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# Flux of upward high-energy muons at the multi-component primary energy spectrum

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Abstract. The atmospheric neutrino-induced upward muon flux are calculated by using the multi-component primary energy spectrum, CORSIKA EAS simulation code for the reproduction of the atmospheric neutrino spectra and improved parton model for charged-current cross sections. The results are obtained at  $10^2 - 10^6$  GeV muon energy range and  $0 - 89^0$ zenith angular range.

## 1 Introduction

Neutrino-induced upward muon flux are both the direct transmitters of information about neutrino flux from energetic astrophysical sources (binary stars, AGN) and direct posterity of atmospheric very high energy neutrino. These problems are being investigated in many modern experiments (see review (Gaisser et al., 1995)) and as a rule, the upward muon flux of atmospheric origin is considered in the capacity of the background flux.

Here, on the basis of multi-component model of primary cosmic rays (Biermann, 1993), QGSJET model of high energy  $A - A_{Air}$  interactions (Kalmykov and Ostapchenko, 1993) and parton model predictions of charged-current cross sections (Quigg et al., 1986) we evaluated the expected background upward muon energy spectra at different zenith angles and studied ( $E_{\mu} - E_{\nu,\overline{\nu}}$ ) and ( $E_{\mu} - E_N$ ) correlations.

# 2 Method of calculations

Upward muon differential energy spectra close to Earth surface at different zenith angles ( $\theta$ ) can be represented as follows:

$$\frac{\partial F(\theta)}{\partial E_{\mu}} = \sum_{A} \int_{E_{\min}}^{E_{\max}} \frac{d\Im}{dE_{A}} dE_{A} \sum_{\xi \equiv \nu_{\mu}, \overline{\nu}_{\mu}} \int_{E_{\mu}}^{E_{A}} \frac{\partial f}{\partial E_{\xi}}$$

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**Fig. 1.** Primary energy spectra according to 2-component origin of cosmic rays. The hatch area contains (Ter-Antonyan and Haroyan, 2000), DICE (Swordy et al., 2000), and CASA-BLANCA (Fowler et al., 2000) data.

$$-dE_{\xi} \int_{t_{\min}}^{t_{\max}} \frac{\partial G_{\xi}}{\partial E_{\mu}} \frac{\partial W}{\partial t} dt \tag{1}$$

where:

 $d\Im/E_A$  - is the primary energy spectrum of the nucleus A;  $\partial f(E_A, A, \theta)/\partial E_{\xi}$  is the atmospheric neutrino ( $\xi \equiv \nu_{\mu}$ ) and anti-neutrino ( $\xi \equiv \overline{\nu}_{\mu}$ ) energy spectrum at the primary nucleus A with  $E_A$  energy and  $\theta$  zenith angle;

 $\partial G_{\xi}(t, E_{\xi})/\partial E_{\mu}$  is the neutrino-induced muon energy spectrum at observation level (close to the Earth surface) at the given neutrino energy  $E_{\nu,\overline{\nu}}$  and depth t of muon production in the Earth;

 $\partial W(E_{\xi}, E_{\mu})/\partial t$  is a probability of charged-current interaction at t depth of the Earth matter.



Fig. 2. Atmospheric neutrino (symbols) and anti-neutrino (lines) energy spectra at different primary proton energy  $(E_p, TeV)$  at zenith angles  $\theta = 0^0$  (left) and  $\theta = 89^0$  (right).

Energy spectra of primary nuclei ( $A \equiv 1 - 59$ ) according to the multi-component model of primary cosmic ray origin (Biermann, 1993) are presented in 2-component form (see Fig.1):

$$\frac{\partial \mathfrak{F}}{\partial E_A} = \Phi_A \left( \delta_{A,1} \frac{d\mathfrak{F}_1}{dE_A} + \delta_{A,2} \frac{d\mathfrak{F}_2}{dE_A} \right) \tag{2}$$

where the first component (ISM) is derived from the explosions of normal supernova into an interstellar medium with expected rigidity-dependent power law spectra

$$\frac{d\Im_1}{dE_A} = \begin{cases} E_A^{-\gamma_1} & : \quad E_A < E_{ISM} \\ 0 & : \quad E_A > E_{ISM} \end{cases}$$
(3)

and the second component (SW) is a result of the explosions of stars into their former stellar winds with expected rigiditydependent power law spectra

$$\frac{d\mathfrak{S}_2}{dE_A} = \begin{cases} E_A^{-\gamma_2} & : \quad E_A < E_{SW} \\ E_{SW}^{-\gamma_2} (E_A/E_{SW})^{-\gamma_3} & : \quad E_A > E_{SW} \end{cases}$$
(4)

where  $\Phi_A$  is a scale factor (*E* in TeV units) from approximations (Wiebel-Sooth and Biermann, 1998),

