Proceedings of ICRC 2001: 483 © Copernicus Gesellschaft 2001

ICRC 2001

Appoximation of individual cascades with energies above the GZK cut-off

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Abstract. Approximation of individual cascades of giant energies is an essential part of energy determination methods for primary particles. Fluctuations of the cascades and their distribution shapes were investigated based on a sufficient number of MC simulated showers, as dependent on primary energy, zenith angle and registration method. Individual cascades were approximated with high accuracy using some special parameters. It enables creating formulae convenient for both processing experimental data and analytical investigation. The latter allows to obtain some physically interpreted characteristics. Quality of this approximation and applicability of the traditional approximation of mean cascades for estimation of individual giant cascades are discussed.

1.Introduction

To study showers of energy $> 5.0 \cdot 10^9$ Gev as physical phenomenon, L.G.Dedenko and G.F.Fedorova have calculated (Antonov et al., 2001) the showers generated by 10÷ 1000 EeV protons.The calculation has been performed for zenith angles of θ =0°, 24° 36′, 44° 24′, 60° in terms of the KGS model taking into account neutral pion interaction in the atmosphere. The present report deals with the approximation of the number of electrons $N(t)$ at depth t. The method and conclusions are spread over the cases of taking into account all charged particles and over the cases of detection of these particle density at the usual scale of distances from the shower core.

The simulation results (taking into account the LPM-effect) showed that 1. Fluctuations of cascades in the vicinity of maximum ($t_m \pm \frac{1}{4}t_m$) are small ($Var_N(t) = \sigma / < N(t) > \leq 1\%, \sigma$ is the standard deviation, <> being the mean) due to the large number of particles ($\sim 10^{10}$) in a shower. At large energies (1000 EeV) and small angles this region covers almost the entire depth of the atmosphere. 2. The greater the distance to the maximum, the greater the fluctuations, and at $t > 2t_m Var N(t) > 100\%$, that is clearly manifested in showers of less energies and greater angles. 3. There are present showers significantly (more than by 10σ) deviating from the means due to features of the nuclear cascade. These deviations are of complex character and cannot be reduced to fluctuations of the first interaction depth or to fluctuations of the depth or value of maximum. These showers are small in number ($\sim 1\%$), but they are no question present and revealed when the statistics is rather large. The existence of these peculiar showers cannot be explained by the drawbacks of the model, probably vice verse. Therefore a large variety of experimentally detected giant showers can be expected.

This behavior of fluctuations (in particular, their depth dependence) is known and in the case of electron-photon cascade of energy $E \sim 1 - 1000$ TeV are studied in detail (Kirillov et al., 1983; Ivanenko et al., 1988). The fluctuations themselves are thought to be deviation of individual cascade from some mean shower. It is significant to note that the same behavior is observed for individual cascade, if the fluctuation is thought to be a deviation from its smoothed value, i.e. a local spread of the value. This fact means that, if the object is identified by a uniformly smoothed function and an accuracy is thought to be a sum of absolute errors, then an accuracy of this object description is a function of the measure of its variability. Thus, the individual cascade description should be very accurate in the maximum region, being sufficiently accurate as the distance from the maximum increases.

Since each giant shower is a rare valuable event, an apparatus is necessary to describe (measure) these showers to an accuracy enabling distinguishing between them and efficient over a rather wide range, i.e. individual cascades should be approximated with an accuracy of $< 1\%$ at $\frac{1}{2}t_m < t < \frac{3}{2}t_m$ and $< 10\%$ at $t > 2t_m$.

2. The Approximation Function Form

The above noted fluctuations leads to smooth bell-like right-asymmetric functions with a smooth region of the maximum (see examples below in fig.3,4,5). The known formula of T.K.Gaisser, A.M.Hillas (1977) obtained for the mean number of particles $N(E,t)$ in a vertical (θ = 0) shower generated by a proton of energy $E \geq 10^{15}$ eV at the depth t_0 is

$$N(E,t') = N_0(E/c)exp(t'_m) \left(\frac{t'}{t'_m}\right)^{t'_m} e^{-t'},$$

t' is measured in units $\lambda = 70$ g/cm^2, the depth of max-

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imum being

$$tm' = 0.51ln(E_0/c) - 1 \,\text{and}\, N_0 = 0.45, c = 0.074GeV,$$

considered (as in (Bird et al., 1995)) to comprise 3 free parameters (N_m, t_m, t_0) enable in many cases description of a cascade in the region $t_m \pm \frac{1}{2}t_m$ with a relative error less than 10% . Assuming the parameter λ = 70 g/cm^2 to be free, this region can be enlarged. An accuracy of the approximation can be increased by introducing additional parameters and complicating the formula with saving the form of the product of an exponent and a power function. But the description of the entire variety of model cascades failed here. A qualitative analysis showed that the formula should be constructed based on a dependence of the form: $e^{-\frac{x^2}{ax+b}}$.

