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Non-extensivity in hadronic interactions at high energies

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Abstract. We examined the effect of generalization of the Boltzmann thermodynamical treatment of the multiparticle production processes, introducing non-extensive statistic, on the average multiplicities of charged particles as well as on the high transverse momenta of newly created particle distributions. It is shown that both can be described in the proposed way.

1 Introduction

The thermodynamical picture of particle creation in hadronic collisions was the first and quite successful attempt to describe it. The best and the most complete theory of such kind was presented in serial of papers by Hagedorn [see (Hagedorn, 1970) and references therein]. His idea of the fireball together with the famous proposition that "all fireballs are equal" were enough to give quite strong predictions concerning produced particle spectra. The formalism he used was based on the phase space shift representation of the interaction which gives a possibility to the reduction of S matrix elements and, finally, leads to the pure statistical (thermodynamic) phase space integrals.

One of the predictions was that the temperature of the "hadronic soup" (well defined) could not exceed a universal constant T_0 [of order of 160 MeV which value comes from examination of identified particle mass spectrum not as a result of fitting procedure using some multiparticle production (e.g., transverse momenta) data!] Just this prediction became one of the reasons why the Hagedorn theory were abundant (for some time) when more sophisticated, jet or QCD based ideas appeared. The temperature of the fireball is defined as the parameter T in the classical Boltzmann exponential term appearing, e.g., in probability weights for phase space average occupation numbers. This define the (asymptotic) form of transverse momentum distributions of particles created from decaying fireball. It was found that at high and

very high interaction energies the predicted exponential fall of high p_t do not agree with the observed one.

Successes of QCD based description of the hard processes giving deep insight into the nature of physics involved, and common believe that this is just the right theory of strong interactions makes the thermodynamical approach only one (very approximative, say simple and naive) of the tools of limited applicability and thus limited significance. But on the other hand the simplicity of the theory and notorious constant lack of the effective QCD theory of soft hadronization processes give a hope that the fireball idea can be enriched, modified and can become important (again).

2 Canonical partition function

The Hagedorn idea was used by Becattini and Heinz (1997) quite recently to describe the identified particle multiplicities in hadronization both, in e^+e^- annihilation and hadronic collisions. The *grand canonical* formalism of Hagedorn was replaced by the *canonical* one, much relevant for studies of small systems (like primary created fireballs) for which the requirement of exact conservation of some quantum numbers is important. In general, thermodynamics of the system is determined by the partition function which can be written as

$$Z\left(Q^0\right) \;=\; \sum_Q \delta(Q - Q^0) \prod_{i,j} x_{jk}^{\nu_{jk}} \;, \tag{1}$$

where x is the Boltzmann exponential factor $x = \exp\left(E_{jk}/T\right)$, j and k enumerate particle types and momentum cells, Q^0 is the initial fireball quantum number vector and Q is the respective vector of the particular state, ν_{jk} is the occupation number. For bosons one has $\nu_{jk} = 0, 1, 2, 3 \ldots$ and $\sum_{\nu_{jk}} x_{jk}^{\nu_{jk}} = 1/(1 - x_{jk})$, while for fermions $\nu_{jk} = 0, 1$ and $\sum_{\nu_{jk}} x_{jk}^{\nu_{jk}} = 1 + x_{jk}$. Thus introducing Fourier transform of δ (and reducing vector Q to 3-dimensional: charge, barion number and strangeness) Eq.(1) becomes

$$Z\left(Q^0\right) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3\phi \, \mathrm{e}^{iQ^0\phi} \times$$

$$\times \exp\left\{ \sum_{j=1}^{n_B} \sum_k \log\left(1 - x_{jk}\mathrm{e}^{-iq_j\phi}\right)^{-1} + \right.$$

$$\left. + \sum_{j=1}^{n_F} \sum_k \log\left(1 + x_{jk}\mathrm{e}^{-iq_j\phi}\right) \right\}, \quad (2)$$

where q_j is the quantum number vector of j-th particle. The next step is to turn the summation over phase space cells to an integration over momentum space

$$\sum_k \longrightarrow (2J_j + 1)\frac{V}{(2\pi)^3} \int \mathrm{d}^3p \qquad (3)$$

and all is ready for detailed numerical calculations.

