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Pion yields and the nature of kaon-pion ratios in high energy nucleus-nucleus collisions: Models versus measurements

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Abstract. The pion densities and the nature of kaon-pion ratios offer two very prominent and crucial physical observables on which sufficient data for heavy nucleus collisions, to date, are available. In the light of two models – one purely phenomenological and the other with a sound dynamical basis – we would try to examine here the state of agreement between calculations and experimental results obtainable from the past and the latest measurements. Impact and implications of all these would also finally be spelt out.

1 Introduction

Multiparticle production in high energy nuclear collisions is still a complete mystry, in so far as the understanding of the dynamics of production of secondaries, especially of the soft varieties, is concerned. Of the various types of particles produced, mesons, especially the pi-mesons, constitute, in all practical terms, the near totality of the secondaries. We would concentrate here only on two important production characteristics of pi-mesons(pions) and K-mesons(kaons) in some nucleus-nucleus collisions. Kaons are also very important because of their strangeness content and this is related with the physics of the postulated QGP signatures (Kapusta, 2001; Vance, 2001). And kaons are the lightest variety of the measurable strange particles. Secondly, kaon production is considered to have a bearing or reflection on the nuclear equation of state(Sturm et al., 2001). In fact, our interest to take up these twin problems was further kindled and intensified by a very recent study of inclusive production of particles in nucleus-nucleus collisions by Kahana and Kahana(Kahana and Kahana, 2001).

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2 The Approaches

In the present work, we will make use of two kinds of theoretical models –one with a sound dynamical basis and the two others of purely phenomenological character – to interpret some of the latest observables which was observed, measured and reported by various groups like NA49, NA35, E802 collaboration etc. (Bormann et al.,1997; Sikler et al. 1999; Alber et al., 1996; Ahle et al.,1996) in the recent past.

2.1 The Sequential Chain Model (SCM)

Here, a particular version of Sequential Chain Model (SCM) (Bhattacharyya, 1988) is used. In this model, the expressions are derived on the basis of the field theoretic considerations for the inclusive cross sections and average multiplicities for various types of secondary pions (of any variety), kaons (of each type) and antiprotons. The average multiplicities for various types of pions and kaons in the reaction of type $p + p \rightarrow c + x$ are given by the following set of relations

$$< n_{\pi^+} >_{pp} \;\cong\; < n_{\pi^-} >_{pp} \;\cong < n_{\pi^0} >_{pp} \;\cong\; 1.1 s^{1/5} \,, \quad (1)$$

$$\begin{aligned} < n_{K^+} >_{pp} \;&\cong < n_{K^-} >_{pp} \;\cong\; < n_{K^0} >_{pp} \\ &\cong\; < n_{\bar{K^0}} >_{pp} \;\cong\; 5 \times 10^{-2}\, s^{1/4} \,. \end{aligned} \quad (2)$$

For any variety of secondary pions (π^+, π^- or π^0) the expression for the inclusive cross section is

$$\begin{aligned} E\frac{d^3\sigma}{dp^3}|_{pp\rightarrow\pi^- x} \;&\cong\; C_{\pi^-}\, exp(\frac{-26.88\, p_T^2}{< n_{\pi^-} >\; (1-x)}) \\ &\times\; exp(-2.38\; < n_{\pi^-} >\; x) \,, \end{aligned} \quad (3)$$

where $|C_\pi| \cong 90$ for ISR energy region,but for $p\bar{p}$ collider energy it will increase and it is different for different energy region.

For high energies and low p_T

$$E\frac{d^3\sigma}{dp^3}|_{pp\to\pi^- x} \cong E\frac{d^3\sigma}{dp^3}|_{pp\to\pi^+ x} \cong E\frac{d^3\sigma}{dp^3}|_{pp\to\pi^0 x} \,. \quad (4)$$

