# Recent results of KLEM method simulations 

D. Podorozhnyi ${ }^{1}$, N. Korotkova ${ }^{1}$, G. Morozov ${ }^{1}$, R. Mukhamedshin ${ }^{2}$, E. Postnikov ${ }^{1}$, T. Roganova ${ }^{1}$, L. Sveshnikova ${ }^{1}$, and A. Turundaevskii ${ }^{1}$<br>${ }^{1}$ Moscow State University, Institute of Nuclear Physics, Russia<br>${ }^{1}$ Institute for Nuclear Research, Moscow, Russia


#### Abstract

Further development of a new method of primary cosmic ray energy determination is presented. The method is based on measurements of spatial density of secondary particles produced in a strong interaction in target and modified by thin converter. This method, named KLEM (Kinematic Lightweight Energy Meter), is aimed to determine primary cosmic ray nuclei energy in extremely wide range $10^{11}-10^{16} \mathrm{eV}$. The first application of the method is planned for satellite project NUCLEON. The results of detailed Monte-Carlo simulation are analyzed.


## 1. Introduction.

Definitive information on primary cosmic ray spectra and composition can be obtained only by direct measurements, when a real primary particle is detected. But direct measurements are usually restricted for high energy range due to impossibility of a significant mass calorimeter launching in the near-Earth orbit. It makes these investigations very expensive. We have proposed a new approach for the investigation of high energy cosmic rays nuclei with $E>10^{11} \mathrm{eV}$ (Bashindzhagyan et al, 1999). This method allows the construction of a relatively lightweight device with large geometric factor. In this paper we present the advanced Monte-Carlo simulation results for the investigation of such problems as the physical base of the method, the preliminary optimization of apparatus design and investigation of the model sensitivity. We have also estimated the influence of limited spatial resolution of microstrip silicon detectors.

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## 2. Physical base of the method

Many years ago Castagnoly (1953) proposed the so called kinematic method of energy determination based on measurements of average value $\left\langle\operatorname{lntg} \theta_{i}>\left(\theta_{i}-\right.\right.$ emission angles of all charged secondaries) in every interaction: $\ln E \sim<\ln \operatorname{tg} \theta_{i}>$. Due to Lorentz transformation the spatial density of secondary particles becomes more narrow in laboratory system with increasing energy. The spatial density is usually analyzed in units $d N / d \eta\left(\eta=-\ln \left(\operatorname{tg} \theta_{i} / 2\right)\right.$. It is well known that $\langle\eta\rangle$ and $\eta_{\text {max }}$ logarithmically increase with increasing energy. Schematic change of $d N / d \eta$ with rise of energy is shown in Fig. 1 by thin line.


Fig. 1.The schematic image of $d N / d \eta$ distribution of charged secondaries in a proton interaction of different energies: above the converter (thin line) and below the converter (thick line).

This method was widely used in emulsion experiments, where neutral secondaries were not detected, that resulted in large fluctuations. Besides, in proton-nucleus interactions slow particles produced in successive
interactions in nucleus deform the left "wing" of $d N / d \eta$ distribution, also enlarging fluctuations. The above mentioned effects lead to large errors in energy determinations $-100-200 \%$. To overcome these problems we proposed a combined method. On the one hand it is based on the measurements of spatial density of not only charged secondaries, but also neutral ones. On the other hand we propose such measuring data processing technique that allows to increase contribution of faster secondaries to energy determination and to eliminate that of slower ones. The principal scheme is the following.

A primary particle interacts in the thin C-target, ( $h_{t} \sim 10 \mathrm{~cm}$ ), where secondary photons (originated via decays of $\pi^{0}$ ) with energy $E_{\gamma}^{i}$ and charged particles with energy $E_{c h}^{i}$ are produced. The converter is a thin lead layer ( $h_{c} \sim 1-2 \mathrm{~cm}$ ) located at some distance ( $H \sim 20 \mathrm{~cm}$ ) from the target and just in front of detecting plane. It converts almost all secondary photons to electrons. According to the cascade theory a number of output electrons for every secondary photons is proportional to $E_{\gamma}^{s}$, where $s \sim 0.1-0.2$ at thickness of converter $\sim 1-2 \mathrm{~cm}$. It results in significant multiplication of charged particles (mainly electrons) below the converter. The multiplication coefficient $M$ depends not only on primary particle energy $(M=3.5$ at 100 $\mathrm{GeV}, M=20$ at 1000 TeV ), but also on distance from the central axis of the cascade because the most energetic secondaries have minimum values of $\theta_{i}$. It causes the change of spatial density $d N / d \eta$ below converter (thick line in Fig. 1). This effect makes the entire procedure more sensitive to primary energy $(E)$.

