Proceedings of ICRC 2001: 3298 © Copernicus Gesellschaft 2001

ICRC 2001

Field line separation in two-component magnetic turbulence

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Abstract. We consider the diffusive separation of magnetic field lines for 2D+slab turbulence, which is known to serve as a reasonable model of the solar wind. In our model, we assume the turbulence is homogeneous, the separation is diffusive (Gaussian), and Corrsin's independence hypothesis holds. We derive a non-perturbative formula for any distance along the mean field direction, allowing us to determine where the Gaussian approximation breaks down. If the random walk of magnetic field lines is diffusive and dominated by the 2D contribution, then field line separation occurs with twice the 2D contribution to the single field line random walk coefficient (fast diffusive separation). If, on the other hand, the slab contribution is dominant, we instead predict two regimes of field line separation: fast diffusive separation at long distances, and at shorter distances, a regime of slow diffusive separation with a rate determined by the autocorrelation of the flux function for 2D turbulence at the initial field line separation. We predict supradiffusive separation in between the two regimes. In the solar wind, it is believed that the 2D contribution dominates, leading to fast diffusive separation. This suggests that recent ACE observations of sharp boundaries between regions of high and low density of solar energetic particles will require an alternative explanation, such as non-homogeneous turbulence.

1 Introduction

Trajectories of individual magnetic field lines and the rate of separation of adjacent field lines are key issues in defining the topology and structure of random magnetic fields in magnetohydrodynamic (MHD) turbulence. In addition, the "random walk" of individual magnetic field lines is often central to understanding the diffusion of energetic particles perpendicular to the mean magnetic field in astrophysical plasmas (Jokipii, 1966; Jokipii and Parker, 1968). There are also circumstances (Jokipii, 1973) in which behavior of distributions of energetic charged particle might be better understood in terms of mutual separation of field lines than by wandering of individual field lines.

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Many issues which are typically envisioned as involving the trajectories of individual magnetic field lines are more properly viewed as determined by the "rate" of separation of adjacent magnetic field lines with distance along the mean field. For example, the long-recognized phenomenon of "channeling" or sudden changes in the fluxes of solar energetic particles (SEP), which has been revisited by recent, detailed measurements (e.g., Mazur et al., 2000), is presumably due to sudden changes in magnetic connection to a spatially localized particle source. This implies that field lines that are adjacent when near the particle source can remain confined to localized flux tubes out to distances $\sim$1 AU along the mean field.

The theory of the separation of adjacent field lines has been examined by Jokipii (1973) and Zimbardo et al. (1984). More recently, this issue has been recognized as relevant to physical processes in the solar corona (Similon and Sudan, 1989), in the heliosphere (Erdős et al., 1997, 1999), and cosmic ray transport and acceleration in the galaxy (Chandran, 2000). In the following sections we re-examine the theory of the separation of adjacent field lines in astrophysical MHD turbulence, in light of improved understanding of solar wind turbulence over the past decade (Matthaeus et al., 1990; Bieber et al., 1994). We consider field line separation in two-component turbulence consisting of a slab component that varies only along the mean field, as well as a two-dimensional component that varies only in the two transverse directions, which has been shown to serve as a good model of solar wind turbulence (Bieber et al., 1996). We proceed using a non-perturbative approach similar to that which has been used previously (Matthaeus et al., 1995; Gray et al., 1996) to examine the field line random walk.