3 Boltzmann statistic

3.1 Average multiplicities

With the known Z the average characteristics of the system can be obtained in an usual way. The average occupation number ν_j representing the mean multiplicity of the particles of the j-th kind is given by (introducing the fictious fugacity λ)

$$\langle n_j \rangle = \frac{\partial}{\partial\lambda} \log Z(Q^0, \lambda) , \qquad (4)$$

what, after some simplifications discussed in Becattini and Heinz (1997), gives

$$\langle n_j \rangle = \sum_{n=1}^{\infty} (-1)^{n+1}\gamma^{ns_j} (2J_j + 1)\frac{V}{(2\pi)^2} \int \mathrm{d}^3p \, x_{jk}$$

$$\times \frac{Z(Q^0 - nq_j)}{Z(Q^0)} . \qquad (5)$$

The last term in Eq.(5) represents classical chemical potential appearing as an effect of the canonical formalism.

To obtain the final set of equations the Boltzmann term $\exp(-\sqrt{p^2 + m_j^2}/T)$ have to be inserted instead of x_{jk} weights in Eqs(2) and (5). All integrals there can be computed within reasonable amount of computer time. It is shown (Becattini and Heinz, 1997) that hadronization volume V, temperature T and strangeness supression γ can be adjusted to the data and reproduce very well measured multiplicities. We have made similar calculations for charged particle multiplicities in $p\bar{p}$ interactions at the CERN SPS ($\sqrt{s} = 200, 546$ and 900 GeV). In the Fig.1 results of the comparison of our results with the data are given. We used for Q^0 the quantum numbers of proton and antiproton assuming two initial fireball creation. Because we limited our analysis to charged particles only (and we fixed the γ factor at 0.5) the data can be reproduces by points along the curve
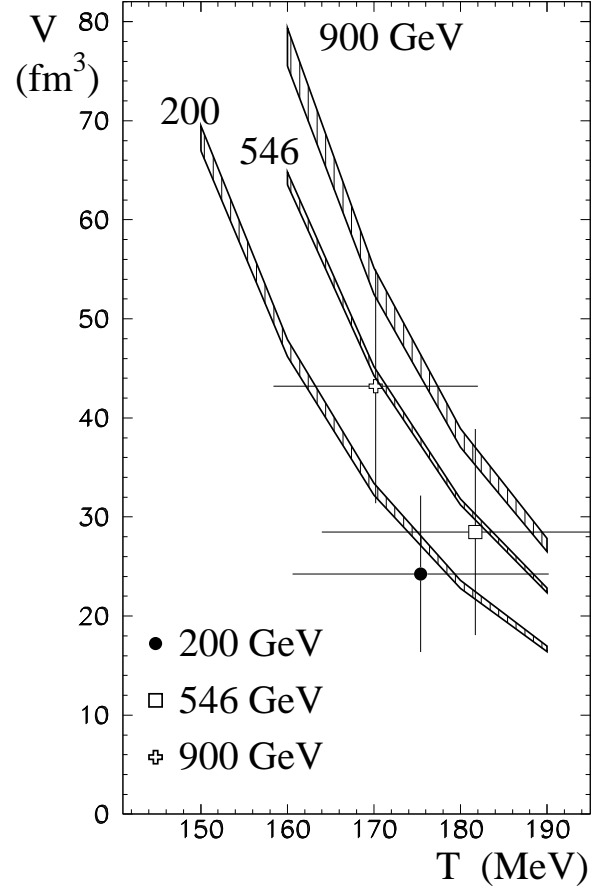

Fig. 1. Regions in ($V \times T$) space resulting to the observed average charged particle multiplicity measured in UA1 and UA5 experiments. Points show results of Becattini and Heinz (1997).

at the two parameter ($V \times T$) space. Dashed regions in Fig.1 represent the 1 s.d. (statistical only) regions allowed by the data. Points (which errors on v and T are strongly correlated) show results of Becattini and Heinz (1997).

