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Radio pulses in dense media: Simulation versus approximations

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Abstract. We calculate radio pulses from high energy showers in ice using the depth development of the charge excess together with fits to the lateral distribution obtained in detailed shower simulations. These approximations allow the calculation of pulses from EeV showers. The results for the spectrum and angular distribution of the electric field are compared to full radio emission simulations. The accuracy and ranges of validity of the approximations are addressed.

1 Introduction

It was about 40 years ago that Askar'yan proposed the detection of very high energy particles through the coherent radio emission from all the charged particles in a high energy shower (Askar'yan , 1961). An excess charge of order 20% (Halzen et al., 1991) is expected from processes that accelerate matter electrons into the shower. Provided the wavelength of the radiation exceeds the characteristic length of the emitting source the emission is coherent and the electric field is proportional to the excess charge. As the number of particles in a shower is also proportional to the shower energy, the emitted power in coherent pulses is expected to scale with the square of the primary energy and this makes it particularly attractive for the study of the highest energy particles.

The radio technique was studied extensively in the context of air showers in the 1960's and 1970's but it was then practically discontinued because of technical difficulties (Weekes, 2001). In the mid 1980's the efforts to build high energy neutrino telescopes renewed the activity in the radio technique as a possible alternative to the other neutrino detection projects. The possibilities included detection in dense media such as ice (Markov and Zheleznykh, 1986; Ralston and McKay, 1990) or sand that can be found in nature in large quantities as well as the possibility to detect showers produced just under the moon surface using radiotelescopes on the Earth (Dagkesamansky and Zheleznykh, 1989).

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As part of this effort a Monte Carlo program was specifically developed for studying the shower development with sufficient accuracy to allow a precise calculation of distant radio signals from TeV electromagnetic showers in ice (Halzen et al., 1991; Zas et al., 1992; Zas, 2001). Improved assessments of the possibility to detect neutrino events with arrays of antennas in Antarctica (Provorov and Zheleznykh, 1995; Frichter et al., 1996; Jelley, 1996) led to experimental efforts in Antarctica (Frichter et al., 1999). Other attempts (Razzaque et al., 2001) to calculate radio pulse distributions from TeV showers have been recently performed using standard simulation packages. Later the results of these calculations were extended to both electromagnetic and hadronic showers of energies above the EeV range, most promising for the radio technique. Cosmic rays of these energies are routinely observed and neutrinos are difficult to avoid in most of the models attempting to explain them. However the energies are too high to be studied by brute force Monte Carlo simulation and it is clear that approximations will have to be made. A specific approximation was developed to tackle the challenge, the *one dimensional* approximation (Alvarez-Muñiz and Zas, 1997, 1998; Alvarez-Muñiz et al., 1999). This approximation has been discussed from a more general theoretical approach in (Buniy, 2000).

In this article we firstly discuss shower results obtained with our simulation (Halzen et al., 1991) in the context of shower theory. For the radio pulses we compare analytical approximations, the one dimensional approximation and results obtained with our full simulations. We then relate the high frequency regime of the spectrum of the radio signals to the lateral distribution. It is our hope that these comparisons may be of help in controlling minor differences between numerical approaches.

2 A shower code designed for radio emission.

The calculation of radio pulses from showers is a complex problem in which the emission from all particles in a high

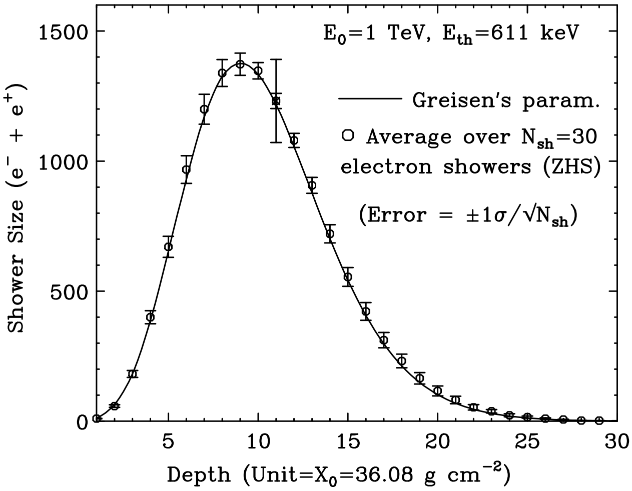

Fig. 1. Depth development of an average electromagnetic shower compared to Greisen's parameterization. Shown is the average number of electrons for 30 1 TeV showers and its error. For 11 radiation lengths the error bar indicates the RMS deviation for the sample.

