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On the description of the turbulent diffusion model

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Abstract.
The Standard Leaky Box model is based on an injection spectrum close to E^{-2} as a natural consequence of strong parallel shock acceleration. Accordingly, an empirical energy dependence of $E^{-0.6}$ for propagation distance in the Galaxy is invoked to interpret the observed spectrum below 10^5 GeV. There are strong arguments, on the other hand, that the interstellar propagation must be based on turbulent diffusion with an energy dependence of $E^{-1/3}$. A theory of origin and transport of cosmic-rays incorporating this concept has already been formulated. In this theory the bulk of cosmic-ray spallation takes place in the shell of stellar winds, and light and heavy nuclei have different propagation parameters. The diffusive parameters are obtained from the primary source spectra and are calculated from the measurement of observed secondary fluxes. The observational data on heavy secondaries such as Sc, Ti and V, and B, determine the parameters of massive stellar wind shells, while measurement of light secondary elements such as $\bar{p}$ together with heavy secondary particles constrain the surrounding region of both low and high mass stars. The inferred propagation parameters also depend on the choice of the initial mass function of stars. Once these parameters are set, they can be checked by measurement of other secondaries such as ^{3}He, ^{2}H, and γ. In this report we discuss how this modeling is done.

1 Introduction

A long-standing problem in cosmic-ray astrophysics is the riddle of the origin and propagation of the particles. In the standard model, most cosmic-ray particles are Galactic in origin, and they are powered by supernova shock waves in the interstellar medium (ISM). The accelerated particles are thought to diffuse in the Galaxy by random magnetic fields and interact with ISM particles over the scale of millions of years before they are detected on earth (Gaisser, 1990; Berezinskii et al., 1990). As byproduct of interactions in the ISM, so-called primary cosmic-ray particles can produce secondary cosmic rays that are normally not synthesized by stars and are under abundant in the Solar System.

In the simplest model, the diffusion of non-decaying particles can be well approximated by the Standard Leaky Box (SLB) model whereby the particles leak out of the Galaxy and the diffusion is characterized by an energy dependent escape length (Gaisser, 1990; Berezinskii et al., 1990). The energy dependence of this length is obtained empirically from secondary-to-primary flux ratios and has approximate energy dependence of $\lambda_{\rm esc}(E) \propto E^{-0.6}$ above few GeV.

The propagation model developed by Biermann (1993), which is based on a new concept for the diffusive transport in a shock region (Biermann, 1994, 1997), adopts the basic notion of turbulent diffusion in the Galaxy with an escape length given by $\lambda_{\rm esc}(E) \propto E^{-1/3}$. The evidence for this form of leakage is summarized by Biermann (1995). Furthermore, massive stars are surrounded by extensive stellar wind shells which are enriched in heavy nuclei and can be the main source of secondary particles and propagation grammage (Völk and Biermann, 1988). The existence of these wind shells and the path-length traversed by particles in the shells are ignored in the Standard Leaky Box model but must be included in any in-depth propagation model. In this paper we briefly discuss the ramification of this turbulent diffusion model by comparing and contrasting its prediction of secondary fluxes with that of the Standard Leaky Box (SLB) model.

2 Standard Leaky Box Model

In the SLB model the cosmic-ray fluxes are assumed to be in equilibrium and there is no time or spatial dependence in the transport equations. For simplicity we assume that the particles do not decay and energies are sufficiently high that ionization energy losses may be ignored. In such case the flux of secondary particles, $I_S(E)$, in typical units of

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$(\mathrm{cm}^2\,\mathrm{s}\,\mathrm{sr}\,\mathrm{GeV/nuc})^{-1}$ is just the product of production and the grammage in the ISM

$$I_S(E) = \frac{Q_S(E)\,\Lambda_S(E)}{4\,\pi}. \tag{1}$$

The production of secondary particles, $Q_S(E)$ with typical units of $(\mathrm{gm}\,\mathrm{s}\,\mathrm{GeV/nuc})^{-1}$, is the sum over interaction of flux particle type i with target particle type j.