As a first step in development of the data processing technique we propose to use for the energy determination a parameter $S$ :

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\begin{equation*}
S=\sum \eta_{i}^{2} \tag{1}
\end{equation*}
$$

where summation is over all the secondaries detected below the converter. $\eta_{i}=-\operatorname{lntg} \theta_{i} / 2 \approx-\ln r_{i} /(H / 2)$, and $r_{i}$ is a distance from axis for every position-sensitive detector located below the converter, $H$ is a distance from interaction point in the target, $N_{i}$ is a number of electrons and pions, detected by each detectors below converter. $\langle S(E)\rangle$ on the one hand characterises the distribution of secondaries for $\eta$ being sensitive to Lorentz factor of primary particle and on the other hand is proportional to multiplicity of secondaries produced in the target and multiplied in the converter. Combination of these two factors allows to obtain simple power law $\left\langle S(E)>\sim E^{0.7-0.8}\right.$ dependence on energy per nucleon, similar for all type of primary nuclei in the entire range of energy up to $10^{16} \mathrm{eV}$ (Bashindzhagyan (1999)). For incident nucleus with mass number A only portion of nucleons $-N_{w}-$ interact with target nucleus C. For forward particles the superposition model is valid, so $d N / d \eta(\mathrm{~A}+\mathrm{C})$ can be expressed as a sum $N_{w} * d N / d \eta(\mathrm{p}+\mathrm{C})$. In this case $\langle S(E)\rangle$ nuclear dependence should be distinguished from that of
proton only by factor $N_{w}$. More complicated problem is the influence of fragments, created in target and passed through converter without interaction, because for them the response of silicon detector is proportional to square charge of fragments. It can enlarge the error in energy determination in individual case. The first series of MonteCarlo calculations shows that accuracy of energy determination $(\delta E)$ in individual event is about $60 \%$ for all nuclei (Bashindzhagyan (1999)).

For the real application of the method we intend to use calibration dependence $\langle S(E)\rangle$ obtained by simulation, so it is necessary to investigate the following problems:

- model sensitivity of $\langle S(E)\rangle$ and $\delta E$;
- dependence of an error in energy determination on angle of primary particle trajectory;
- optimal thickness of converter and target;
- dependence of an error in energy determination on spatial resolution of silicon microstrip detectors;
- sufficiency of accuracy in energy determination for the probable identification of some peculiarities of energy spectra, such as "knee" or "peak".


## 3. Monte-Carlo simulation

This simulation was performed with the GEANT program complex, including for hadron-nucleus interaction model FLUKA for $E<50 \mathrm{GeV}$ and QGSM (Kalmykov (1993) for $E>50 \mathrm{GeV}$. Additionally we have made series of calculation using new Monte-Carlo code SPHINX (Mukhamedshin, (2001)) presented in this conference. QGSM and SPHINX have some differences in inclusive spectra of forward secondaries and SPHINX code includes the electromagnetic processes with taking into account Landau-Pomeranchuk effect. Comparison of these two models simulation results, in particular calibration dependence $\langle S(E)\rangle$ and accuracy of energy determination will be presented at the Conference. A few simulations were carried out using only FLUKA for all energies.

Calculations were performed for the following design of installation: thickness of carbon target $h_{t}=10 \mathrm{~cm}$, thickness of converter $h_{c}=1-5 \mathrm{~cm}$, the height of air gap between the target and the converter $H \sim 20 \mathrm{~cm}$. Three types of projectile nuclei (protons, carbon, iron) were considered at incident angles $\vartheta=0-45^{\circ}$. It was simulated 15 groups of events ( $\sim 400$ in every group): 6 groups of protons with $E=10^{11}-10^{16} \mathrm{eV}$ through order of energy, 5 groups of carbon nuclei with $E=10^{11}-10^{15} \mathrm{eV} /$ nucl, 4 groups of Fe nuclei with $\mathrm{E}=10^{11}-10^{14} \mathrm{eV} /$ nucl.

