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An efficient Monte Carlo scheme for relativistic shock acceleration

R. J. Protheroe

Department of Physics and Mathematical Physics, University of Adelaide, SA 5001, Australia

Abstract. Obtaining good statistics for accelerated particle spectra, angular distributions and time distributions by simple Monte Carlo techniques is inefficient in relativistic shock acceleration because simulation of diffusion in the downstream region is very computer time-consuming. In this paper, I present Monte Carlo results for the angular and time distributions of particles returning to an ultrarelativistic shock after propagating downstream for the case of a random walk with isotropic scattering. These distributions have been parametrized and collectively represent an "event generator" for simulating quickly and efficiently the downstream part of an acceleration cycle.

1 Introduction

Acceleration of charged particles at relativistic shocks may occur in a variety of astrophysical locations, e.g. shocks in relativistic flows in active galactic nucleus jets and in the relativistically expanding fireball of a gamma ray burst. The nature of scattering plays an important role in shaping the angular distribution of particles crossing the shock, and hence the acceleration rate and spectrum (e.g. Kirk and Schneider 1987, Heavens and Drury 1988, Ellison et al. 1990). In the case of oblique shocks the main acceleration process may be shock drift acceleration (Begelman and Kirk 1990). The acceleration time scale and efficiency for various scattering models is discussed by Bednarz and Ostrowski (1996, 1999). For recent reviews of relativistic shock acceleration see Baring (1997) and Ostrowski (2001).

One of the simplest propagation models to implement in a Monte Carlo simulation is a random walk with isotropic scattering after each step. This model is very probably unrealistic in the upstream region for most relativistic astrophysical shocks, but it might possibly apply downstream if the downstream plasma flow is highly turbulent, resulting in a fully tangled magnetic field, although this is far from certain (Bednarz and Ostrowski 1996). Nevertheless, I adopt it here because of its simple implementation in a Monte Carlo code; the purpose of the present work being to develop numerical schemes which will allow the rapid simulation of acceleration to extremely high energies. The techniques developed will be applied in an accompanying paper (Protheroe 2001) to more realistic scattering models in future work, which may also include in an approximate way shock modification by non-linear affects.

The approach I use here is to consider particles of a particular momentum injected from upstream to downstream at the shock (I use unprimed variables for the upstream frame, singly-primed variables for the downstream frame, and doubly-primed variables for the shock frame). For injection at a particular angle θ'_1 to the shock normal I obtain: (1) the probability of returning to the shock $\mathrm{Prob.}(\mathrm{return}, \theta'_1)$, (2) the angular distribution of particles crossing the shock from downstream to upstream at a particular angle θ'_2 to the shock normal $p(\cos\theta'_2; \theta'_1)$, (3) the distribution of time spent downstream t'_d before returning for angles θ'_1 and θ'_2, $p(t'_d; \theta'_2, \theta'_1)$. These distributions enable the downstream part of shock acceleration to be simulated quickly and efficiently by sampling (a) $\cos\theta'_2$ from $p(\cos\theta'_2; \theta'_1)$, (b) t'_d from $p(t'_d; \theta'_2, \theta'_1)$ for a particle crossing from upstream to downstream at angle θ_1.

2 Acceleration cycle and definitions

I consider a strong relativistic shock, i.e. Lorentz factor of shock in upstream frame $\Gamma_1 \gg 1$, propagating through a relativistic plasma (ratio of specific heats: 4/3). In this case, the downstream flow Lorentz factor is $\Gamma_2 \approx \sqrt{9/8}$ and velocity is $u_2 \approx c/3$. I define the direction of flow in the shock frame to be in the positive x''-direction, and the shock to be at $x'' = 0$ ($x'' < 0$ is upstream and $x'' > 0$ is downstream). The shock geometry and an acceleration cycle are illustrated in Fig. 1.

