Mean square radius of EAS electrons

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Abstract. Detailed theoretical study of the mean square radius of extensive air shower electrons has been made in connection with further development of scaling formalism for electron lateral distribution function. A very simple approximation formula, which allows joint description of all our results obtained in wide primary energy range and for different observation depths is presented. The sensitivity of the mean square radius to variations of basic parameters of hadronic interaction model is discussed.

1 Introduction

In a series of publications [A. A. Lagutin et al. (1997, 1998, 1999), see also R. I. Raikin et al. (this proceedings)] we described model-independent scaling property of lateral distribution function (LDF) of electrons in the air showers. These results allow to reproduce the electron LDF up to $r \sim (25 - 30) R_{\text{m.s.}}$, where $R_{\text{m.s.}}$ is mean square radius of electrons. In particular, it was shown that if one use the mean square radius as a radial scale parameter of lateral distribution function instead of Moliere unit, then normalized electron LDF becomes invariable on the primary energy and the age of cascade. In other words, the dependence of the shape of lateral distribution function on energy, observation depth, chemical composition of primaries and even features of hadronic interactions is well described by the variation of single scale parameter $R_{\text{m.s.}}$. Unfortunately, such a sensitive characteristic can not be estimated well directly from experimental data so far as it demands high precision measurements of lateral distribution of electron component in wide radial distance range (from several meters to several thousands meters from the core). Thus the detailed theoretical study of $R_{\text{m.s.}}$, including its sensitivity to the hadronic interaction model, is necessary for the comparison of our LDF with experimental data of different arrays.

In this paper we present our latest results on the mean

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square radius of electrons in extensive air showers (EAS). Detailed analysis of these results shows that there is a possibility of very simple joint description of all data obtained for primary energies $E_0 = (10^{14} - 10^{18})$ eV and atmospheric depths $t = (614 - 1030)$ g/cm^2. We present such formula here together with initial results on the sensitivity of $R_{\text{m.s.}}$ to variations of basic parameters of hadronic interaction model.

2 Calculation methods

In order to simulate extensive air showers of superhigh energies we resort to the combination of full Monte-Carlo treatment of hadronic cascade with analytical expressions of the results of numerical calculations of electromagnetic subshowers. Two different approaches were used to obtain the characteristics of pure electromagnetic cascade initiated by primary photon: the semi-analytical Monte-Carlo method and the method based on the numerical solution of adjoint cascade equations. For detailed description of these methods see A. V. Plyasheshnikov et al. (1988); A. K. Konopelko, A. V. Plyasheshnikov (1997); A. A. Lagutin (1993); A. A. Lagutin et al. (1998). It should be noted here that the main advantage of semi-analytical Monte-Carlo method is high accuracy in tracking very distant particles, because complete Monte-Carlo treatment is applied for low-energetic part of cascade with threshold energy $E_{\text{th}} = 0.1$ MeV. On the other hand, the numerical solution of adjoint cascade equations taking into account the deflection of photons in the multiple Compton scattering is preferable for independent calculations of moments of lateral distribution function (such as mean square radius) and gives very high performance in fluctuation problem and sensitivity problem.

For hadronic part of the cascade we used simplified algorithmic generator (A. A. Lagutin et al., 1997, 1998) based on the quark-gluon string model (A. B. Kaidalov et al., 1986; Yu. M. Shabelski, 1986) with extrapolation to 10^{18} eV. Though our generator is very simple, especially in comparison with modern high-developed codes, the main features of

interactions, that make a major impact on shower development (including fluctuations of full and partial inelasticity coefficients) are well reproduced by our program [see A. A. Lagutin et al. (1998); R. I. Raikin (2000)]. Besides this code is well adapted for the investigation of sensitivity of results to variations of basic parameters of hadron-nucleus interaction model.

Due to complete Monte-Carlo treatment of hadronic cascade, the computation time depends strongly on primary energy. Nevertheless, the simplicity of hadronic generator together with rejection of tracking muons in combination with analytical expressions used for electromagnetic component allowed us to perform very speeding calculations, that ensure maximum 1.5% relative statistical error in $R_{\mathrm{m.s.}}$ estimation.

Thus we calculated cascade curves and lateral distributions of electrons in $(10^{14} - 10^{18})$ eV proton-induced EAS at six atmospheric depths from 614 to 1030 g/cm^2 in standard atmosphere. Since we implemented analytical expressions of normalized average LDF and mean square radius for each partial electromagnetic subshower taking into account fluctuations of shower size caused by both hadronic and electromagnetic parts of a cascade, the mean square radius of EAS was calculated in the following way:

$$R_{\mathrm{m.s.}} = \left[\frac{\sum\limits_{i} \left(N_e R_{\mathrm{m.s.}}^2 \right)_i}{\sum\limits_{i} (N_e)_i} \right]^{1/2} , \qquad (1)$$

were index i denotes characteristics obtained for i^{th} individual extensive air shower, the sum goes over all simulated EAS of fixed primary energy.