To approximate individual cascades, use was made of the formula

$$N(t) = N_m e^{-\frac{(t-t_m)^2}{a(t-t_m)+b}}, a(t-t_m)+b \geq 0, \qquad (1)$$

where N_m, t_m are the location of the maximum, a is the asymmetry parameter, b is the area parameter (interpretation of a and b will be given in the section Parameters).

This distribution is determined over the half-line, having exponential asymptotics. N_m and t_m were calculated as the location of the maximum of parabolic approximation by the method of least squares 5 points close to the cascade maximum. The ways of determination of a and b are somewhat free. The most general rule in determining their values is a linear approximation (by method of least squares) of the function:

$$D(t_i) = (t_i - t_m)^2/(ln(N(t_i)/N_m)), \qquad (2)$$

where $N(t_i)$ are the known values of the cascade being approximated. An example of this function for two values of energy and angle (E=10 EeV, θ = 60°; E=1000 EeV, θ = 0°;) is shown in Fig.1. An examination of (1,2) and Fig.1 shows that (1) describe the cascade beginning not so well, and for (2) the points about the maximum ($\sim N(t_i)/N_m$ >0.9) would be excluded.

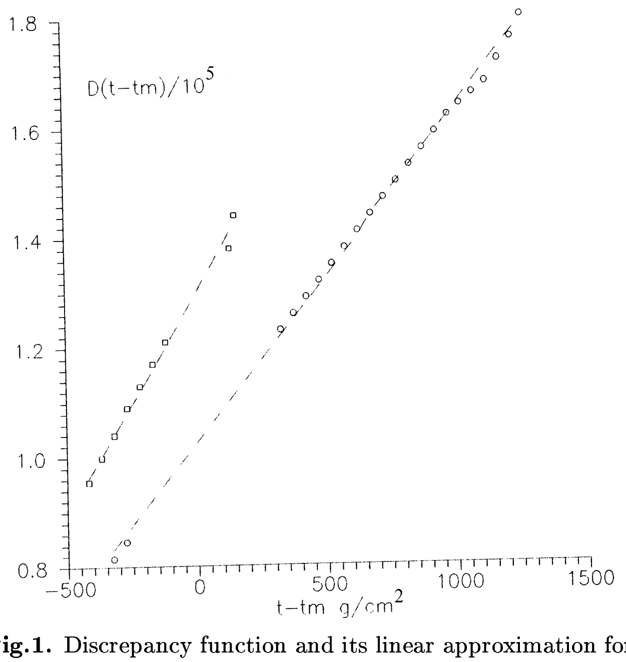

Fig.1. Discrepancy function and its linear approximation for two individual cascades, squares - E = 1000 EeV, θ =0 (the upper pair); circles - 10 EeV, θ =60° (the lower pair).

Use will be made of (1), taking in (2)

$$t > \frac{1}{2}t_m. \qquad (3)$$

3. Accuracy

Fig.2 shows the depth dependence of a relative error of the approximation for E = 10 EeV, θ = 60° as more difficult case for approximation. The solid lines are for the mean and for maximal σ - relative errors of approximation of the entire amount (500) of cascades with of this energy and angle. The dashed line is for relative errors of typical individual cascade from Fig.1.

The mean $> || <$ error is calculated as the mean of absolute values and is finally presented with a sign of usual mean value. A difference between the mean errors and the errors for typical individual cascade is due to the influence of the above noted peculiar showers. Averaging over depth in the same manner for the presented individual shower, obtain $> |\sigma| <= 2.05\%$, $|max\sigma| = 11.8\%$, for the entire set: $> |\sigma| <= 2.56\%$, $> |max\sigma| <= 12.5\%$.