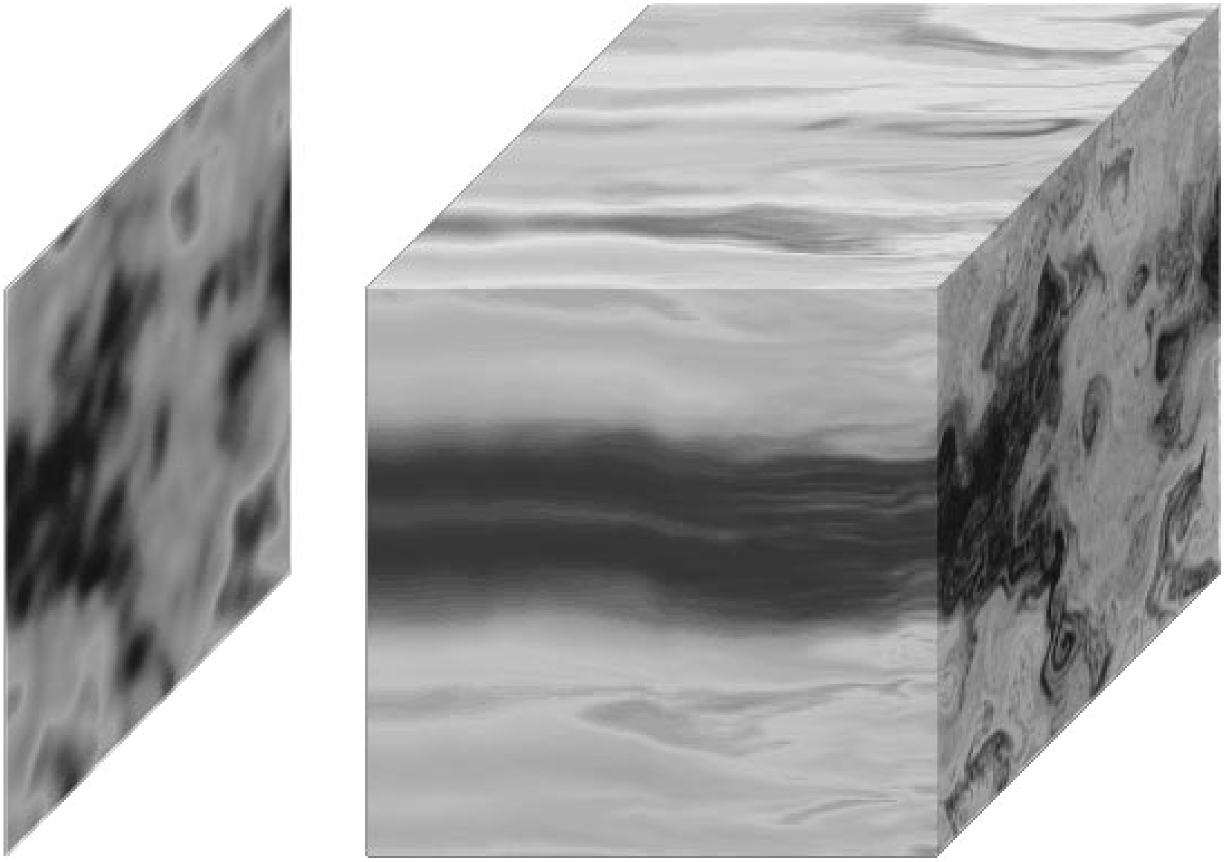

Fig. 1. Field line motion in a realization of 2D+slab turbulence. In the left part of the figure, the shading indicates the value of $a(x, y)$, the potential function for the 2D component. In the right part, that initial shading is convected along random field lines [no longer indicating $a(x, y)$]. It can be seen that some regions exhibit strong mixing, while others are still organized as distinct flux tubes.

2 Separation of Magnetic Field Lines in 2D+Slab Turbulence

Space does not permit a complete derivation here, so we stress the formulation, assumptions, and results. In the 2D + slab model of magnetic turbulence, we assume $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}(x, y, z)$, where $\boldsymbol{B}_0 = B_0\hat{z}$, $\boldsymbol{b} \perp \hat{z}$, and $\boldsymbol{b} = \boldsymbol{b}^{2D}(x, y) + \boldsymbol{b}^{slab}(z)$. In general, we can write $\boldsymbol{b}^{2D}(x, y) = \boldsymbol{\nabla}\times(a(x, y)\hat{z})$, where $a\hat{z}$ is the vector potential for the 2D component of magnetic turbulence. Figure 1 illustrates the initial distribution of $a(x, y)$ and the motion of field lines for a realization of such turbulence with an 80:20 ratio of 2D to slab component energies, as found in the solar wind (Bieber et al., 1996).