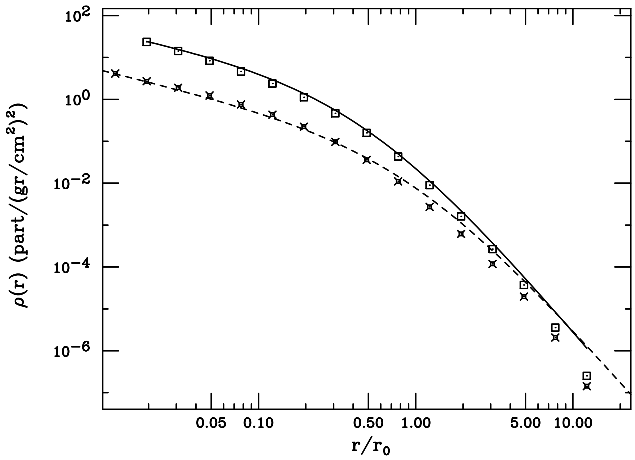

Fig. 2. Lateral distribution function of charged particles obtained from Monte Carlo (squares) and of the charge excess (crosses) for an electromagnetic shower at maximum compared to the NKG parameterization with $s = 1$ (full line) and $s = 1.1$ (dashed line). Here r_0 is the theoretical Molière radius (10.4 g cm^{-2}). The reference radius for the NKG parameterization is 0.5 in these units.

energy shower have to be carefully added in a coherent fashion, taking into account the velocity and direction, position and time of each of the particles involved. For a medium such as ice, having a refraction index $n = 1.78$ for radio waves, electrons of energy exceeding 107 keV emit Čerenkov radiation. In the coherent regime in a dense medium such as ice it is the excess electrons that give the largest contribution. The simulation of electromagnetic showers in ice and the subsequent calculation of the frequency spectrum of the radio pulse was tackled by Halzen, Stanev and Zas in (Zas et al., 1992) with a result that has become known as the *ZHS* Monte Carlo.

The shower code treats pair production and bremsstrahlung considering different screening regimes and including the Landau Pomeranchuk Migdal (LPM) effect (Landau et al., 1953) which becomes very important in ice at EeV energies (Alvarez-Muñiz and Zas, 1997). It deals with interactions with atomic electrons which are responsible for the charge excess, namely Möller, Bhabha, Compton and electron positron annihilation. For the lateral distribution it approximates multiple elastic scattering keeping the first two terms in Molière's expansion (Molière, 1948). The code selects an energy crossover below which bremsstrahlung is considered as a continuous loss, depending on the low energy threshold for the particles to be followed which is fixed externally.

The results obtained for the longitudinal development of the showers are consistent with the Greisen parameterization as shown in Fig. 1. The lateral distributions are consistent with three dimensional theoretical shower results, parameterized through the NKG (Greisen et al., 1956) provided the theoretical Molière radius is divided by a factor of 2, in agreement with other simulations (Hillas, 2000), as shown in Fig. 2. The excess charge depends on threshold and is of order 20%. The lateral distribution of the excess charge follows that of the total charge rather closely.

Relative timing for each particle track is followed with great care because it enters in the phase factor contributing to the overall signal. Particle delays are measured with respect to a plane perpendicular to the shower axis that starts in phase with the primary particle and moves parallel to it at speed c. Time delays arising from angular deviations, multiple scattering and subluminal velocities are considered.

The pulse is calculated using an expression in the Fraunhofer approximation for the frequency spectrum of the electric field $\boldsymbol{E}(\omega)$ resulting at a distance R (position $\mathbf{x}$) from a particle of uniform velocity $\boldsymbol{v}$, producing a track δl in the time interval $[t_1, t_1 + \delta t]$:

$$\boldsymbol{E}(\omega, \mathbf{x}) = \frac{e\mu_\mathrm{r}\ i\omega}{2\pi\epsilon_0 \mathrm{c}^2}\ \frac{\mathrm{e}^{ikR}}{R}\ \mathrm{e}^{i(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\mathrm{t}_1}\ \boldsymbol{v}_\perp \left[\frac{\mathrm{e}^{i(\omega-\boldsymbol{k}\cdot\boldsymbol{v})\delta t}-1}{i(\omega-\boldsymbol{k}\cdot\boldsymbol{v})}\right]$$
$$\simeq \frac{e\mu_\mathrm{r}\ i\omega}{2\pi\epsilon_0 \mathrm{c}^2}\ \boldsymbol{\delta l}_\perp\ \frac{\mathrm{e}^{ikR}}{R}\ \mathrm{e}^{i(\omega - \boldsymbol{k}\boldsymbol{v}_1)\mathrm{t}_1}, \qquad (1)$$

with c the speed of light in vacuum, μ_r the relative permeability of the medium and ϵ_0 the permittivity of the vacuum. The wave vector $\boldsymbol{k}$ points in the observation direction at an angle θ to the track, and has a modulus $|\boldsymbol{k}| = k = n\omega/\mathrm{c}$. The suffix $\perp$ applied to vectors refers to their projection onto a plane perpendicular to $\boldsymbol{k}$.