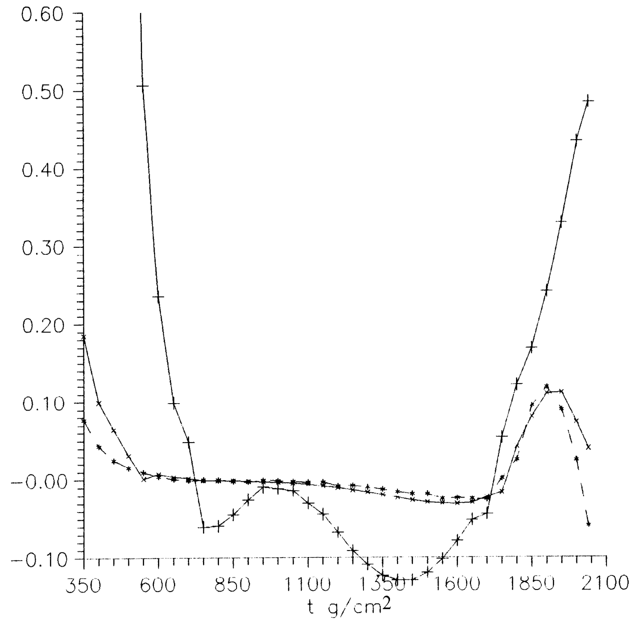

Fig.2. The depth dependence of relative error of the approximation (see notations in the text) for E= 10 EeV, θ =60°.

This figure confirms the above qualitative statement about the approximation accuracy and shows that the physically informative region $t < \frac{1}{2}t_m$ should be investigated separately.

Table 1 lists (in %) the means of a relative error $> |\sigma| <$ obtained when approximating all the simulated cascades (500 for each event)

Table 1

%	0°	24°36′	44°24′	60°
10 EeV	.25	.29	.40	2.56
100 EeV	.28	.35	.49	1.21
1000 EeV	.37	.34	.69	4.00

The mean maximal deviation is about five times greater than the corresponding $> |\sigma| <$. As a preliminary fact, it should be noted that this relative error increases with energy as the above mentioned measure of cascade variability.

4. Parameters

Formula (1) can be seen to be similar to the Gaussian distribution, coinciding with it at $a = 0$. It can be also seen that at $t < t_m$ (1) has more narrow distribution and at $t > t_m$, wider distribution than that Gaussian one. Thus, distribution (1) is the right-asymmetric, this asymmetry measure being proportional to a. A similar consideration shows $\sqrt{b}$ to be the measure of the distribution width. To determine more precisely these qualitative considerations, it should be noted that parameters a and b can be determined by requiring the approximation to coincide with the initial data at two points. In particular, if these points are taken so that $N(t_1) = N(t_2) = (1/c)N_m$, an obvious interpretation of a and b is obtained. This is especially simple at $c = e$: determinant of the obtained linear system for a and b is equal to $t_2 - t_1$, i.e. to a width of the distribution $N(t)$ at the level e, $a = (t_2 - t_m) - (t_m - t_1)$, i.e. asymmetry measure of a cascade, $b = (t_2 - t_m)(t_m - t_1)$, i.e. to a product of the left and right widths. Fig.3 showers an example of the same cascade as in figs.1 and 2 and presents the interpretation of the parameters. These parameters values are $t_m = 776$, $N_m = 72.5 \cdot 10^8$, $a = 60.6$, $b = 103 \cdot 10^3$.

The known physically interpreted and widely used characteristic - the area under the cascade curve - can be easily and rather precise obtained from (1).

Integrating (1) in t and leaving only one term in expansion of $\mathbf{K}_1(z)$, a useful relation

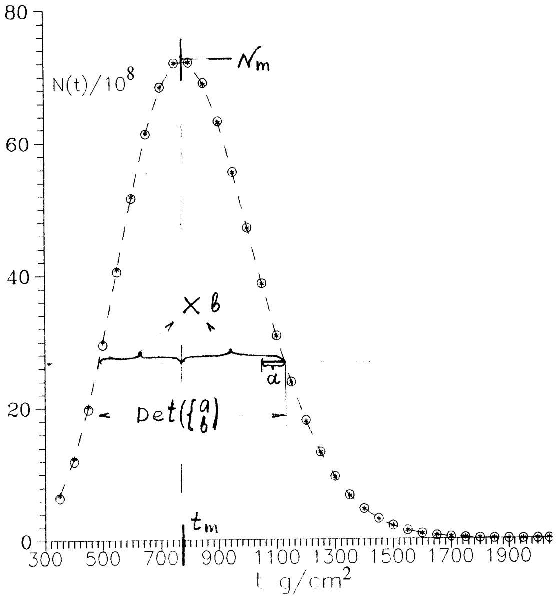

Fig.3. Approximation of individual cascade and interpretation of the approximation parameters.

$$\int_{t_m - \frac{b}{a}}^{\infty} N(t)dt \approx \sqrt{\pi} N_m \sqrt{b} \qquad (4)$$

can be obtained to an accuracy of $\sim 1\%$, which can be treated as the interpretation of the parameter b and one of the methods of obtaining its value. The direct formula for a in this case is obvious.

