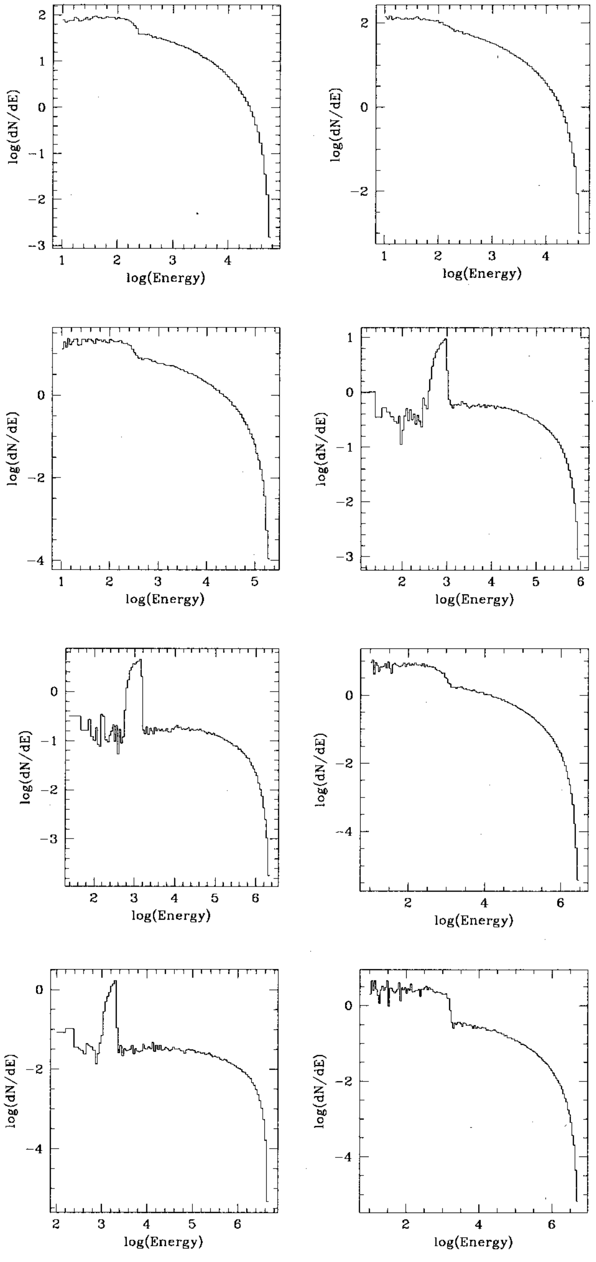
$0.98 \leq V_1 \leq 0.9997$ which correspond to Lorentz factors that of 5-40. In this investigation of shock drift acceleration in higly relativistic shock fronts, we use a simple picture of following the particle's trajectory in the respective fluid frames where the $\boldsymbol{E}$ is still zero having as a main aim to investigate the particle's spectral shape (measured in the shock frame) and the energy gain of the particle in every shock crossing. The physical picture of such a situation refers to the astrophysical environments of Active Galactic Nuclei Jet shock formations given the fact that the shock-magnetic field geometry is likely to be quasi-perpendicular, and the shock is sweeping along magnetic field lines at speeds in excess of the speed of light c. Thus, our results are applied to such objects where relaticistic velocities of Lorentz gammas up to 40 are observed or thought to occur.