Similarly, for kaons of any specific variety (K^+, K^-, K^0 or $\bar{K^0}$) We have

$$E\frac{d^3\sigma}{dp^3}|_{pp\to K^- x} \;\cong\; C_{K^-}\; exp(\frac{-\,1.329\, p_T^2}{< n_{K^-} >^{3/2}}) \times\; exp(-\,6.55\; < n_{K^-} >\; x)\,, \quad (5)$$

with $|C_{K^-}| \cong 11.22$ for ISR energies.
And at low p_T

$$E\frac{d^3\sigma}{dp^3}|_{pp\to K^- x} \;\cong\; E\frac{d^3\sigma}{dp^3}|_{pp\to K^+ x} \;\cong\; E\frac{d^3\sigma}{dp^3}|_{pp\to K^0 x \text{ or } \bar{K^0} x} \,. \quad (6)$$

The term, rapidity distribution, plays another key role to address some other salient features of the reaction dynamics and the properties of the production of the secondary particles. The rapidity distribution for the secondaries in $p+p \to c+x$ types of reactions is derived from the inclusive production cross sections. For the pions it would be

$$\frac{dN}{dy}|_{pp\to\pi x} = 50.375 s^{\frac{1}{5}} \exp\left(-5.28\; s^{-0.3}\; m_T\;\sinh y_{cm}\right) \quad (7)$$

We are using here a standard connector prescribed by Wong (Wong, 1994)to switch over from nucleon – nucleon to nucleus – nucleus collisions.The method is shown in the Appendix.

2.2 Hadron / Nucleus – Nucleus collisions: A New Phenomenological Model

Very recently we(Bhattacharyya and De, 2001) have checked and forwarded a phenomenological fit to the rapidity density for the production of pions in a host of nucleus – nucleus collisions at high energies in the following manner:

$$\frac{dN}{dy}|_{AB} \;=\; (A.B)^{f(y)}\;\frac{dN}{dy}|_{pp} \quad (8)$$

where $f(y) = \alpha + \beta y + \gamma y^2$ with some reaction- specific values for α, β, γ which we will not present here in any detail.And the chosen form of $\frac{dN}{dy}|_{pp}$ in expression (4) above is given by

$$\frac{dN}{dy}|_{pp} \;=\; C[1 \;+\; \exp(\frac{y-y_0}{\Delta})\;]^{-1} \quad (9)$$

where the letters (symbols) have their contextual significance and meaning; and the values of them for calculational purposes were obtained from Thome et al. (Thome et al.,1977). The name given here to the model in expression (8) is Bhattacharyya – De Model (BDM).

2.3 The HSD Approach

In the HSD approach(Geiss et al., 1998) the high energy inelastic hadron – hadron collisions are described by the Fritiof model, where two incoming hadrons do emerge from the reaction as two excited colour – singlet states, i.e. strings. The energy and momentum transfer in this model are assumed to happen instantaneously at the collision time. With the phenomenological description of the soft processes, the global properties of heavy ion collisions could be described satisfactorily, as stated by Geiss et al.The baryon – baryon collisions are described using the explicit cross sections given in the work of Geiss et al. in the for invariant energies, $s^{1/2} < 2.65$. Some additional channels involving resonance production are to be included for cases of energies, $s^{1/2} < 2.65$.

3 The Results

With these theoretical backgrounds, we now test our models in the light of the some experimental results as reported by various groups.

3.1 Rapidity Densities of Secondary Pions

Here, a tabular picture for the experimental results vs. theoretical predictions are presented.

Systems	$\frac{dN}{dy}\|^{\pi}(data)$	$\frac{dN}{dy}\|^{\pi}_{SCM}$	$\frac{dN}{dy}\|^{\pi}_{BDM}$
p+S	1.3 ± 0.2	1.29	1.40
p+Au	1.6 ± 0.1	1.91	2.32
S+S	25 ± 1	31	25.75
S+Ag	40 ± 2	44	46.62
S+Au	47 ± 5	45	44.84
Pb+Pb	150 ± 1	150.6	156

TABLE-1:Rapidity Distribution for Pions for Various Interactions in $3 < y < 4$. First Five Sets of Data Are Taken From NA35 Collaboration (Alber et al., 1996) and the last one are from NA49 Collaboration (Sikler et al.,1999). Theoretical Fits are taken from the SCM and the BDM.
The fig.1 presents the rapidity distributions for the cases of Pb+Pb and S+S done theoretically by the BDM against the experimental results.