Following Jokipii and Parker (1968) and Jokipii (1973), we start the calculation with the defining equation of a magnetic field line,

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}, \tag{1}$$

and express the change in, say, the x-coordinate of a field line over a distance Δz along the mean magnetic field as

$$\Delta x \equiv x(\Delta z) - x(0) = \frac{1}{B_0}\int_0^{\Delta z} b_x[x(z'), y(z'), z']dz'. \tag{2}$$

Then $\langle\Delta x^2\rangle$ can be expressed as a double integral over z' and $z'' = z' + \Delta z'$. Next, we consider the lateral coordinates of two different field lines, $x_1(z)$ and $x_2(z)$, defining $X \equiv x_1 - x_2$ and $Y \equiv y_1 - y_2$. Without loss of generality, we consider $X(z = 0) = X_0$ and $Y(z = 0) = 0$. Various quantities are illustrated in Figure 2.

Instead of specifying the power spectra of 2D and slab components, we derive general formulae on the basis of certain assumptions. One assumption is Corrsin's independence hypothesis (Corrsin, 1959; Salu and Montgomery, 1977), in which the Lagrangian correlation function is taken to be the Eulerian correlation function, R_{xx}, weighted by the conditional probabilities of finding $\Delta x' \equiv x(z'') - x(z')$ and $\Delta y' \equiv y(z'') - y(z')$ after a given $\Delta z'$. Another assumption is that those conditional probability distributions are Gaussian, and the variances are diffusive, i.e., proportional to $|\Delta z'|$. Our answer tells us whether or not this is the case, i.e., the range of applicability of our results.

The complete expression for $\langle \Delta X^2 \rangle$ is

$$\begin{aligned}\langle \Delta X^2 \rangle &= \frac{8\Delta z^2}{\langle \Delta x^2 \rangle} \frac{1}{2\pi B_0^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_{xx}^{2D}(k_\perp)}{k_\perp^2} \\ &\quad \times \Bigg(1 - g\left(\langle \Delta x^2 \rangle k_\perp^2/2\right) \\ &\quad - e^{ik_x X_0} \Big\{ g\left(\langle \Delta X^2 \rangle k_\perp^2/2\right) \\ &\quad - \frac{1}{2} g'\left(\langle \Delta X^2 \rangle k_\perp^2/2, \langle \Delta x^2 \rangle k_\perp^2/2\right) \\ &\quad - \frac{1}{2} g\left[\left(\langle \Delta X^2 \rangle + \langle \Delta x^2 \rangle\right) k_\perp^2/2\right] \Big\} \Bigg) \\ &\quad \times dk_x dk_y, \end{aligned} \tag{3}$$

where $\langle \Delta x^2 \rangle = 2D_\perp \Delta z$ and

$$\begin{aligned} D_\perp &= \frac{1}{\sqrt{2\pi}} \frac{1}{B_0^2} \int_{-\infty}^{\infty} \frac{[1 - \cos(k_z \Delta z)]}{k_z^2 \Delta z} P_{xx}^{slab}(k_z) dk_z \\ &\quad + \frac{1}{2\pi} \frac{1}{B_0^2} \frac{1}{D_\perp} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_{xx}^{2D}(k_\perp)}{k_\perp^2} \\ &\quad \times \left[1 - g(D_\perp k_\perp^2 \Delta z)\right] dk_x dk_y. \end{aligned} \tag{4}$$

Here P_{xx}^{2D} is the Fourier transform of R_{xx}^{2D}, $g'(x, y) \equiv (e^{-x} - e^{-y})/(y - x)$ is a two-dimensional low-pass filter which approaches 1 only when $x \ll 1$ and $y \ll 1$, and $g(x) \equiv (1 - e^{-x})/x$ is a similar, one-dimensional low-pass filter. In terms of diffusion coefficients, with $D_{sep} \equiv \langle \Delta X^2 \rangle/(2\Delta z)$, we have

$$\begin{aligned} D_{sep} &= \frac{2}{D_\perp} \frac{1}{2\pi B_0^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_{xx}^{2D}(k_\perp)}{k_\perp^2} \\ &\quad \times \Bigg(1 - g\left(D_\perp k_\perp^2 \Delta z/2\right) \\ &\quad - e^{ik_x X_0} \Big\{ g\left(D_{sep} k_\perp^2 \Delta z/2\right) \\ &\quad - \frac{1}{2} g'\left(D_{sep} k_\perp^2 \Delta z/2, D_\perp k_\perp^2 \Delta z/2\right) \\ &\quad - \frac{1}{2} g\left[\left(D_{sep} + D_\perp\right) k_\perp^2 \Delta z/2\right] \Big\} \Bigg) dk_x dk_y. \end{aligned} \tag{5}$$