2 The Numerical Method

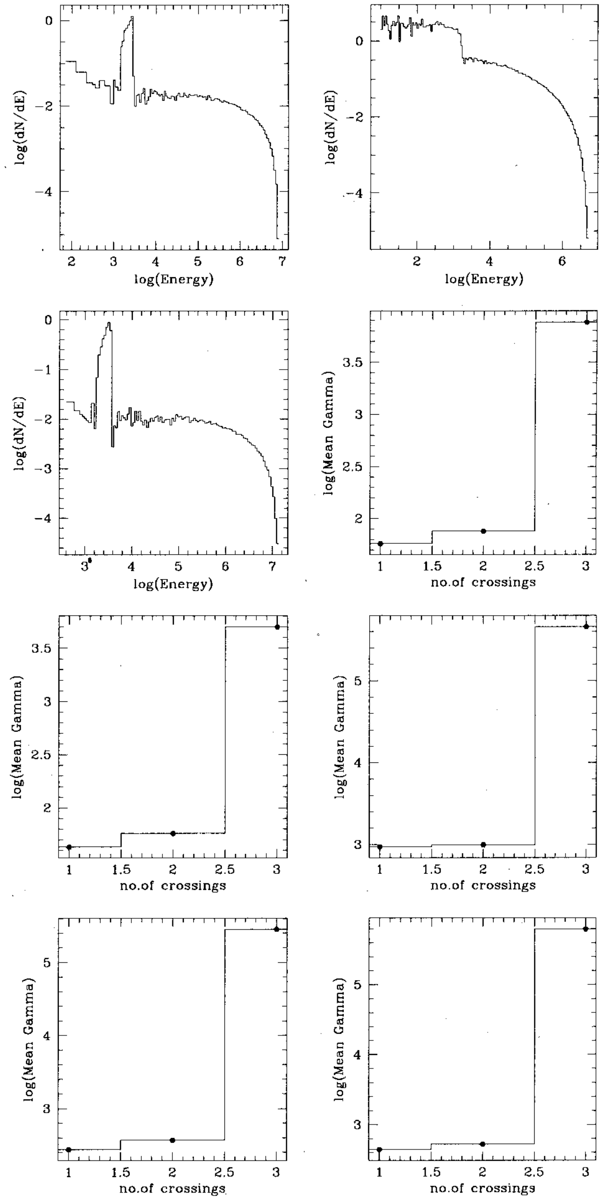
For this kind of investigation we employ a Monte Carlo scheme. We consider the motion of a particle of momentum p in the magnetic field B. In the super-luminal situation it is not possible to transform to a single frame where $\boldsymbol{E}$=0 (de Hoffmann-Teller frame). So for our investigation the most convenient frames of reference to use are the fluid frames (where still the electric field is zero), and the shock frame which we will use only as a frame of 'coordination' as we follow along the particles's helix-trajectory throughout the simulation. We use $\sim 8 \times 10^5$ particles in order to keep reasonable statistics throughout the simulations and balance the time needed for the codes to be run as they are very CPU consuming. We consider a large angle scatterring which is calculated in the respective fluid rest frames. Initially we inject the particles 100λ from the shock and we follow their guiding center in the upstream frame according to a mean free path, $\lambda_{||} = \lambda_o p cos\theta$, where θ is pitch angle, until -after the suitable transformation to the shock frame- the particle reaches the shock at x_{sh}=0. At that point we change the 'picture' to a helix trajectory -in the fluid frame- where velocity coordinates of the particle are calulated in a three dimensional space such that: $u_{x_1} = u_1 cos\theta_1 cos\psi_1 - u_1 sin\theta_1 cos\phi_1 sin\psi_1$, $u_{y_1} = u_1 cos\theta_1 cos\psi_1 + u_1 sin\theta_1 cos\phi_1 sin\psi_1$ and $u_{z_1} = -u_1 sin\theta_1$ $sin\phi_1$ where θ_1 is the pitch angle ψ_1 is the angle between the magnetic field and the shock normal. We follow the trajectory in time using $\phi_1 = \phi_o + \omega t$ where $\phi_1 \in (0, 2\pi)$ where t is time from detecting shock presence at x_{sh}, y_{sh}, z_{sh} by using $dx = x_{sh} + u_{x_1} t$, $dy = y_{sh} + u_{y_1} t$, $dz = z_{sh} + u_{z_1} t$ and by assuming that $t = R/Hc$, where R is larmor radius, $H \geq 100$ and c is the speed of light where in our investigation equals m_o=1. The particle's gyroradius ω is given by the relation $\omega_1 = eB_1/\gamma_1$. B_1 is the magnetic field, γ_1 is the particle's gamma and e is the particle's charge, in gaussian units. For a matter of convenience though, the last relation is transformed in units of c/sec. We note here that '1' refers to the upstream fluid frame while '2' to the downstream fluid frame. All the above relations apply to the downstream case by only changing respectively the signs from 1 to 2. All the calculations are performed in the upstream or downstream frames. Because of the peculiar properties of the helix we need to establish where a particle -in the upsteam frame- of a particular θ, ϕ *first* encounters the shock. When this happens, we choose to go back a whole period: $T_1 = 2\pi/\omega_1$ by reversing signs of the helix velocity coordinates and keep checking troughout the simulation to see if the shock encounters the shock again. The starting point for transform to the downstream frame is the *furthest* upstream shock crossing. By making the suitable relativistic tranformations to the downstream fluid frame by calculating the (x_2, y_2, z_2, t_2) and $(u_{x_2}, u_{y_2}, u_{z_2})$ coordinates, the momentum p_2 and the gamma γ_2 of the particle we follow the trajectory of the particle for a whole period $T_2 = 2\pi/\omega_2$ by checking to see whether the particle meets the shock again, by transforming to the shock frame. If the particle meets the shock than the suitable transformations to the upstream frame are made and we follow the particle's trajectory as described above. If the particle never meets the shock, than its guiding center is followed, the same way we mentioned earlier at the upstream side right after the injection, and it is left to leave the system if it reaches a well defined E_{max} momentum boundary or, a spatial boundary that of 100λ or so.