3.2 Comparison of K/π Ratios: Data vs. Models

In this subsection we would like to present the calculated remarks in two tabular forms.The theoretical outputs are taken from two models: one set is based on the SCM, and the other from a hydrodynamical one, named as HSD model by Geiss et al. (Geiss et al., 1998). We made an extensive study on the nature of K/π ratios in a previous work(Bhattacharyya et al., 2000).This apart, the behaviour of the ratios of K/πin several nucleus – nucleus collisions are given in figures 2 to 4. The relation for this figures would be(Bhattacharyya et al., 2000)

$$\frac{K}{\pi} \;=\; 4.5\times 10^{-2}\;(AB)^{\alpha}\;(\sqrt{s})^{0.1}\,. \quad (10)$$

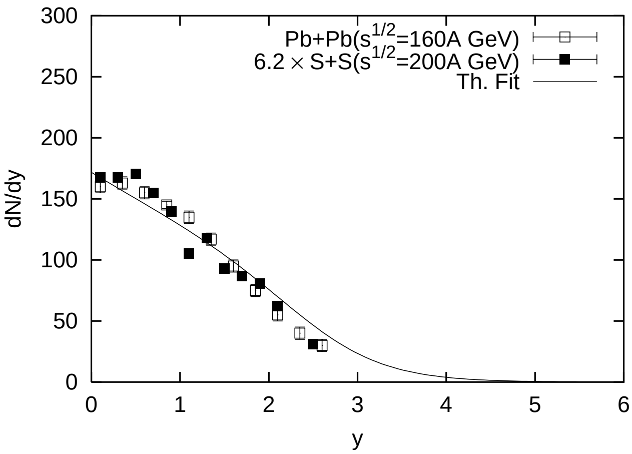

Fig. 1. Presentation of the plot of the rapidity Distribution for Pb+Pb and S+S collisions for various values of y, the rapidity variable . The Solid Line is based on the Theoretical BDM against the obtained data sets (Bachler et al., 1999; Geiss et al., 1998).

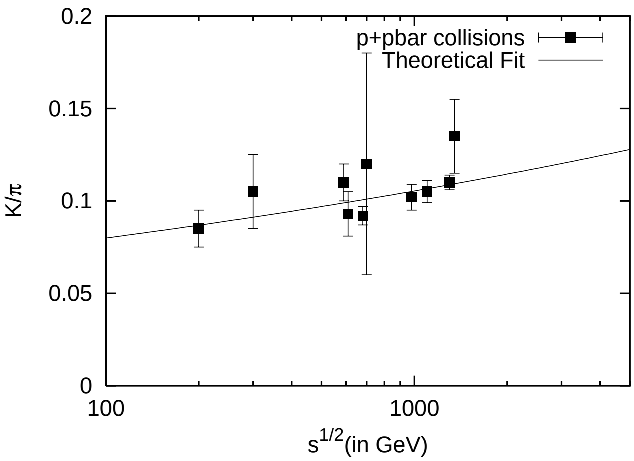

Fig. 2. Presentation of the plot of K/π ratio at different center of mass energies for $p + \bar{p}$ collisions; The solid line indicates the SCM-based results against the data-sets taken from Bocquet et al.(Bocquet et al.,1996)

α depends on the nature of the system.

The figure 2 given here shows strikingly the nature of rise of K/π ratio for $p + \bar{p}$ scattering at high energies.

Systems	$\frac{<K>}{<\pi>}(data)$	$\frac{<K>}{<\pi>}\|_{SCM}$	$\frac{<K>}{<\pi>}\|_{HSD}$
p+p	0.08 ± 0.02	0.06	0.08
S+S	0.15 ± 0.015	0.10	0.139
S+Au	0.13 ± 0.015	0.14	0.132
Pb+Pb	0.14 ± 0.02	0.11	0.15

TABLE 2.1: Strangeness at SPS Energies. The $< K >$ $/ < \pi >$ Ratios Obtained by the Theoretical SCM and HSD Approaches Are Compared with the Corresponding Experimental Ratios Taken From NA49 Collaboration (Bormann et al., 1997).