3 Results

The results of our calculations of the mean square radius of electrons in vertical extensive air showers generated by primary protons of $E_0 = (10^{14} - 10^{18})$ eV for atmospheric depths $t = (614 - 1030)$ g/cm^2 are presented in Fig. 1. One can see from this figure, that at fixed atmospheric depth $R_{\mathrm{m.s.}}$ decreases with primary energy. That is, our results indicate narrowing lateral distribution of electrons with primary energy in whole considered energy range. Though *narrowing rate* $\mathrm{NR} = \partial R_{\mathrm{m.s.}}/\partial \log E_0$ becomes smaller when energy increases, it is still essential at 10^{18} eV. It can be understood as a reaction to the decrease of ages of powerful partial electromagnetic subshowers, which give main contribution to the EAS electron yield.

The behaviour of mean square radius with atmospheric depth is, however, more complicated. The basic tendency is obvious expansion of shower in process of development realized in increasing mean square radius. Nevertheless, there is distinct maximum in $R_{\mathrm{m.s.}}(t)$ obtained for relatively low primary energies, that shifts deeper into the atmosphere when energy increases. The analysis of our results shows that such a behaviour can not be explained by only the influence of the increase of air density. There is another one mechanism
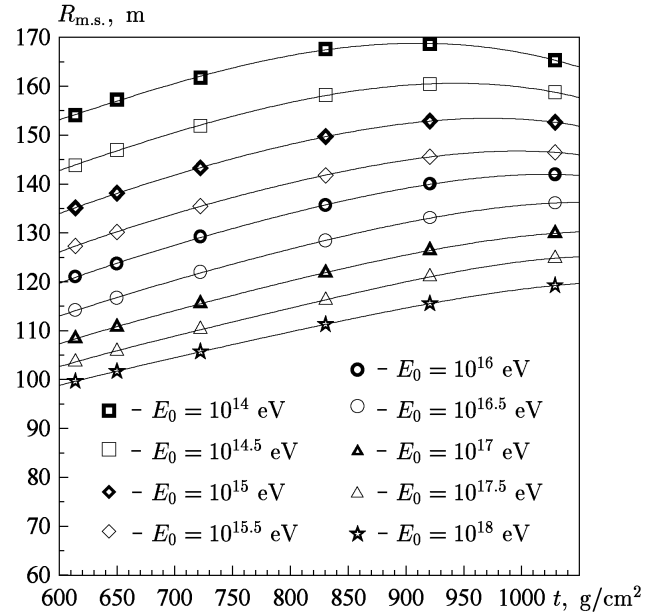

Fig. 1. Mean square radius of electrons in vertical proton-initiated EAS. Symbols – our calculational data for $E_0 = (10^{14} - 10^{18})$ eV and $t = (614 - 1030)$ g/cm^2. Curves – polynomial approximation

of the narrowing of average lateral distribution related to the correlation between shower size $[(N_e)_i]$ and width $[(R_{\mathrm{m.s.}})_i]$ in individual showers at fixed primary energy.

In Fig. 2 we show this correlation by plotting $(R_{\mathrm{m.s.}})_i$ vs. $(N_e)_i$ at sea level (a) and at 614 g/cm^2 (b) for individual showers of 10^{14}, 10^{16} and 10^{18} eV (500 showers of each energy were included in data set). It is seen from part a) of a figure that at sea level $(R_{\mathrm{m.s.}})_i$ and $(N_e)_i$ demonstrate strong anticorrelation. Since relatively large and simultaneously narrow individual showers of considered energy give more essential contribution to average LDF [see also eq. (1)], the expansion of the average shower is slowing down with increasing fluctuations when atmospheric depth increases. As analysis shows, such anticorrelation exists at observation depths located deeper than level t_{opt}, where fluctuations of $(N_e)_i$ are minimal (according to our results, $t_{\mathrm{opt}} = t_{\mathrm{max}} + 55$ g/cm^2, where t_{max} is depth of maximum of average cascade curve). Thus we should expect existence of such a maximum at higher energies in case of artificial showers, which continue their development below the sea level assuming the further exponential growth of air density together with increasing fluctuations. On the other hand, for atmospheric depths above t_{opt} such as, for example, 614 g/cm^2 for $E_0 = 10^{18}$ eV (see Fig. 2b) mean square radius correlate with shower size and this mechanism works conversely.