Equation (5) is more useful for physical interpretation, while equation (3) is useful for calculations, because it allows one to circumvent the implicit nature of the equation by expressing Δz as an explicit function of $\langle \Delta x^2 \rangle$ and $\langle \Delta X^2 \rangle$. Fortunately the low-pass filters g and g' allow an interpretation of the general behavior of D_{sep} as a function of Δz, which is confirmed by numerical evaluation of equation (3).

In the limit $\Delta z \to \infty$, all expressions with g and g' tend to zero, and we recover the simpler random walk formulae of Jokipii and Parker (1968) (for slab turbulence) and Matthaeus et al. (1995) (for composite 2D+slab turbulence),

$$D_\perp = D_\perp^{slab} + \frac{(D_\perp^{2D})^2}{D_\perp} \tag{6}$$

$$D_\perp^{slab} = \sqrt{\frac{\pi}{2}} \frac{P_{xx}^{slab}(0)}{B_0^2} \tag{7}$$

$$(D_\perp^{2D})^2 = \frac{1}{2\pi} \frac{1}{B_0^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{P_{xx}^{2D}(k_\perp)}{k_\perp^2} dk_x dk_y, \tag{8}$$

where the implicit formula for $D_\perp$ can be solved in closed form. For field line separation, there is no explicit contribution from slab turbulence, which can be understood in that slab turbulence, which is independent of x and y, moves field lines 1 and 2 in tandem. However, slab turbulence does have an effect insofar as it contributes to the field line random walk; in the limit $\Delta z \to \infty$, we find

$$D_{sep} = 2 \frac{(D_\perp^{2D})^2}{D_\perp}. \tag{9}$$

We refer to this as fast diffusive separation.

Paradoxically, increased slab turbulence increases $D_\perp$ and *decreases* the coefficient of diffusive separation [as well as the 2D contribution to $D_\perp$; see eq. (6)]. An interpretation of this effect is that rapid lateral excursions due to slab turbulence quickly decorrelate the “random flights” in the relative excursions of the two field lines, ΔX and ΔY. The random flights depend on 2D turbulence and hence x and y, which are rapidly changing due to slab turbulence. This yields a shorter “mean free path” in the motion of one field line relative to another, hence the lower coefficient of diffusive separation.

If the 2D component dominates the field line random walk, $D_\perp \approx D_\perp^{2D}$, as expected in the solar wind, then we have $D_{sep} \approx 2D_\perp$. This behavior persists down to lower Δz until the perpendicular diffusion (and separation) are on the order of the perpendicular coherence length, where our approximations begin to break down and we enter a free-streaming régime (ultimately, field lines are nearly straight, and $\langle \Delta x^2 \rangle \propto \Delta z^2$). Thus once diffusion sets in, only fast diffusive separation is expected, with the two field lines essentially moving independently.

On the other hand, if the slab component dominates the field line random walk, we predict more interesting behavior: for small initial separations, there can be two régimes of diffusive separation. This behavior can be understood by examining the effects of the low-pass filters when Δz is not extremely large, and verified by direct calculation (see Figure 3). Slow diffusive separation occurs for Δz above the coherence length (so the field lines undergo a random walk) but a field line separation less than a coherence length perpendicular to the mean magnetic field. The coefficient of slow diffusive separation varies linearly with the autocorrelation of $a(x, y)$, the potential function for 2D turbulence, at the initial separation. Next comes a régime of superdiffusion, when the field line separation is on the order of the perpendicular coherence length, followed by fast diffusive separation, as before, for large field line separations.