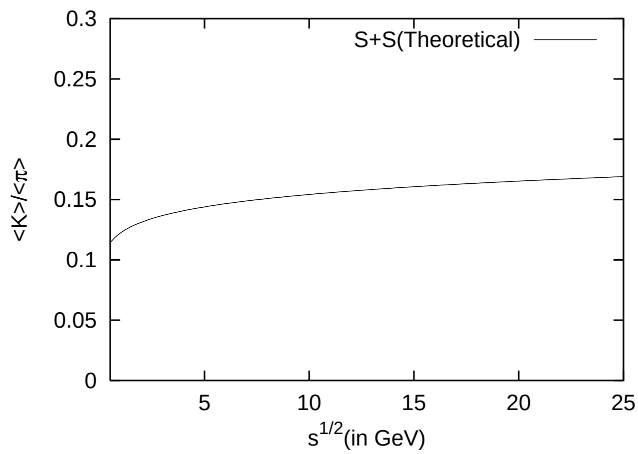

Fig. 3. Nature of very slow rise of secondary K/π in the S+S collisions as predicted by the SCM with c.m. energy

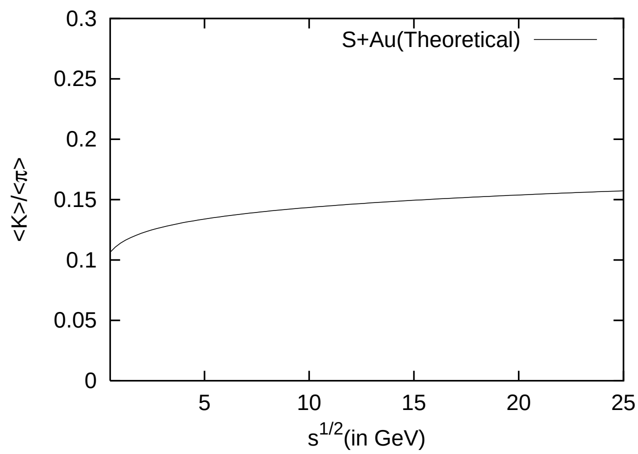

Fig. 4. The SCM -based predicted nature of the k/π for S+Au collisions with c.m. energies.

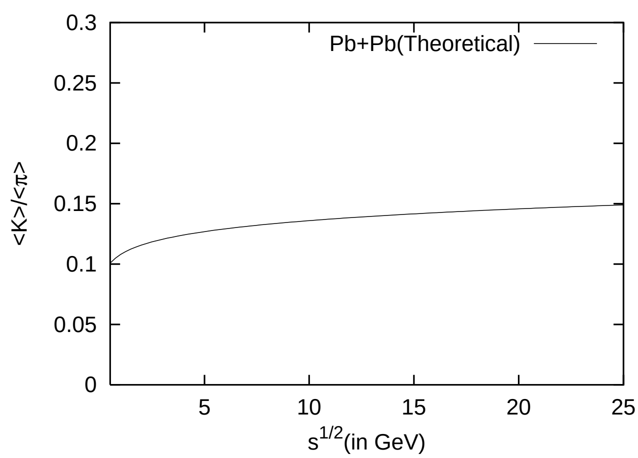

Fig. 5. The theoretical prediction in the light of the SCM on the nature of K/π for Pb+Pb Collisions with c.m energies.

Systems	$\frac{<K>}{<\pi>}(data)$	$\frac{<K>}{<\pi>}\vert_{SCM}$	$\frac{<K>}{<\pi}\vert_{HSD}$
p+BE	0.059 $\pm$ 0.01	0.64	0.59
Si+Al	0.12 $\pm$ 0.01	0.10	0.071
Si+Au	0.17 $\pm$ 0.02	0.13	0.084
Au+Au	0.18 $\pm$ 0.01	0.097	0.095

TABLE 2.2: Stangeness at AGS Energies. The $< K > / < \pi >$ yields for different systems at AGS Energies Obtained Within the SCM and HSD approaches Are in Comparison With the Experimental Data Taken From E802 Collaboration (Ahle et al., 1996).