In this connection it is necessary to note, that $R_{\mathrm{m.s.}}$ estimated as (1) describes the shape of average lateral distribution of electrons and deviates distinctly from *average mean square radius* $\langle R_{\mathrm{m.s.}} \rangle$, which one can calculate using data
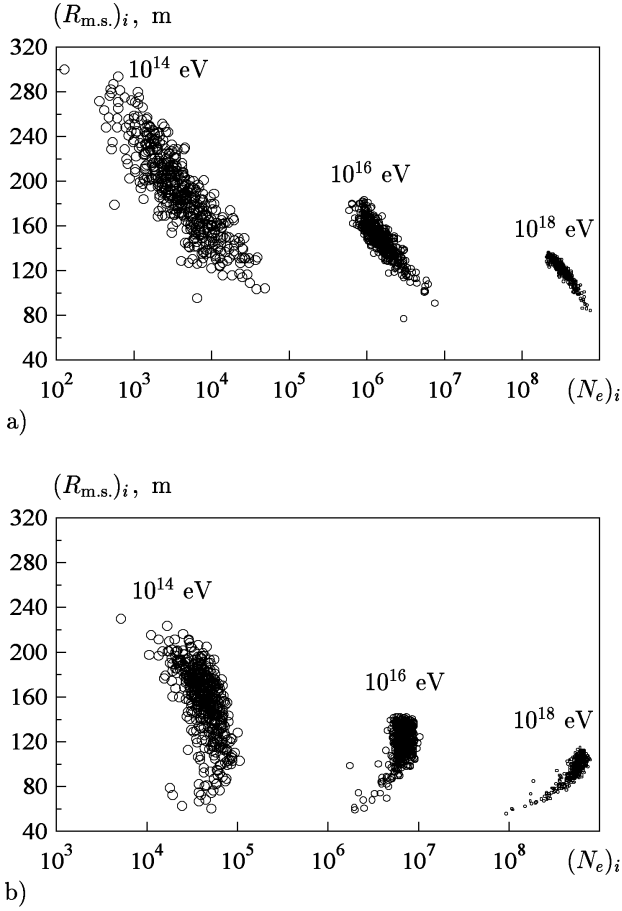
Fig. 2. Correlation between $(N_e)_i$ and $(R_{\text{m.s.}})_i$ in individual extensive air showers of $E_0 = 10^{14}$, 10^{16} and 10^{18} eV at $t = 1030$ g/cm^2 (a) and $t = 614$ g/cm^2 (b)

about individual showers as

$$\langle R_{\text{m.s.}} \rangle = \frac{1}{n} \sum_{i=1}^{n} (R_{\text{m.s.}})_i.$$

The difference becomes essential when observation level is located far from t_{opt}. For example, at sea level 10^{14} eV individual showers are (on average) significantly wider than average shower ($\langle R_{\text{m.s.}} \rangle$ is 15% larger than $R_{\text{m.s.}}$).

It is important that, though mean square radius is affected by number of different factors, a very simple approximation of all our results based only on the information about longitudinal shower development is possible. On Fig. 3 the whole set of our calculational data is presented in the form $F_R(s') = R_{\text{m.s.}}(s') \times \rho_0/\rho(t)$, where $\rho(t)$ is air density at depth t, $\rho_0 = 1.225$ g/cm^3, $s' = t/(t_{\text{max}} + 100 \text{ g/cm}^2)$. It is seen that functions $F_R(s')$ for different primary energies are precisely superposed with each other in overlapping s' intervals. The fit of $F_R(s')$ with maximum relative error of 1% was obtained using method of maximum likelihood as $F_R(s') = a + b \cdot \text{arctg}(s' - 1)$, where a and b – free parameters. Thus we can propose an analytical expression for

$R_{\text{m.s.}}(E_0, t)$ as follows:

$$R_{\text{m.s.}}(E_0, t) = \frac{\rho_0}{\rho(t)} 173.0 \times$$

$$\times \left[0.546 + \frac{2}{\pi} \text{arctg} \left(\frac{t}{t_{\text{max}} + 100 \text{ g/cm}^2} - 1 \right) \right], \text{ m.} \quad (2)$$

Our results on t_{max} for proton-induced showers were approximated as

$$t_{\text{max}}(E_0) = 740 + 65 \lg(E_0/10^{18} \text{ eV}), \text{ g/cm}^2. \quad (3)$$

Assuming the validity of the superposition model for showers initiated by a nucleus of energy E_0 and mass A, we obtain the relation

$$R_{\text{m.s.}}^A(E_0, t) = R_{\text{m.s.}}^p(E_0/A, t),$$

where upper indexes denote the type of primary particle.