Correspondence to: rprother@physics.adelaide.edu.au

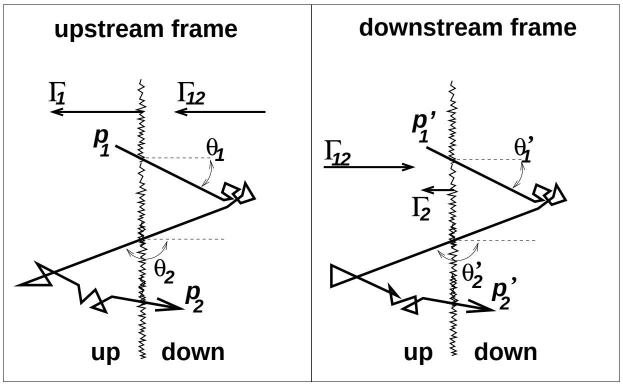

Fig. 1. Shock geometry and an acceleration cycle (p is particle momentum in upstream frame). Note: $+x$ direction is defined differently in some papers (i.e. in some previous work $\cos\theta = 1$ points upstream, whereas it points downstream in the present work).

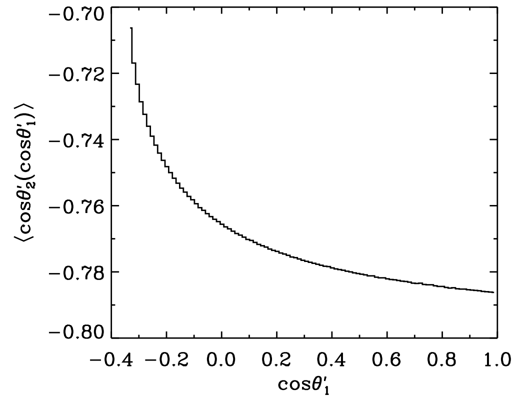

Fig. 3. Average value of $\cos\theta'_2$ on returning to the shock after crossing the shock from upstream to downstream with $\cos\theta'_1$.

3 Simulation of motion downstream

The scattering is modeled by isotropic scattering after traveling a distance (free path) sampled from an exponential distribution with a given mean free path $\lambda'_{\rm sc}$, e.g., in the case of Bohm diffusion $\lambda'_{\rm sc} = k r'_{\rm gyro}(p') \propto p'$, where $k \geq 1$ is a constant. The probability of returning to the shock, the time spent downstream, and the angular distribution on returning to the shock all depend on the injection angle downstream θ'_1 (see Fig. 1). In this simulation, a particle is deemed to have escaped downstream if it goes beyond 8 scattering mean free paths downstream of the shock. The probability of returning to the shock obtained by Monte Carlo simulation is plotted as a function of $\cos\theta'_1$ in Fig. 2, and the average value of $\cos\theta'_2$ is plotted as a function $\cos\theta'_1$ in Fig. 3. Note that in order to cross the shock from upstream to downstream $\cos\theta'_1$ must be between $-\beta_2$ and 1, where $\beta_2 = 1/3$.

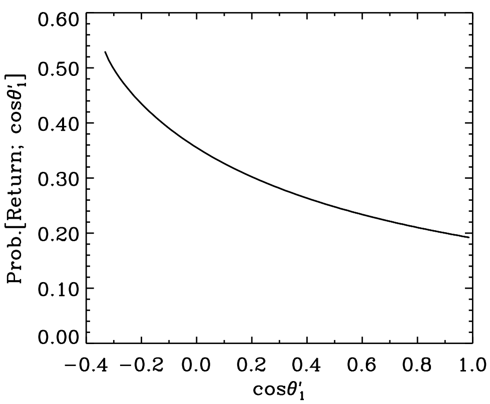

Fig. 2. Probability of returning to the shock after crossing the shock from upstream to downstream with $\cos\theta'_1$.

In the following sections I explore the angular and time distributions of returning particles, and provide a parametrization of these distributions which will facilitate efficient Monte Carlo simulation of relativistic shock acceleration.