The last expression in Eq. 1 is a good approximation provided the phase in the first term of the square brackets is small. It illustrates that the electric field amplitude is proportional to the tracklength and that radiation is polarized in the direction of $\boldsymbol{\delta l}_\perp$, the apparent direction of the track as seen by the observer. The Fraunhofer condition for a finite track corresponds to $R >> [v\ \delta t \sin\theta]^2/\lambda$. It should be pointed out that numerically Eq. 1 can always be used provided δt is chosen to be sufficiently small.

For the numerical calculation of the electric field amplitude, charged particle tracks are divided in *subtracks* and the contribution of each of these subtracks is calculated using Eq. 1. As the expression corresponds to a rectilinear track interval from a particle with uniform velocity, an average velocity is calculated using the time and position information of the end points of the corresponding subtrack. We have implemented the prescription in three different ways, depending on the way the charged particle tracks are divided in subtracks for calculating their contribution to the total signal. We have previously referred to them as approximations (a), (b) and (c) (Alvarez-Muñiz, 1995).

Approximation (a) corresponds to applying Eq.1 to the whole electron or positron track from the instant the particle is created to the instant it is annihilated or becomes subthreshold. For approximation (b) the subtracks are defined as portions of the track in between the main interactions, namely bremsstrahlung, Möller, Bhabha or electron positron annihilation. Lastly for approximation (c) the subtracks of approximation (b) are further subdivided according to an internal track splitting algorithm used for an improved calculation of multiple scattering effects to the low energy particles accounting for continuous ionization losses. In both approximations (b) and (c) the subtracks become shorter as the particle energy is smaller, in (b) because the interactions with atomic electrons have a decreasing mean free path with energy and in approximation (c) because of the algorithm designed for calculating multiple scattering with continuous energy losses which also becomes more important for low energies.

The frequency spectrum of the radioemission in the Čerenkov direction calculated under the three approximations is compared in Fig. 3. Approximation (a) qualitatively reproduces the overall behavior fairly well but gives an underestimate which increases with frequency being about 30% (a factor of 2) at 1 GHz (10 GHz). The convergence between approximations (b) and (c) is a reassuring sign for the numerical technique but these approximations are obviously much more demanding in CPU time. For this reason approximation (a) has been chosen as the default approximation used by us when no explicit mention is made otherwise. As the good transmission properties of clear ice corresponds to frequencies below the GHz scale it is sufficient for many applications.

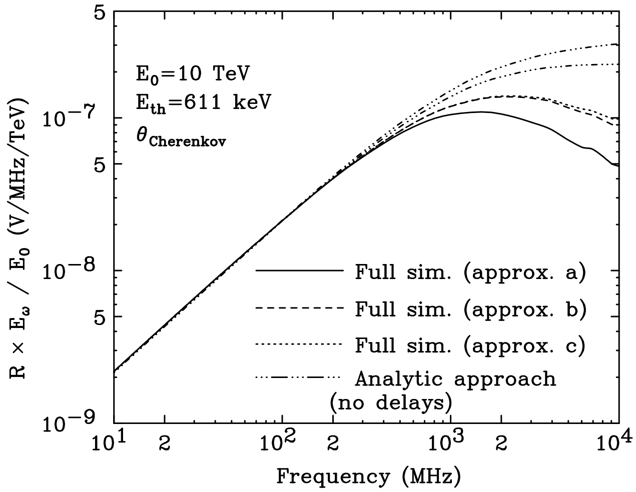

Fig. 3. Frequency spectrum of the electric field amplitude for observation at the Čerenkov angle. The three approximations (a), (b) and (c) are compared to the analytic results of Eq.(5) using two extreme NKG parameterizations.

3 The one dimensional approach

As an alternative to the simulation the equations for the pulse emitted by a charge distribution can also be solved directly (Askar'yan , 1961; Ralston and McKay, 1990; Buniy, 2000). It is easy to show that the Fourier transform of the electric field relates to the spatial distribution of the electric charge through:

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \frac{i\omega\, e\mu_{\rm r}}{2\pi\epsilon_0 {\rm c}^2} \int dt' d^3\boldsymbol{x}'\, \frac{e^{i\omega t' + ik|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} \boldsymbol{J}_\perp(\boldsymbol{x}',t') \quad (2)$$

where $\boldsymbol{J}_\perp(\boldsymbol{x}',t')$ is a divergenceless component of the current, in the Fraunhofer limit transverse to the direction of observation $\boldsymbol{x}$. We now make the following approximation for the charge distribution in the shower using for $\boldsymbol{x}'$ cylindrical coordinates z', ρ', ϕ' with z' along the shower axis:

$$\boldsymbol{J}_\perp(\boldsymbol{x}',t') = Q(z')\, \boldsymbol{c}_\perp\, \delta(z'-ct')\, f(z',\rho') \quad (3)$$

where we have assumed that the shower has a lateral distribution given by the function $f(z',\rho')$ which is normalized according to:

$$\int_0^{2\pi} d\phi' \int_0^\infty d\rho' \rho' f(z',\rho') = 1 \quad (4)$$

so that $Q(z')$ gives the total charge at a given depth. $Q(z')$ can be suitably parameterized by Greisen's expression and $f(z',\rho')$ by an NKG type parameterization which is a slowly varying function of z'. We use the standard result of Ref. (Fenyves et al., 1988) and a modified result which uses an age increased by 0.1 to account for a flatter lateral distribution of the excess charge.