4 Concluding Remarks

The results depicted in the above Tables (Table 1,Table 2.1 and Table 2.2) and the figures (figures 1-5) reflect and reveal the undernoted realities in our observations made here: (i) In explaining the rapidity density for production of pions, the majority of the produced secondaries, the SCM works quite agreeably with data; and, in our rating, it performed better than the BDM, a purely phenomenological model so far. (ii) The agreement between measured data on K/π ratio in $p+\bar{p}$ reactions and the theoretical plot(shown in fig.2.) obtained on the basis of expression(10)here[with A=B=1 for $p+\bar{p}$ collisions] is strikingly encouraging for the future prospects of SCM. (iii) But, the agreement between the data on kaon-pion ratios and both the SCM and HSD models at CERN-SPS are just modestly satisfactory. However, in explaining this latter set of data, the SCM is visibly far more successful than the HSD model which is also just a phenomenological approach. So, the present work essentially boils down to the affirmation of the triumph of the chosen dynamical model, the SCM.(iv) The predictive graphs in Fig.3-5 could vindicate this point in future.

But a point must be made here. True, on the behaviour of K/π ratios we did not differentiate here precisely between what is known as "global(g) kaon-to-pion($R_g =< K > / < \pi >$) ratio" and what goes by the name of "event-by-event"(ev) ratio of kaon to pion, represented symbolically by $R_{ev} = K/\pi$. According to the philosophy of event-by-event analysis(Yang and Cai, 2001), it is correct that the conditions to produce conjectured QGP might be reached in every event; but the fact that a phase transition is a critical phenomenon implies immediately that it may occur in a small sub-sample of events. The fluctuations in such events would be averaged out in the conventional ensemble analyses. In contrast, the event-by-event analysis which became possible only with the advent of the large acceptance detectors(LADs), helps us to find or sort out the very interesting or anomalous event candidates with some specific dynamical properties. So, this event-by-event method could provide dynamical information which is not available from the analysis of traditional inclusive spectra or just average multiplicity.

The limitations of the present work are, thus, as follows: (a) We have not reckoned here with either the statistical or the dynamical fluctuations aspects. (b) There is no correlation of any kind between the two types of particles viz., pi-mesons and K-mesons. (c) So, on extrapolation of (a) and (b) here, one has to state that this study fails to throw light on the questions posed by Yang and Cai(Yang and Cai, 2001).

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Appendix A The Approximate Estimation of dN/dy in Central Rapidity Region for Nucleus-Nucleus Collisions

We proceed in the route built up by Wong(Wong,1994) to bridge the gap between nucleon-nucleon and nucleus-nucleus collisions.

Let us consider nuclear reaction reaction $A + B \rightarrow C + X$, where A and B are projectile and target nucleus respectively. For two unequal nucleii, the relationship between the rapidity distributions for nucleus-nucleus and nucleon-nucleon collisions would be

$$\frac{dN}{dy}|_{AB} \simeq 1.28 \frac{AB}{A^{2/3} + B^{2/3}} \frac{1}{1 + a(A^{1/3} + B^{1/3})} \times \exp(-\frac{b^2}{2\beta^2}) \frac{dN}{dy}|_{pp} \quad \text{(A1)}$$

where a is the parameter that is to be chosen and b is the impact parameter. The term β^2 satisfies the following relation; $\beta^2 = \beta_A^2 + \beta_B^2 + \beta_p^2$. Here, $\beta_A = r_0' A^{1/3}/\sqrt{3}$ with $r_0' = 1.05 fm$, and β_p,the thickness function parameter for nucleon – nucleon collisions, is 0.68 fm.

For two equal nuclei, the relation would then become

$$\frac{dN}{dy}|_{AB} \simeq 0.64 A^{\frac{4}{3}} \frac{1}{1 + 2aA^{\frac{1}{3}}} \exp(-\frac{b^2}{2\beta^2}) \frac{dN}{dy}|_{pp} . \quad \text{(A2)}$$

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