4 Angular distribution of particles returning to the shock from downstream to upstream.

Particles were injected at the shock at 100 discrete values of $\cos\theta'_1$ in the range -1/3 to 1, and their directions on returning to the shock were binned in 100 bins with $\cos\theta'_1$ in the range -1 to -1/3. The distribution of $\cos\theta'_2$ on returning to the shock after crossing the shock from upstream to downstream for three different $\cos\theta'_1$ values are shown by the histograms in Fig. 4(a). I have performed a least squares fit to the $\cos\theta'_2$ distributions, for all $\cos\theta'_1$ values simulated, using a function of the form

$$p(\cos\theta'_2; \cos\theta'_1) = (1+\alpha)(3/2)^{(1+\alpha)}(-\cos\theta'_2 - 1/3)^{\alpha},$$

and these fits for $\cos\theta'_1 = -0.3, 0,$ and 1.0 have been added to Fig. 4(a) and are seen to be in reasonable agreement with the histograms. The exponent $\alpha(\cos\theta'_1)$ giving the best fit is plotted as a function $\cos\theta'_1$ in Fig. 4(b).

For particles downstream which return to the shock, the time spent downstream, t'_d, depends not only on the direction in which the particle crossed the shock from upstream to downstream, θ'_1, but also on the direction of the particle on returning to the shock, θ'_2 (see Fig. 1). The average time spent downstream is plotted as a function of $\cos\theta'_1$ in Fig. 5. Note that all times downstream are measured in units of the downstream mean scattering time, $t'_{\rm sc} = \lambda'_{\rm sc}/c$.

5 Distribution of time spent downstream

Examples of the distribution of time spent downstream are given in Fig. 6(a)—(d) for various combinations of directions of crossing the shock from upstream to downstream and from downstream to upstream. The distribution of the downstream

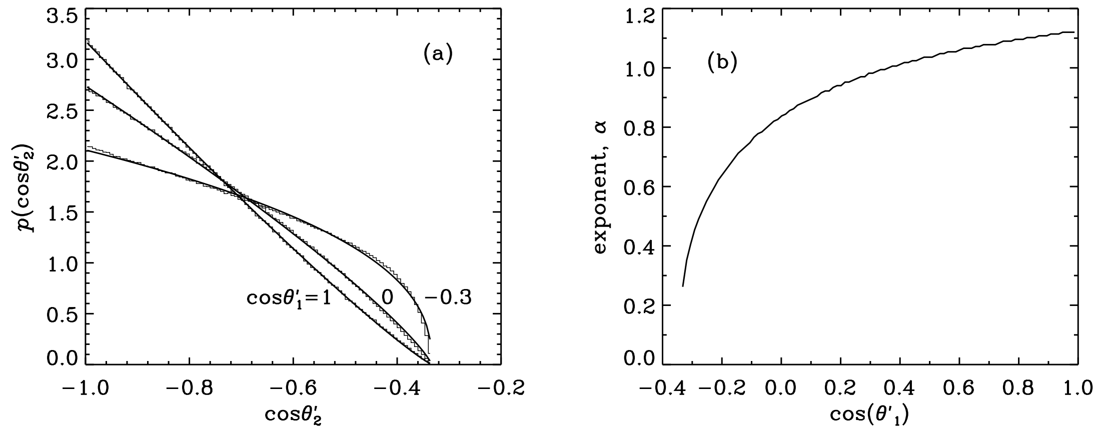

Fig. 4. (a) Distribution of $\cos\theta'_2$ on returning to the shock after crossing the shock from upstream to downstream with $\cos\theta'_1 = -0.3, 0,$ and 1.0 as labelled. The histograms show the Monte Carlo results and the curves show the fits. (b) Exponent α in the fit to the $\cos\theta'_2$ distributions as a function of $\cos\theta'_1$.