 $E_{ISM} = R_{ISM} \cdot Z$  and  $E_{SW} = R_{SW} \cdot Z$  are the corresponding rigidity-dependent critical energies (Biermann,

1993) and Z is the charge of nucleus A.

The values of spectral parameters from (2-4) are:  $\gamma_1 = 2.78$ ,  $\gamma_2 = 2.65$ ,  $\gamma_3 = 3.28$ ,  $R_{ISM} \simeq 200$  TV,  $R_{SW} \simeq 2000$  TV and the fraction of each component:  $\delta_{A,1} = 1 - \delta_{A,2}$ ,  $\delta_{A,2} = (2ZR_{ISM})^{(\gamma_2 - \gamma_{A,0})}$  at  $\gamma_{A=1,0} = 2.75$  and  $\gamma_{A>1,0} = 2.66$  (Wiebel-Sooth and Biermann, 1998) is obtained by normalization of (2-4) with approximation of balloon and satellite data (Wiebel-Sooth and Biermann, 1998) at  $E_A = 1$  TeV and  $\chi^2$  minimization of KASCADE (Classtetter et al., 1999) and ANI (Chilingaryan et al., 1999) EAS size spectra with the expected ones using 2-component representation (2-4) (Ter-Antonyan and Biermann, 2001).

Values of primary energy limits in the expression (1) are chosen  $E_{min} = 2E_{\mu}$  and  $E_{max} = 10^9$  GeV. The expected primary energy spectra for different nuclei, all-particle and nucleon energy spectra by 2-component model are shown in the Fig. 1. A contribution of the third primary extra-galactic component is ignored.

Atmospheric neutrino (anti-neutrino) energy spectra  $\partial f(E_A, A, \theta) / \partial E_{\xi}$  at given six A group (A = 1, 4, 12, 16, 28, 56) and  $E_A = 3.16 \cdot 10^2, 10^3, \dots 10^9$  GeV,  $\theta = 0, 30, 60, 89^0$  parameters of primary nuclei were calculated in the framework of QGSJET interaction model (Kalmykov



Fig. 3. Atmospheric neutrino-induced upward muon differential energy spectra.

and Ostapchenko, 1993) by CORSIKA562 EAS simulation code (Heck et al., 1998). Intermediate values of tabulated spectral data are evaluated by interpolation with preliminary linearizations. It should be noted that the neutrino energy spectra depend only on primary nucleon energy ( $E_A/A$ ) with accuracy < 5% in  $A \equiv 1 - 56$  range.

The atmospheric neutrino (anti-neutrino) energy spectra at different primary proton energies ( $E_p = 3.16 \cdot 10^2 - 10^9$  GeV) and zenith angles ( $\theta = 0^0, 89^0$ ) are presented in Fig. 2. Neutrino-induced muon energy spectra are determined as

$$\frac{\partial G_{\xi}}{\partial E_{\mu}} = \frac{1}{\sigma_{\xi}(y < y_{\max})} \frac{d\sigma_{\xi}}{dy} \frac{e^{bt}}{E_{\xi}}$$
(5)

where

$$\frac{d\sigma_{\xi}}{dy} = \int_{x_{\min}}^{1} \frac{\partial^2 \sigma_{\xi}}{\partial x \partial y} dx \quad , \tag{6}$$

$$\sigma_{\xi}(y < y_{\max}) = \int_{0}^{y_{\max}} \frac{\partial \sigma_{\xi}}{\partial y} dy \tag{7}$$