For this individual cascade, one can easily obtain, using (4), the mean energy lost by an electron traversing 1 g/cm^2.

$< E_e >$ MeV/(g/cm^2) $\approx E/(\sqrt{\pi} N_m \sqrt{b}) = 2.4$ MeV/g/cm^2

The above described properties and varying (1) in a and b

$$\Delta N = \frac{\partial N}{\partial a}\Delta a + \frac{\partial N}{\partial b}\Delta b =$$

$$= N_m \cdot exp\left(-\frac{x^2}{ax+b}\right) \cdot \left(\frac{x}{ax+b}\right)^2 (x\Delta a + \Delta b)$$

$$\delta N = |\frac{\Delta N}{N}| = \left(\frac{x}{ax+b}\right)^2 (xa\delta a + b\delta b) \approx 0.25\delta a + 1.0\delta b,$$

where $x = t - t_m$, showed that: 1. calculation is stable (1); 2. the parameters a and b are available to be used in the methodics of determination of energy of a primary; 3. the product $N_m\sqrt{b}$ is the most stable.

5. Examples

The direct application of the above presented approximation to experimental data on $N(t)$ of a shower detected by Fly-Eyes (Bird et al., 1995; Halzen et al., 1995) yielded: $t_m = 815$ g/cm^2, $N_m = 22.1 \cdot 10^{10}$, $a = 74.8$, $b = 124 \cdot 10^3$. When calculating $D(t_i)$ by (2) two points around the maximum were excluded. Fig.4 and table 2 are given to illustrate this.

Table 2

t_i(g/cm^2)	210	261	325	395	465	538	624	714	802	892	983	1070
$N(t_i)/10^{10}$	.230	.570	1.50	3.70	7.80	10.8	14.7	19.5	23.1	20.5	18.0	14.9
$Appr N(t_i)/10^{10}$	.210	.533	1.41	3.28	6.32	10.5	15.9	20.3	22.1	21.1	18.0	14.0

to be continued

(By the way, one of the simulated showers of energy 320 EeV and angle 44° 24′ has as its parameters those rather close to the above obtained ones, though the mean values for this energy differ from the obtained ones by somewhat 1σ.) This concrete example illustrates just the idea of the application of the method proposed. The results themselves are rather rough requiring more precise determination as the experimental errors are disregarded at all and the values of $N(t_i)$ themselves (the second line in the table) can be insufficiently correct.

The method and results were noted above to spread over the case of detection of particle density. For example Fig.5 shows the approximation of depth dependence $\rho_\mu(600)$, the muon density in an individual shower with energy of 100 eV and zenith angle of $\theta = 44°24'$. The relative error of this approximation is also given here, similar to this in Fig.2, but in per cent.

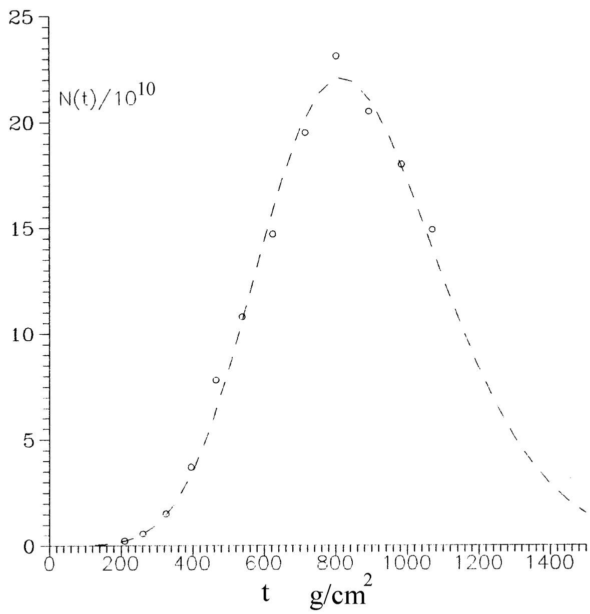

Fig.4. Approximation of the Fly's-Eye shower (October 15, 1991) disregarding errors of the initial data.