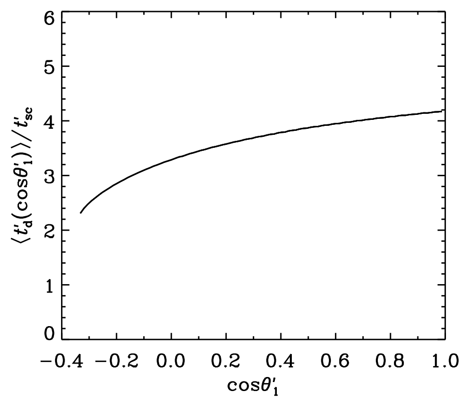

Fig. 5. Average time spent downstream in units of the downstream scattering time, $t'_{\rm sc}$, before returning to the shock after crossing the shock from upstream to downstream at angle θ'_1.

time obtained from the full Monte Carlo simulation has been fitted by the sum of two exponential distributions,

$$p(t'_d;\theta'_2,\theta'_1) = \frac{a}{\tau_1}\exp(-t'_d/\tau_1) + \frac{1-a}{\tau_2}\exp(-t'_d/\tau_2),$$

where a, τ_1 and τ_2 are functions of (θ'_2,θ'_1).

I determine the functions $a(\theta'_2,\theta'_1)$, $\tau_1(\theta'_2,\theta'_1)$ and $\tau_1(\theta'_2,\theta'_1)$ from the first three moments of the distributions $p(t'_d;\theta'_2,\theta'_1)$, i.e. $\langle {t'_d}^n(\theta'_2,\theta'_1)\rangle$ for $n = 1, 2, 3$, by analytic solution of the three simultaneous equations

$$\begin{aligned}
\langle t'_d\rangle &= a\tau_1 + (1-a)\tau_2 \\
\langle {t'_d}^2\rangle &= 2[a{\tau_1}^2 + (1-a){\tau_2}^2] \\
\langle {t'_d}^3\rangle &= 6[a{\tau_1}^3 + (1-a){\tau_2}^3]
\end{aligned}$$

where for brevity I have dropped the explicit dependence on (θ'_2,θ'_1).

I have performed a least squares quadratic fit of the form

$$\langle {t'_d}^n(\theta'_2,\theta'_1)\rangle = b_{n0}(\cos\theta'_1) + b_{n1}(\cos\theta'_1)\cos\theta'_2 + b_{n2}\cos^2\theta'_2$$

to $\langle {t'_d}^n(\theta'_2,\theta'_1)\rangle$ obtained from the full Monte Carlo simulation, and found the dependence of the coefficients of $\cos\theta'_1$ can be well represented by

$$b_{nj}(\cos\theta'_1) = c_{0nj} + c_{1nj}\cos\theta'_1 + c_{2nj}\cos^2\theta'_1$$

where the coefficients c_{inj} are given in Table 1.

Table 1. The coefficients c_{inj}

		$i=0$	$i=1$	$i=2$
$n=1$	$j=0$	0.52938775	1.6197228	-0.81077940
$n=1$	$j=1$	-3.9592780	0.021014775	-0.011342470
$n=1$	$j=2$	-0.50736811	0.0	0.0
$n=2$	$j=0$	-1.1471720	16.573265	-6.1717835
$n=2$	$j=1$	-29.217302	-10.154183	5.4653394
$n=2$	$j=2$	14.860891	0.0	0.0
$n=3$	$j=0$	11.353390	246.78429	-50.493735
$n=3$	$j=1$	-202.20586	-362.75654	188.74139
$n=3$	$j=2$	634.22088	0.0	0.0

Comparison of the distributions obtained from the fits to $\langle {t'_d}^n\rangle$ described above is made with the examples of the distribution of time spent downstream from the Monte Carlo simulation given in Fig. 6(a)—(d), and seen to reproduce the Monte Carlo results well.