and  $\partial^2 \sigma_{\xi}/\partial x \partial y$  are corresponding inclusive charged-current cross sections for the reaction  $\nu_{\mu}(\overline{\nu}_{\mu}) + N \rightarrow \mu^{+}(\mu^{-}) +$ anything. Here  $\xi \equiv \nu_{\mu}, \overline{\nu}_{\mu}, x = Q^2/2M\nu$  and  $y = \nu/E_{\xi}$  are the scaling variables,  $Q^2$  is an invariant momentum transfer between an incident neutrino and an outgoing muon,  $\nu = E_{\xi} - E_{\mu}^{*}$  is the energy loss in a laboratory frame, M is the nucleon mass,  $x_{\min} = 5 \cdot 10^{-6}$ ,

$$y_{max} = 1 - E_{\mu}^* / E_{\xi}, \tag{8}$$

$$E_{\mu}^{*} = (a + bE_{\mu}) \exp[b(t_{\max} - t)]/b - a/b$$
(9)

is the muon energy in production point t, parameters  $a = 0.002 \text{ MeV} \cdot \text{cm}^2/\text{g}$  and  $b = 4 \cdot 10^{-6} \text{ cm}^2/\text{g}$  (Gaisser et al.,



Fig. 4. Integral upward muon energy spectra. The data (Gaisser et al., 1995) obtained at  $-0.3 < \cos \theta < 0.3$ .

1995) determine muon energy losses:  $-dE_{\mu}/dt = a + bE_{\mu}$ . The probability of charged-current interaction in expression (1) at given depth t of the Earth matter has a form:

$$\frac{\partial W}{\partial t} = \sigma_{\xi} N_A exp(-\sigma_{\xi} N_A t) \tag{10}$$

where  $\sigma_{\xi} \equiv \sigma_{\nu,\overline{\nu}}(y < y_{max})$  is a total charged-current cross section (7) and  $N_A$  is the Avogadro number.

The depth limits of effective  $(y < y_{max})$  charged-current interactions in the Earth matter (t) are

$$t_{\min} = t_{max} - L(E_{\xi}, E_{\mu}),$$
 (11)

$$t_{\max}(\theta) = \int_0^{z_{\max}} \rho(r(z)) dz.$$
(12)

Here  $\rho(g/cm^2) = 13.8 - 1.73 \cdot 10^{-8} r$ (cm) is our approximation of the Earth density at the radius r,  $z_{\text{max}} = 2R \cos \theta$ ,  $R = 6.371 \cdot 10^8$  cm is the Earth radius,

$$L = (1/b) \ln[(a + bE_{\xi})/(a + bE_{\mu})]$$
(13)

is a rock thickness for muon detection with energy  $E_{\mu}$  on the Earth surface at y = 0.

### **3** Results

Calculations of upward muon energy spectra according to expression (1) are performed using numerical integration. The charged-current cross section for muon production by neutrino was taken from the renormalization-group-improved parton model (Quigg et al., 1986) with quark structure functions for  $x > 10^{-4}$  from (Eichen et al., 1984). We limited the range of  $x < 10^{-4}$  by  $x_{min} = 5 \cdot 10^{-6}$  because the check of



**Fig. 5.**  $(E_{\mu} - \langle E_N \rangle)$  and  $(E_{\mu} - \langle E_{\nu,\overline{\nu}} \rangle)$  correlations.

valence quark structure functions at very small x values by quark number sum rules

$$\int dx \ u_v(x, Q^2) = 2 \tag{14}$$

$$\int dx \ d_v(x, Q^2) = 1 \tag{15}$$

showed that the obtained errors of sum rules became more than 1 - 2% at  $x < x_{min}$ .

Upward muon differential energy spectra at different zenith angles are presented in Fig. 3. An accuracy of multidimensional integration (1) was less than 3%. Corresponding upward muon integral energy spectra (with accuracy 4 - 5%) in comparison with the same calculations (Minorikawa et al., 1995) are presented in Fig. 4.

Correlations of muon energy at observation level (near the Earth surface) with primary nucleon and parent-neutrino energies are shown in Fig. 5. Some irregularities of presented data at very high energies ( $E_{\mu} > 3 \div 5 \cdot 10^5$  GeV) in Fig. 3-5 are explained by the insufficient simulation sampling for atmospheric neutrino energy spectra (fig. 2) at corresponding energies and accuracies of multi-dimensional integrations.

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