It is now straightforward to plug the excess charge distribution into Eq. 2 to obtain an expression for the amplitude of the electric field spectrum. In the Fraunhofer approximation after integration in the angle ϕ' it can be shown to become:

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \frac{i\omega\, e\mu_{\rm r}}{2\pi\epsilon_0 {\rm c}^2}\, \sin\theta\, \frac{{\rm e}^{ikR}}{R}\, \hat{\boldsymbol{n}}_\perp \int dz'\, Q(z')\, {\rm e}^{ipz'}$$
$$2\pi \int_0^\infty d\rho' \rho' f(z',\rho') J_0\left[\frac{\omega n}{c}\rho' \sin\theta\right] \quad (5)$$

where $\hat{\boldsymbol{n}}_\perp$ is a unitary vector perpendicular to $\boldsymbol{x}$ and J_0 is a Bessel function of the first kind. We have introduced for convenience the parameter $p(\theta,\omega) = (1 - n\cos\theta)\,\omega/c$ in Eq. 5. Provided that ω is small the second integral can be ignored, because the Bessel function is unity and the last integral becomes precisely the normalization condition 4 which is also unity. The electric field amplitude is inversely proportional to R and becomes proportional to the Fourier transform of the depth distribution of the charge excess. Since ω is assumed to be small the parameter p in the Fourier phase factor is also small and the Fourier integral becomes the excess tracklength of the shower, in agreement with results of the simulation.

The one dimensional approximation relates directly the amplitude of the pulse to the Fourier transform of the excess charge distribution. It is appropriate for low frequencies and can be used for EeV showers that have elongated features corresponding to the LPM effect (Alvarez-Muñiz and Zas, 1997). It is valid provided the wavelength of the radiation exceeds the lateral distribution of the shower, typically below 300 MHz for ice, and it provides an accurate description of the angular distribution of the Čerenkov peak. The approximation is equivalent to assuming all particles travel parallel to the shower axis with speed c, that is to neglecting all time delays (Alvarez-Muñiz et al., 1999).

At frequencies of about 1 GHz the approximation has to be corrected for the effect of the lateral distribution and timing differences between particles. The second integral in Eq. 5 accounts for the effect of the lateral distribution alone. The electric field amplitude in the Čerenkov direction remains proportional to the excess tracklength because $p = 0$ but there are deviations from the naive one dimensional approximation because of this integral. In Ref. (Alvarez-Muñiz et al., 1999) the ratio of the simulated amplitude to that obtained in the one dimensional approach was shown to be fairly independent of shower energy and discussed as a phenomenological correction factor that would allow the extension of the frequency range to above 1 GHz. It is in fact a form factor, which is unity when the wavelength exceeds the characteristic lateral spread of the shower, but it drops as the wavelength decreases, providing the characteristic cutoff of the frequency spectrum in the Čerenkov direction (Buniy, 2000).

The results of plugging Eq. 3 and using the lateral distribution function as described are also shown in Fig. 3. The results of the full simulation in approximations (b) and (c) including the time delays and the angular deviations of the shower particles have the effect of producing a maximum in the frequency spectrum at the 2-3 GHz range. The phenomenological factor discussed in (Alvarez-Muñiz et al., 1999) accounts for these effects.

4 Summary and Conclusions

We have critically discussed the simulation package for calculation of coherent radio pulses in ice and shown that the shower results are fairly consistent to well established results for electromagnetic showers. We have discussed the one dimensional approximation, its validity range and its possible extension to the GHz range. We have shown that analytical approximations that only use depth and lateral distribution functions, thus ignoring the sphericity of the shower front, tend to overestimate the radiopulse at high frequencies. This high frequency limit of the spectrum of the electric field amplitude has been shown to be very sensitive to both the lateral distribution and to the time delays of the shower particles. It can vary between 1 and 5 GHz depending on such details. Relatively fine division of the electron tracks is needed in the simulations to be able to reproduce this feature with confidence. For practical purposes the amplitude of the electric field in ice at frequencies above the 1 GHz scale is not that relevant because the attenuation distance becomes shorter it dependens strongly on ice temperature. However these results are necessary for comparisons with other calculations.

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