4 Discussion

Expression (2) together with scaling parameterization of LDF (A. A. Lagutin, R. I. Raikin, 2001; R. I. Raikin et al., this proceedings) allows one to obtain the reliable data on normalized average lateral distribution function of EAS electrons up to $r \sim (25 - 30)R_{\text{m.s.}}$ ($\sim 2000 - 3000$ m) in wide primary energy interval and for any practically important atmospheric depth from mountain level to sea level. Nevertheless, there is always vital question related with applicability of any theoretical result in the field of EAS research: how strong it depends on hadronic interaction model used in calculations?

In our papers [see, for example, A. A. Lagutin et al. (1997, 1998); R. I. Raikin (2000)] it was shown that scaling property of electron LDF is very poorly sensitive to variation of basic parameters of model. Thus mean square radius carries the whole information about the influence of features of hadronic interactions on the shape of LDF as well as about mass composition and shower development. Since the key parameters of hadronic interactions most influential for the longitudinal development of EAS are inelastic cross section of p-air collisions and inelasticity (J. Knapp, 1999), we have performed the basic test of sensitivity of expression (2) by the change of these parameters in our code. We used p-air cross sections from different well-known hadronic models (J. Knapp, 1999) (including MOCCA'92 cross section, which demonstrates highest increasing rate with primary energy), and independently varied inclusive energy spectra of secondaries in such a way that spectrum-weighted moments of inclusive spectra change by $\pm 20\%$ [see G. Battistoni (1999)]. Our results show, that within the limits defined by widespread hadronic models formula (2) remains valid.

It is also important, that if one extrapolate expression (2) to ultrahigh primary energies ($E_0 > 10^{18}$ eV), then values of variable s' corresponding to $t \approx (920 - 1030)$ g/cm^2 will be kept within interval well presented in our data set by the
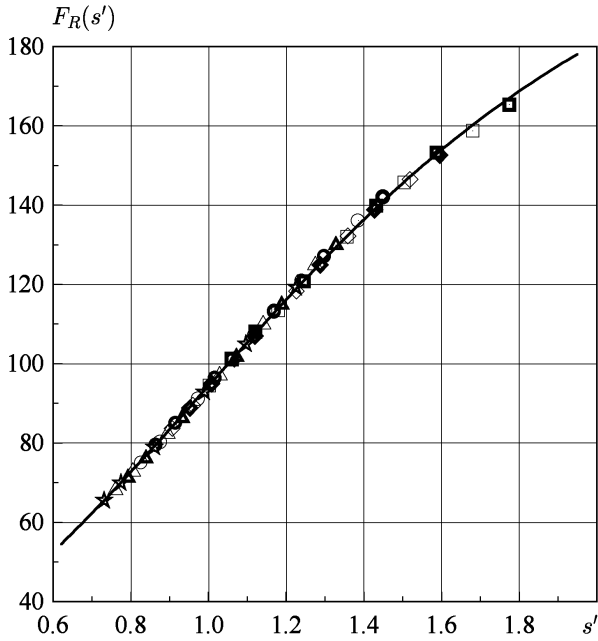
Fig. 3. Function $F_R(s') = R_{\mathrm{m.s.}}(s') \times \rho_0/\rho(t)$, where $s' = t/(t_{\max} + 100 \ \mathrm{g/cm^2})$. Symbols are same as in Fig. 1. Curve – fitting function (2)

values calculated for moderate energies and higher observation levels [for example, $s' = 0.95$ when $E_0 = 10^{20}$ eV and $t = 920 \ \mathrm{g/cm^2}$, assuming the validity of formula (3)]. Though such extrapolation is rather formal, we expect that it can give reliable predictions at least in the energy range, where Landau-Pomeranchuk-Migdal and magnetic field effects of hadron-initiated showers and also π^0-air inelastic interactions are negligible. Of course, special calculations are needed to confirm the reliability of extrapolation of formula (2).

5   Conclusion

This paper represents the further development of scaling formalism (A. A. Lagutin et al., 1997, 1998, 1999) for the description of lateral distribution function of EAS electrons. The calculations of the mean square radius of electrons in extensive air showers have been made for primary energies $E_0 = (10^{14} - 10^{18})$ eV and atmospheric depths $t = (614 - 1030) \ \mathrm{g/cm^2}$. Very simple approximation formula for $R_{\mathrm{m.s.}}(E_0, t)$, that gives the one-valued relation between depth of maximum of average cascade curve and the shape of average lateral distribution of electrons is presented. It can be practically implemented during comparisons of experimental data of large ground-based air shower arrays with results obtained by fluorescent detectors.

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