6 Summary

For the case of random walk plus isotropic scattering, the distributions given in this paper provide a quick and efficient method for simulation of propagation of particles downstream

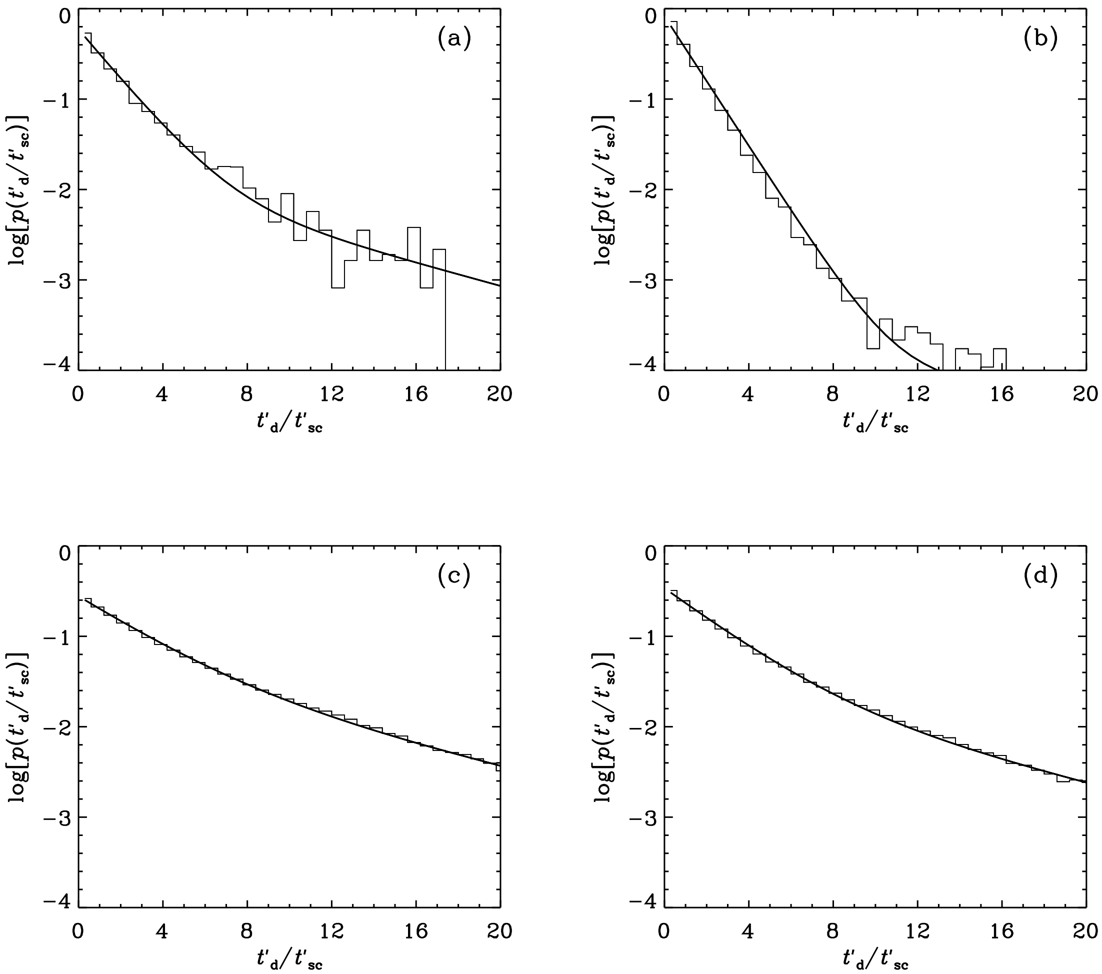

Fig. 6. Examples of distributions of time spent downstream for various combinations of $(\cos\theta_1', \cos\theta_2')$: (a) (0.3267,-0.3367), (b) (-0.2067, -0.3367), (c) (0.5933, -0.9967), and (d) (-0.8600, -0.6700). Solid curves: distributions obtained from the fits to $\langle {t_d'}^n \rangle$ described in the text.

in shock acceleration by ultrarelativistic shocks. In effect they collectively represent an "event generator" for simulation of the downstream part of the acceleration cycle. An application of these distribution is described in an accompanying paper (Protheroe 2001). In future work I plan to investigate the development of similar downstream event generators for more realistic scattering models.

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