At first we considered parameter $S$ (1) (every secondary particle is detected and spatial resolution of silicon detectors is not taken in account). The accuracy of energy determination depends on fluctuation of $S$ and on slope of a curve representing its energy dependence $<S(E)>$. For power functional dependence $<S(E)>\sim E^{\beta}$ relative error depends on $\beta: \delta E=(1 / \beta) \delta S$ - the more $\beta$ the better accuracy.

Performed calculations revealed that optimal thickness of the converter $\left(h_{c}\right)$, at which $\beta$ is maximum, $h_{c} \sim 2 \mathrm{~cm}(\beta=$ $0.80,0.77,0.75$ for incident proton, C nucleus, Fe nucleus correspondingly).

For nuclei with angle of trajectory $\vartheta>0^{\circ}$ we modify parameter $S$ with taking into account an ellipse form of the lateral distribution and an increase of effective thickness of detectors. After these corrections the $\langle S(E)>$ dependence for inclined events was found to be close to that for vertical events with the thickness of converter $h_{c} \cos \vartheta$. One can note that 1 cm converter for vertical events is the same as 2 cm converter for events with $\vartheta=60^{\circ}$. So the thickness of converter $h_{c}=1-2 \mathrm{~cm}$ can be considered as optimal.

For every event with real energy $E$ the "measured" energy $E_{\text {meas }}$ can be obtained by using the calibration dependence $\langle S(E)\rangle$. We analyzed errors of energy determination in terms $\delta\left(E_{\text {meas }} / E\right)$ or $\delta\left(\lg E_{\text {meas }} / E\right)$ being more suitable for the analysis of power spectra.


Fig. 2. Probability density function $W\left(\lg \left(E_{\text {meas }} / E\right)\right)$ for incident protons, C and Fe nuclei: $W$ - direct function (black circles), $W^{*}$ inverse function with a priori spectrum $\mathrm{E}^{2.7}$ (open circles).

The examples of direct probability density functions $W\left(\lg \left(E_{\text {meas }} / E\right)\right)$ for primary protons, C nucleus and Fe nucleus are presented in Fig. 2. One can see the noticeable "tail" of distribution in the range of small values of $E_{\text {meas }} / E$, which leads to high enough average value of $\delta \lg \left(E_{\text {meas }} / E\right)=0.46,0.49,0.54$ for p, C nucleus, Fe nucleus correspondingly. But it is obvious that at measurement of power spectra, the contribution of such kind of events will be suppressed. For the analysis of real situation we calculated the inverse probability density function $W^{*}\left(\lg \left(E_{\text {meas }} / E\right)\right) d E$. It means the probability of real energy $E$ when measured energy is fixed at $E_{\text {meas }}$. If a priori spectrum of incident particles $F(E)=E^{-\gamma}$, then $W$ and $W^{*}$ can be expressed in accordance with Bayes theorem:
$W^{*}\left(E_{\text {meas }}, E\right)=W\left(E, E_{\text {meas }}\right) F(E) / \int W\left(E, E_{\text {meas }}\right) F(E) d E$.
The calculated inverse probability density function $W^{*}\left(\lg \left(E_{\text {meas }} / E\right)\right)$ for a priori spectra $F(E) \sim E^{-2.7}$ is denoted in Fig. 2 by thick line. One can see that contribution of "tail" is significantly suppressed (as $\left.\left(E_{\text {meas }} / E\right)^{I .7}\right)$. The
average value of errors in this case are $\delta \lg \left(E_{\text {meas }} / E\right)=0.22$, $0.23,0.25$ for $p, \mathrm{C}$ nucleus, Fe nucleus.

For the case of measurements of pure power a priori spectrum, the measured spectrum is related with the real one by formula $F\left(E_{\text {meas }}\right)=\left\langle\left(E_{\text {meas }} / E\right)^{\gamma-1}\right\rangle F(E)$ (Murzin, (1965)), and the measured spectrum resembles the real one by its slope, if $W\left(E, E_{\text {meas }}\right)$ does not depend on energy. If $<\left(E_{\text {meas }} / E\right)>\sim 1$, the measured spectrum is always higher by intensity then the real one. Therefore the absolute value of error is not so important. But for the probable identification of some peculiarities of energy spectra, such as "knee" or "peak", the accuracy of energy determination becomes very important. Is obtained accuracy $\delta \lg \left(E_{\text {meas }} / E\right) \sim 0.22-0.25$ sufficient for this aim?