3.2 Transverse momenta

Equation (2) can be used also to obtain the transverse momentum distribution for each kind of particle (of given mass). In the Hagedorn approach it has an assymptotic form (for $p_t \gg m, T$)

$$f(p_t) \sim p_t^{3/2} \exp\left(-p_t/T\right) , \qquad (6)$$

We have obtained transverse momentum distributions of charged particles integrating numerically the Eq.(2). The high p_t tails can be compared with data from SPS experiments (Albajar et al., 1990) and it is shown in Fig.2. As is clearly seen observed high values of transverse momentum are inconsistent with exponential behaviour expected for the Boltzmann statistic.
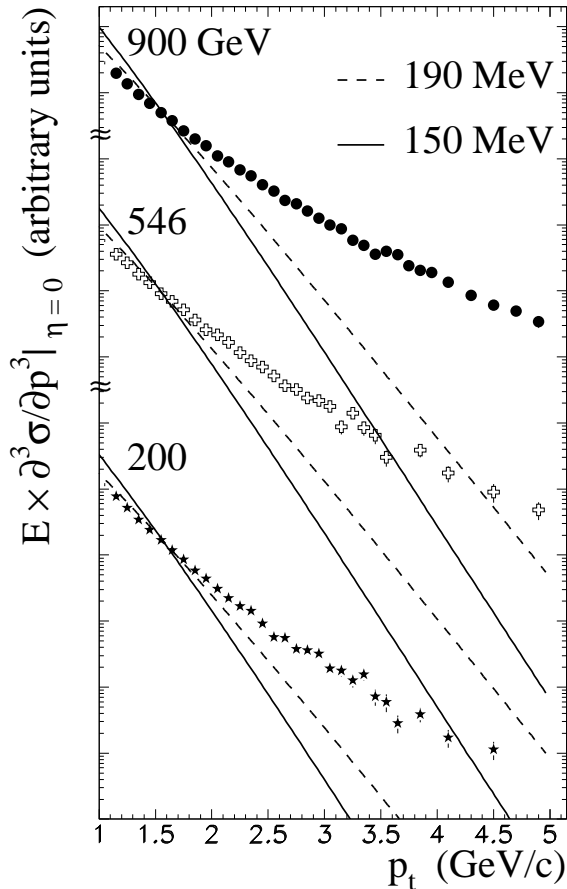
Fig. 2. Distributions of high transverse momenta measured at 200, 546 and 900 GeV compared with results of our calculations using Boltzman statistics. The lines are for $T = 190$ MeV (upper curve of each energy) to 150 MeV (lower one).

Fig. 3. Contours (as in Fig.1) in $(V \times T)$ space corresponding (within 1 s.d.) to average multiplicities observed at 200, 546 and 900 GeV for $q = 1.02$ (upper three regions) and $q = 1.05$ (lower ones).

4 Non-extensive statistics

Important modification of thermodynamical approach can be made introducing non-extensive (Tsallis) statistics. The idea was widely discussed by Beck (2000). Shortly speaking it can describe a lot of physical phenomena as different as, e.g., hydrodynamical turbulence, systems with long-range interactions, non-Poissonian fluctuations, fractality or other self-similar granulity of the system. The idea is to replace the Boltzmann exponential term with

$$\exp(-x/T) \longrightarrow (1 - (1-q)x)^{1/(1-q)} , \qquad (7)$$

where q is called non-extensivity parameter. For $q \rightarrow 1$ the Tsallis statistics tends to the ordinary Boltzmann form. It is shown by Bediaga *et al.* (1999) that high transverse momenta in e^+e^- annihilation into hadrons can be satisfactory described by the Tsallis factor with q changing from 1.02 at the energy of 14 GeV to 1.215 at 161 GeV (T_0 changes from 130 MeV to 110 MeV, respectively). We have introduced the change proposed in Eq.(7) to $p\bar{p}$ interactions at the SPS energies.
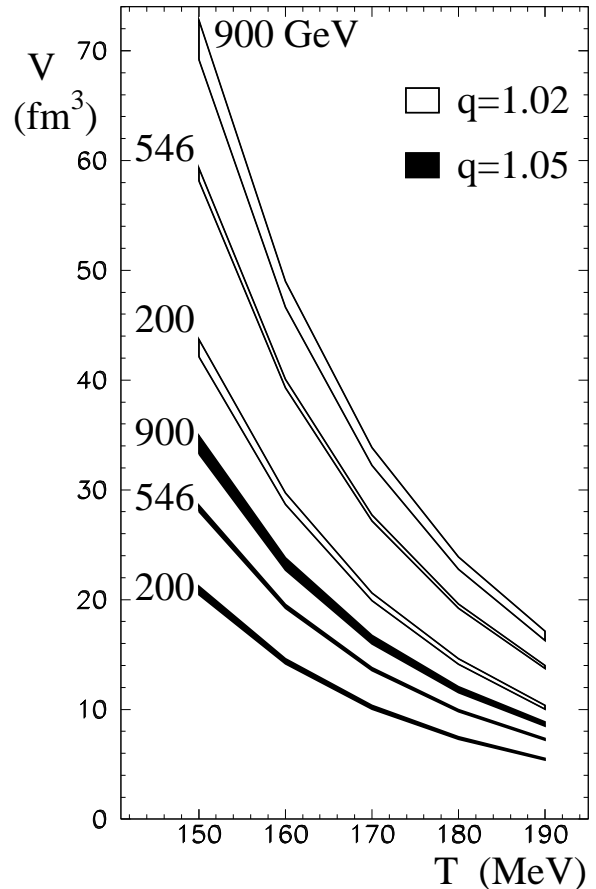
4.1 Charged particle multiplicities