3 Conclusions

Our main results are as follows: We predict that there can be two distinct régimes of diffusion in the field-line separation, connected by a period of superdiffusion (see Figure 3). Slow diffusive separation occurs for field-line separations less than a coherence length perpendicular to the mean magnetic field. The rate of slow diffusion depends on the autocorrelation of

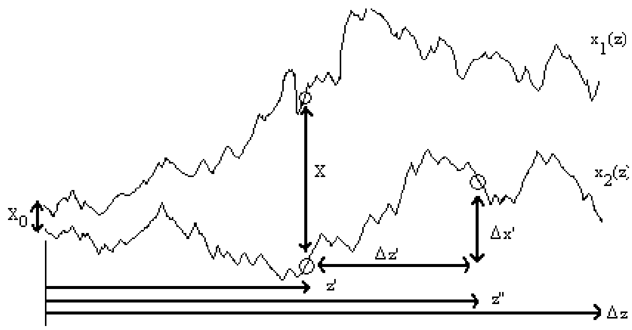

Fig. 2. Schematic of two random field lines and the definitions of various quantities.

the flux function for 2D turbulence at the initial separation. Fast diffusion, for field-line separations greater than the perpendicular coherence scale, is not simply at twice the rate of perpendicular diffusion for individual field lines, because the slab component alone does not lead to any change in the field-line separation. Instead, we obtain a more interesting formula for the fast-diffusion rate. For an initially low separation, we predict a nearly constant "escape distance."

In the solar wind, it is believed that the 2D contribution dominates, leading to fast diffusive separation. This suggests that recent ACE observations of sharp boundaries between regions of high and low density of solar energetic particles will require an alternative explanation, such as non-homogeneous turbulence.

Acknowledgements. This work was partially supported by a Basic Research Grant from the Thailand Research Fund.

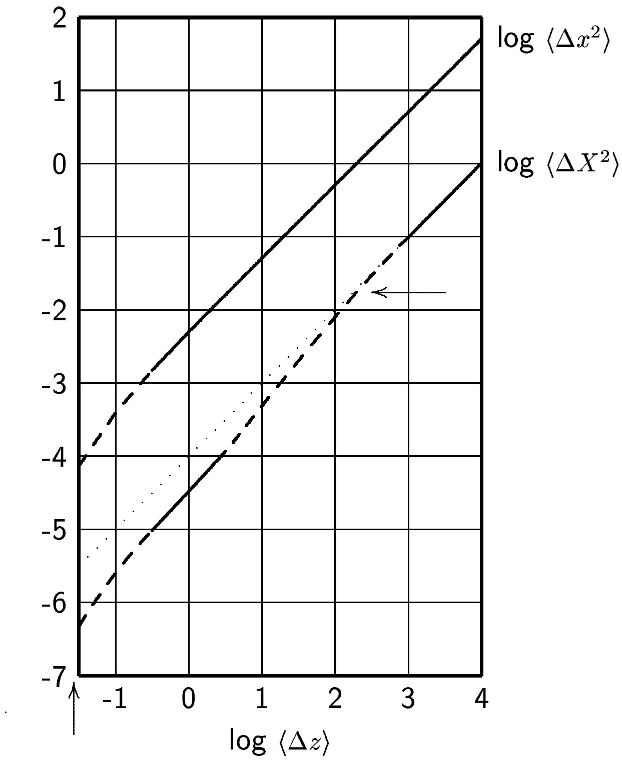

Fig. 3. Results for field line separation in a slab-dominated field line random walk, for initial separation $X_0 = 0.001$. Arrows indicate the parallel and perpendicular coherence lengths. In this plot, a separation or random walk of slope 1 (solid lines) is diffusive, while a greater slope (dashed lines) indicates superdiffusion. At high Δz, the field line random walk $\langle \Delta x^2 \rangle$ is slab-dominated, while the separation of two field lines is given by $\langle \Delta X^2 \rangle$ of twice the 2D contribution to $\langle \Delta x^2 \rangle$ (fast diffusive separation; extended as a dotted line to guide the eye). At lower Δz there is another régime of diffusive separation (slow diffusive separation). In between, there is supradiffusive separation when $\langle \Delta x^2 \rangle$ is on the order of the perpendicular coherence length. There is a free-streaming régime for Δz below the parallel coherence length.

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