3 Results

In this section we present the results of our Monte Carlo simulations for $\psi_1 = 48°, 75°, 89°$, for large angle scattering and gamma lorentz factors ranging from 5 to 40 and for compression ratio equal to 4. Respective plots in figure 1 show the spectral shape of the accelerated particles at the shock frame for $\psi_1 = 48°, 75^o, 89^o$ and gamma Lorentz factor ranging from 10 up to 40. As the trajectory of the particle is followed along the simulation we add the corresponding momentum of the particle in the corresponding bin each time the particle crosses the shock. Figure 2 shows the energy gain of the particle versus the no. of the helix-trajectory shock crossings for the same gamma Lorentz factor and for $\psi_1 = 75°, 89°$ between the magnetic field and the shock normal. We observe that the spectral shape falls away from power law and descends rapidly to an upper cut off. The energy gain figures show that the particle in every trajectory-shock crossing gains a considerable amount of energy. As we may see particles with energy gain significantly greater than γ^2 obviously have crossed the shock more than once. Our caluclations show that $\sim 60\%$ of the particles cross the shock three times (downstream-upstream-downstream). All figures, those showing the spectral shape as well those showing the energy gain per trajectory-shock crossing do not give us a definite indication showing whether only 'drift' acceleration mechanism could actually take place in highly relativistic astrophysical environments, such as Active Galactic Nuclei Jets where Lorentz gamma flows of the order > 5. High energy particles, due to high relativistic flows, cross the shock only few times and are swept away by the flow they do not have the opportunity to gain as large amount of energy as the diffusive acceleration proves to do (see Meli & Quenby, 2001). Nevertheless, because the 'drift' acceleration mechanism is controversial as far as its *exclusivity* is concerned, detailed investigation of the mechanism could help to clarify the situations where this mechanism may be actually at work and what its consequences in relation to different astrophysical situations. Such a work is under way.



. **Fig. 1.** In the two top plots we observe the spectral steeppening for upstream gamma flow equal to 10 and $\psi_1 = 48°$ and $75°$ respectively. The next two figures show the spectral shape for upstream gamma=15,20 and $\psi_1 = 75°$ respectively. The next two show a similar spectral shape for gamma=25,30 and $\psi_1 = 75°$, where we can observe that near the injection there is a distinctive structure of the spectrum shape. The last two figures are for gamma=30,35; $\psi_1 = 89°$ and $\psi_1 = 75°$ respectively.



. **Fig. 2.** Two top plots for upstream gamma flow equal to 35 and 40 for $\psi_1 = 89°$ and $75°$ respectively. The next two show (left) the spectral shape for gamma=40 and $\psi_1 = 89°$, (right) the energy gain of the particle versus the number of the trajectory-shock crossings for gamma=10 and $\psi_1 = 89°$. The next two show the energy gain versus the number of the trajectory-shock crossings for gamma=10,25 and $\psi_1 = 75°$ respectively, while the last two show the same but for upstream gamma flow equal to 30 and 40 and $\psi_1 = 75°$.

4 Conclusions

We have presented Monte Carlo simulations for highly relativistic oblique super-luminal shock acceleration, where a transformation to a frame of reference where $\boldsymbol{E}$=0 cannot apply. For that single reason we have explicitly followed the particles helix-trajectory in upstream and downstream fluid frames where $\boldsymbol{E}$ is still zero and by constantly checking its position in the shock frame. We have presented results showing the spectral shape for a variety of shock inclinations, for different Lorentz gamma flows and compression ratios. We note that our Monte Carlo simulations of 'drift' acceleration expect to apply in highly relativistic astrophysical environments where gamma flow $> 5 - 40$ given that the shock-magnetic field geometry is likely to be quasi-perpendicular, and the shock is sweeping along magnetic field lines at speeds greater than the speed of light. First results indicate that the particle gains a considerable amount of energy versus the trajectory-shock crossing. The spectral shape has a clear smooth shape following a steepening power law. It seems though that the particles do not have the opportunity to cross the shock many times as they are swept away by the relativistic flow thus confining their energy gain to smaller amounts in comparison with the diffusive acceleration mechanism (Meli and Quenby, 2000; Meli and Quenby, 2001). There is under way a detailed investigation of the particles trajectory using pitch angle diffusion and higher Lorentz gamma factors.

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