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Field line random walk for non-axisymmetric magnetic fluctuations

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Abstract. The problem of the random walk of magnetic field lines in a turbulent magnetic field is reconsidered for the case of magnetic fluctuations that are non-axisymmetric with respect to the mean magnetic field. This extension is trivial for quasilinear theory and equivalently for the case of one-dimensional slab fluctuations. For the general case of transverse fluctuations of arbitrary amplitude the problem is more complex. Here we derive the magnetic field diffusion coefficient for two-component fluctuations, a superposition of slab plus two-dimensional fluctuations that has proven useful for various solar wind applications. Using homogeneity and diffusion approximations and Corrsin's independence hypothesis, we derive non-perturbative, analytic formulae, including closed-form expressions for special cases of interest in the heliospheric transport of charged particles, such as solar modulation of cosmic rays and particle acceleration at a nearly perpendicular shock.

1 Introduction

Although diffusion of charged particles perpendicular to the magnetic field in a collisonless astrophysical plasma remains an incompletely solved problem (Giacalone and Jokipii, 1999; Mace et al., 2000), it is clear that it is closely related to the simpler problem of random walk of magnetic field lines in the presence of magnetic turbulence (Jokipii, 1966; Jokipii and Parker, 1968). The classic quasilinear calculation (Jokipii, 1966) shows that field line random walk is associated with the "power at zero wavenumber" of the turbulence, as convenient shorthand for a diffusion coefficient proportional to the product of the energy density in the fluctuations and the correlation scale. In the usual treatments, the magnetic fluctuation properties are assumed to be axisymmetric with respect to the mean magnetic field. However there are indications that the variances of the fluctuation vectors may be non-axisymmetric in some cases of interest (Jokipii, 1973; Jokipii et al., 1995; Burger and Hattingh, 1998). Particular interest in the non-axisymmetric perpendicular diffusion and field line random walk derives from recent studies that suggest a possible role of enhanced latitudihal transport of cosmic rays at high heliographic latitudes (Jokipii et al., 1995; Burger and Hattingh, 1998). The generalization to non-axisymmetry is immediate in quasilinear theory, since the diffusion coefficient is linear in the variances, but in general this is not the case. In the present paper we develop a theory for the non-axisymmetric field line random walk in a more general non-perturbative scheme (Matthaeus et al., 1995). The approach is useful for general transverse turbulence, although we apply it explicitly here to a two-component model of fluctuations which has been useful in solar wind and cosmic ray scattering studies (Matthaeus et al., 1990; Bieber et al., 1994, 1996). Our principal results are a general framework for the non-axisymmetric field line random walk as a set of coupled bi-quadratic equations, and perhaps more useful, closed solutions for several cases. The latter should find immediate application in heliospheric scattering problems such as cosmic ray modulation.

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2 Calculation of Field Line Diffusion for Non-axisymmetric 2D+Slab Turbulence

In the 2D + slab model of magnetic turbulence, we assume $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}(x,y,z)$, where $\boldsymbol{B}_0 = B_0\hat{z}$, $\boldsymbol{b} \perp \hat{z}$, and $\boldsymbol{b} = \boldsymbol{b}^{2D}(x,y) + \boldsymbol{b}^{slab}(z)$. In general, we can write

$$\boldsymbol{b}^{2D}(x,y) = \boldsymbol{\nabla} \times [a(x,y)\hat{z}], \tag{1}$$

where $a\hat{z}$ is the vector potential for the 2D component of magnetic turbulence.

Let $A(k_x,k_y)$ be the Fourier transform of the autocorrelation $\langle a(0,0)a(x,y)\rangle$. Then axisymmetry of the 2D turbulence implies that A depends only on the magnitude $k_\perp = \sqrt{k_x^2+k_y^2}$, i.e., is constant along circles in (k_x,k_y) space. To consider non-axisymmetric 2D turbulence, we suggest a form in which A is instead constant along ellipses in (k_x, k_y) space, with an ellipticity parameter ξ defined as the aspect ratio as shown in Figure 1. [Thus the x- and y-directions are defined by the principal axes of A in (k_x, k_y) space.] This form of non-axisymmetry permits the axisymmetric case, $\xi = 1$, and still allows us to solve the problem analytically.

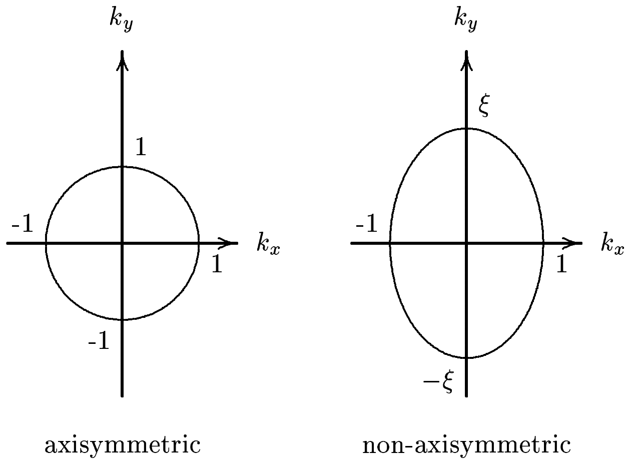

Fig. 1. Contours of constant power $A(k_x, k_y)$ of the 2D potential function for the axisymmetric case and for the type of non-axisymmetric 2D turbulence we consider.