The mean relative error for the total number of model cascades of the above noted energy and angle (statistics of 500) is $> |\sigma| <= 1.25\%$ and the mean maximal deviation is $> |max\sigma| <= 5.2\%$. A comparison with Figs.2 and 3 shows the same properties of density approximation as that of the particle number.

In the case of small zenith angles the cascade variability decreases and the depth range narrows that enhances the conditions of fitting, making the approximation more accurate.

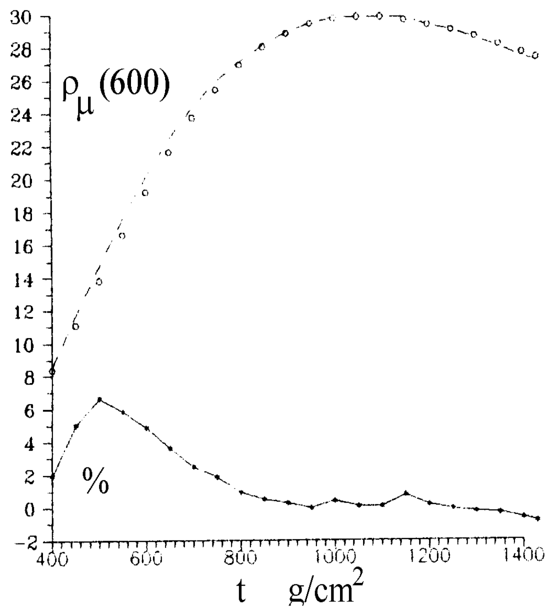

Fig.5. An example of approximation (the dashed line) of the muon density $\rho_\mu(600)$ in an individual shower (circles) and this approximation relative error (stars, the solid line), the relative error is given in per cent.

6. Summary

An analysis of the simulation results shows that the presented approximation describes the individual cascades with relative error of $\sim 0.1\%$ in the maximum region and $\sim 1-2\%$ at $t > \frac{1}{2}t_m$. The informative region $t < \frac{1}{2}t_m$ requires special study. The above consideration shows the approximation parameters to be interpreted.

The product $N_m \cdot \sqrt{b}$ is the less fluctuating and the most regular and can be considered as a candidate to be an invariant in a methodics of primary energy determination.

The proposed method of constructing the approximation is convenient for introducing (e.g. through (2)) a weigh coefficients method to take into account a spread in accuracy of the initial data.

A more precise fit can be obtained (especially in a narrower region) considering the above used parameters as independent ones, and their values as initial for the best fit.

7. Discussion

Physical results revealed with this approximation can be treated as presumable, since the model independence of these results is uncertain. However a general methodical idea can be presented. Individual cascade is usually compared with a mean one as a pattern deviations from the mean being treated as fluctuations. However, for $N(t)$ there is obvious difference between typical individual cascade and the traditional mean cascade: the curve N(t) for the individual cascade should be higher and more narrow than that for this mean cascade. Using a mean individual cascade (a cascade which parameters are the mean values of parameters of the cascades under consideration) as a pattern of a cascade for comparison means elimination of a constructive error of the pattern itself. The fluctuations (as a difference from the mean individual cascade) are just described by the used theoretical-probability distribution functions with unshifted means. It was always unexplicitely assumed that the usually used mean cascade has the same property with the some accuracy, but for giant showers their fluctuations can be shadowed by this systematic error of the mean, therefore the mean and individual cascades should be described more accurately. The curve N(t) of a mean individual cascade (as of an individual cascade in general) is higher and more narrow than that of a usual mean. These means differ in maximum by $\sim 2\%$, intersect at $\sim t_m \pm \frac{1}{4}t_m$, but at $t \sim \frac{1}{2}t_m$ and $2t_m$ differ by 10-20%, then the difference increases as t_m is being left behind. The approximation proposed enables seeing this difference and investigation of the fluctuations.

Preliminary analysis of the distribution functions of the parameters showed a narrow peak shape in the region of their means and a very wide ($\sim 10\sigma$) region of their variation. Thus, more than 90% of the cascades are described by fluctuations of a mean individual cascade though there exist some strongly deviating cases which can be of interest. Therefore it is impossible to conclude, using one parameter, about belonging of a concrete cascade to a definite class (e.g., energy range). To solve this type problems, a methodics is needed which takes into account the overall variation of the parameters describing the individual cascade more accurately than the known fluctuations which could be neglected.

Acknowledgements The authors are grateful to L.G.Dedenko for initiating of the present work and to G.F.Fedorova for performing special calculations.

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