Fig. 3. A priori spectra with "knee" and "peak" simulated by Monte-Carlo method (full lines) and these spectra "measured" by KLEM method (dotted lines).

For illustration we present in Fig. 3 a priori spectra with "knee" and "peak", simulated by Monte-Carlo method and these spectra "measured" by our method. Intensity is multiplied by $\mathrm{E}^{2.7}$. "Knee" and "peak" are reproduced well enough. The shift of intensity and the shift of energy region of "peak" and "knee" can be taken into account by calculations.

## 4. Opportunities of the method with taking into account the detection procedure.

The presented results are obtained for ideal method of registration - the $X, Y$ coordinates of every secondary particle are supposed to be measured. How can the real spatial resolution of microstrip silicon detectors change these results? To exclude the technical details of the real detectors we considered the worst case. It was suggested that under converter we have two planes of silicon strip detectors, oriented in perpendicular directions - along $X$ axis and $Y$-axis. Every plane consists of strips $50 \mu \mathrm{~m}$ pitch and 30 cm long. Every strip has readout channel in one direction $X$ or $Y$. It means that we measure the integral ionization with the step $50 \mu \mathrm{~m}$ along $X$-axis and along $Y$ axis. In this case we changed parameter $S$ (1). Instead of measured angle $\theta_{\mathrm{i}}$ its projection on observation plane was calculated. Taking into account $\eta_{i}=-\ln \left(\operatorname{tg}\left(\theta_{i} / 2\right)\right)=$ $=-\ln \left(2 H / R_{i}\right)$ we introduce $\phi^{X}{ }_{i}=\ln \left(2 H / X_{i}\right), \phi^{Y}{ }_{i}=\ln \left(2 H / Y_{i}\right)$. Here $H$ is the distance from point of interaction to detectors plane, $X_{i}$ is the distance between position of strip and the central $X$-axis of the cascade of secondary particles. $Y_{i}$ is the same for the $Y$ detector plane. $N_{i}^{X}, N_{i}^{Y}-$ an integral number of particles detected by every strip. Then $S_{2}$ is an analogue of parameter $S(1)$ :
$S_{2}=1 / 2\left(\sum \phi^{X}{ }_{i} N_{i+} \phi_{i}^{Y} N_{i}\right)$
The series of simulation with taking into account the registration procedure revealed that $\left\langle S_{2}(E)\right\rangle$ is a power dependence $\sim E^{\beta}$ (Fig. 4) and $\beta=0.78$ for protons, $\beta=0.79$ for C nucleus, $\beta=0.71$ for Fe nucleus. These values are very close to those obtained above for the case when all coordinates of secondary particles are registrated $(\beta=0.82$, $\beta=0.82, \beta=0.74$ correspondingly).


Fig. 4. Calibration dependence $\left\langle S_{2}(E)\right\rangle$, obtained with taking into account the spatial resolutions of microstrip detectors (p circles, C - squares , Fe - triangles)

Probability density functions $W$ and $W^{*}$ are also similar to those shown in Fig. 2. The accuracy of energy determination in logarithmic scale $\delta \lg \left(\mathrm{E}_{\text {meas }} / \mathrm{E}\right)=0.22$, $0.219,0.265$ for $p, \quad \mathrm{C}, \mathrm{Fe}$ primary particles correspondingly. It means that main fluctuations in the described method of energy determination are related with physical fluctuations of multiple production processes, but not with the registration method. We have also considered the fluctuations of ionization losses in detectors and the fluctuations caused by noise of detectors and electronics. The total contribution of these fluctuations should not exceed $20 \%$. This value is much smaller than physical fluctuations of multiplicity in one strip.

## 5. Summary

The considered method of energy determination of primary cosmic ray nuclei can be applied in wide energy range $10^{11}-10^{16}$. It permits to reconstruct primary energy spectra that may have some peculiarities. We plan to develop new energy estimation algorithms, which are based on the present-day theory of experimental data analysis and interpretation. In particular, such technique using multivariate correlation analysis and measurement reduction theory will be worked out. On the base of this method light enough devices for primary cosmic ray measurements beyond the $10^{14} \mathrm{eV}$ in satellite investigations can be constructed.

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[^0]:    Correspondence to: D.Podorozhnyi
    dmp@eas.sinp.msu.ru Fax:+7-095-939-0896