Considering the magnetic correlation function of the 2D component, $R_{ij}^{2D}(x,y) \equiv \langle b_i^{2D}(0,0) b_j^{2D}(x,y)\rangle$ (we assume homogeneity of the turbulence), and its Fourier transform, $P_{ij}^{2D}(k_x, k_y)$, eq. (1) implies that

$$\begin{aligned} P_{xx}^{2D}(k_x,k_y) &= k_y^2 A(k_x,k_y) \\ P_{yy}^{2D}(k_x,k_y) &= k_x^2 A(k_x,k_y). \end{aligned} \tag{2}$$

Previous work (Matthaeus et al., 1995) has identified an important scale length for the 2D contribution to perpendicular diffusion, called the “ultrascale” or “mesoscale,” which we define here as

$$\begin{aligned} \tilde{\lambda} &\equiv \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A dk_x dk_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(P_{xx}^{2D}+P_{yy}^{2D}) dk_x dk_y} \\ &= \frac{\langle a^2\rangle}{(1-f_s)b^2}, \end{aligned} \tag{3}$$

where $b^2 \equiv \langle b_x^2 + b_y^2\rangle$, and f_s is the fraction of the turbulent energy in the slab component (the remainder being in the 2D component).

Space does not permit a complete presentation of our derivation, which follows the non-perturbative approach of Matthaeus et al. (1995) (see also Ruffolo and Matthaeus, these proceedings). We again assume homogeneity, diffusion, and Corrsin’s independence hypothesis. In the limit $\Delta z \to \infty$, we recover the Jokipii and Parker (1968) results for slab turbulence:

$$D_i^{slab} = \frac{\ell_i f_{si} b^2}{B_0^2} \quad (i = x, y), \tag{4}$$

where ℓ_i and f_{si} are the correlation length and fraction of turbulent energy, respectively, of the i-component of slab turbulence. In the limit that the slab fraction goes to zero, we have the field line diffusion coefficients

$$\begin{aligned} D_x^{2D} &= \xi D_\perp^{2D} \\ D_y^{2D} &= \frac{1}{\xi} D_\perp^{2D} \\ D_\perp^{2D} &= \frac{\tilde{\lambda} b}{\sqrt{2} B_0}. \end{aligned} \tag{5}$$

[Note that $D_i^{slab} \propto (b/B_0)^2$ while $D_i^{2D} \propto (b/B_0)$.] Then the general result for the combined, long-distance field line diffusion coefficients of non-axisymmetric 2D+slab turbulence is

$$\begin{aligned} &\left(D_y + \frac{\sqrt{D_x D_y}}{\xi}\right)(D_x - D_x^{slab}) = I \\ &\left(D_x + \xi\sqrt{D_x D_y}\right)(D_y - D_y^{slab}) = I, \end{aligned} \tag{6}$$

where $I = 2(D_\perp^{2D})^2$.

These equations are straightforward to solve numerically for a given case of interest. Furthermore, it is possible to scale diffusion coefficients according to $D_\perp$ and anisotropies according to ξ (not shown here) to reduce the above to equations that depend on only two parameters, allowing their limiting behavior to be readily elucidated.

3 Applications to Various Physical Limits

While the coupled bi-quadratic equations (6) are not difficult to solve, for certain physical limits there are closed-form solutions with interesting interpretations. There are certain physical inputs one should specify for a given application, e.g., for solar modulation in different parts of the heliosphere, or anomalous cosmic ray acceleration at different parts of the solar wind termination shock. The “user” of this calculation should specify:

B_0, the mean magnetic field,
b, the root-mean-squared turbulent magnetic field,
f_s, the slab fraction of turbulent energy,
$\eta^2 \equiv f_{sx}/f_{sy}$, the slab anisotropy,
ℓ_x, the correlation length of b_x^{slab},
ℓ_y, the correlation length of b_y^{slab},
$\tilde{\lambda}$, the ultrascale (of 2D turbulence), and
ξ, the anisotropy of 2D turbulence ($\xi^2 = D_x^{2D}/D_y^{2D}$).

In many applications, direct measurements of these quantities are not available, so one must make educated guesses, or *ad hoc* approximations. Therefore, we present solutions of the general equations (6) for specific limits and approximations. [Naturally, the simplest approximation is that either slab or 2D turbulence can be neglected, in which case equations (4) or (5) suffice.]

If η and ξ are not known, a simple approximation is to set them equal, in which case equations (6) decouple and reduce to the form obtained by Matthaeus et al. (1995):

$$D_i = \frac{D_i^{slab}}{2} + \sqrt{\left(\frac{D_i^{slab}}{2}\right)^2 + (D_i^{2D})^2} \quad (i = x, y). \qquad (7)$$

Furthermore, if all other input values are fixed but η and ξ both tend to 0 or ∞, then it can be shown that the 2D contribution dominates. Thus if one employs such limits, say in the outer heliosphere where field fluctuations might become increasingly anisotropic (e.g., if "frozen in" the solar wind), then one must also use $D_i \propto b/B_0$.

Suppose that $\eta \to \infty$, while ξ is fixed. (If η instead goes to zero, the roles of x- and y-components are reversed.) If $D_x^{slab} \ll D_x^{2D}$, then the diffusion coefficients tend to 2D values. If, on the other hand, $D_x^{slab} \gg D_x^{2D}$, then $D_x \approx D_x^{slab}$ and $D_y \approx 2D_x^{2D}D_y^{2D}/D_x^{slab}$, which is much lower than D_y^{2D}. This is similar to the "paradox" discussed earlier; here, increased slab turbulence in the x-direction leads to *decreased* y-diffusion.

Finally, we consider the case where $\xi \to \infty$ for fixed η. Let $D_\perp^{slab}$ be the geometric mean of D_x^{slab} and D_y^{slab}. If $D_\perp^{slab} \gg D_\perp^{2D}$, then $D_i \approx D_i^{slab}$. If $D_\perp^{slab} \ll D_\perp^{2D}$, then $D_y \approx D_y^{slab}$ while $D_x \approx 2D_x^{2D}D_y^{2D}/D_y^{slab}$, which is again much lower than D_x^{2D}.

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