In the Fig.3 results of our calculations are shown for two values of non-extensivity parameter, 1.05 and 1.02 in the same form as earlier in Fig.1 for Boltzmann statistics ($q = 1$).

The interaction volumes acceptable (and/or temperatures) are much smaller, but they still can represent the hadronization volume. The problem, how to sum up volumes if initial fireball fragment to a few moving with relativistic velocities along the interaction axis is not trivial [see discussion in (Becattini and Heinz, 1997)] specially for non-extensive statistics (when the entropy is not extensive quantity).

4.2 High transverse momenta

Similarly, as for $q = 1$ case shown in Fig.2, the transverse momentum distributions for $q = 1.02$ and $q = 1.05$ are presented in Fig.4.

First thing clearly seen is that the overabundance of high p_t in Fig.2 is explainable by Tsallis statistics with proper values of q. The changes of q with interaction energy are slightly masked by the possible change of the temperature parameter
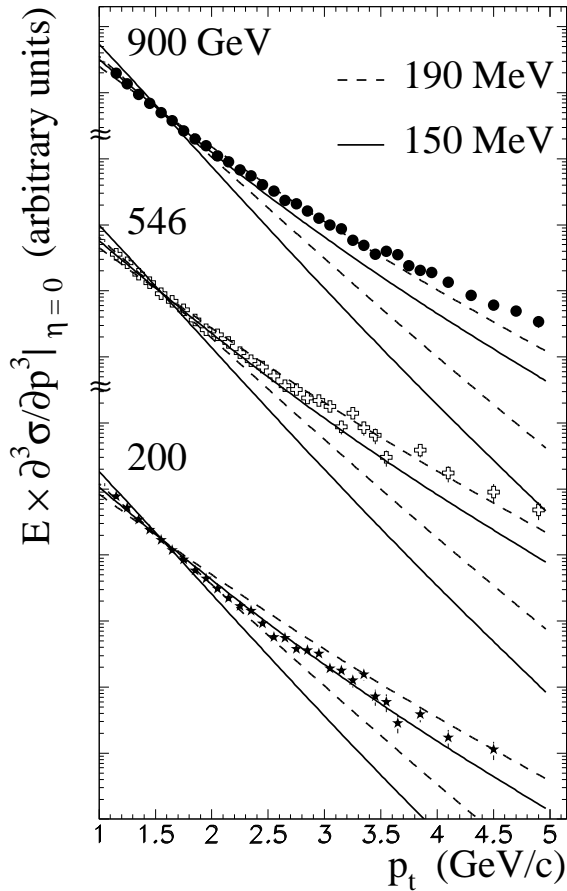
Fig. 4. Transverse momenta distributions obtained using Tsallis statistics with $q = 1.05$ (harder spectra) and $q = 1.02$ (softer ones) for 200, 546 and 900 GeV compared with SPS data. For each energy and q value the upper (dashed) line represents distribution for T=190 MeV, while lower (solid) for T=150 MeV.

T, but the evidence of the rise of the non-extensivity parameter over the SPS energy range is seen.

5 Conclusions

We have shown that the introduction of non-extensive statistics to the thermodynamical theory of multiparticle production in hadronic collisions can give a consistent description of particle multiplicities as well as high transverse momenta, not explainable in the classical Boltzmann–Hagedorn approach.

Proposed treatment of hadronization can put some new light on the particle creation physics and it is somehow complementary to the jet type non-perturbative QCD models. As such it deserved some attention and additional (extensive) studies

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