$$Q_S(E) = \frac{4\,\pi\,\sum_i \sum_j \int I_i(E') \frac{d\sigma_{ij\,\rightarrow\,S}}{dE}(E' \rightarrow E) n_j dE'}{\sum_j n_j\, m_j}. \tag{2}$$

In the above equation n_j is the number density of the j particles and m_j is their mass. The integration is over all possible initial energy per nucleon E'. The grammage $(\mathrm{gm/m}^2)$ is the column density of matter traversed in the ISM before the particle either escapes from the Galaxy or interacts and is destroyed in collision with the ISM, producing, for example, tertiary particles.

$$\Lambda_S = \left(\frac{1}{\lambda_{\mathrm{esc}}(E)} + \frac{\sum_j n_j\, \sigma_{Sj}}{\sum_j n_j\, m_j} \right)^{-1}. \tag{3}$$

If the interaction cross section is small, the grammage is determined by the escape length λ_{esc}. This is typically the case for light particles. On the other hand, heavy particles such as V, Sc, and Ti have large interaction cross sections and therefore higher chance of interacting before escaping from the Galaxy.

In production calculation of boron and sub-iron elements the energy per nucleon is approximately conserved in interactions, and one may write

$$\frac{d\sigma_{ij\,\rightarrow\,S}}{dE}(E' \rightarrow E) = \sigma_{ij\,\rightarrow\,S}(E)\,\delta(E - E'), \tag{4}$$

so that the secondary flux is simplified to

$$I_S(E) = \frac{\sum_i \sum_j I_i(E) \sigma_{ij\,\rightarrow\,S}(E) n_j}{\sum_j n_j\, m_j} \Lambda_S(E). \tag{5}$$

When there is only one primary element responsible for the secondary production, and at high energies when the cross section becomes energy independent, one may consider the ratio of secondary to primary fluxes in terms of the grammage, $I_S(E)/I_P(E) \propto \Lambda_S(E)$. For example boron is assumed to be produced in interaction of carbon with the ISM, and I_B/I_C data has been used to infer escape length $\lambda_{\mathrm{esc}}(E)$ from the Galaxy (If more than one primary is involved in the production, then scaling laws are often used to express the escape length in terms of a flux ratio) (Garcia-Munoz et al., 1987). For the ISM with the standard composition, The boron to carbon ratio comparison gives $\lambda_{\mathrm{esc}}(E) \propto E^{-0.6}$ for energies above few GeV. This combined with the prediction of linear parallel shock acceleration models that $Q_P(E) \propto E^{-2}$ (Berezinskii et al., 1990; Gaisser, 1990) has been hailed as a success for SLB model to explain the approximate index of 2.6 of cosmic-ray primaries.

3 Turbulent Diffusion Model

Although the SLB model correctly predicts the B/C ratio, it has several shortcomings and by no means is the only viable propagation model on hand. The SLB model, for example, predicts a diffuse gamma-ray spectrum in the Galaxy which is softer than observed (Hunter et al., 1997) by the EGRET instrument. The spatial dependence of the gamma-ray flux is also incompatible with predictions of the SLB model. Another shortcoming of this model is the lack of clear theoretical understanding of the grammage energy dependence. Such dependence is obtained only from the observed data.

In a model proposed by Biermann (1993) and developed by Biermann and Strom (1993) and Biermann and Cassinelli (1993), primary cosmic-ray nuclei are accelerated by supernova shocks and interact with the wind shells of stars to produce the secondary particles (Völk and Biermann, 1988). In this model cosmic-ray particles encounter very little grammage in the ISM, but see most their grammage in the wind shells. Based on theoretical arguments of Biermann (1993) as well as observational evidence (see for example Biermann (1995) and the references therein), the diffusion in the ISM is turbulent with the Kolmogorov power law spectrum (Kolmogorov, 1941; Landahl and Mollo-Christensen, 1986). In this turbulent diffusion model, the ISM grammage has the form

$$\Lambda_{\mathrm{ism}}(E) = a\,E^{-1/3} \tag{6}$$

over all relevant energy scales. In this model, one has to distinguish between different types of stellar population which are surrounded by different amounts of wind-shells. Massive stars like Wolf-Rayet stars have extensive wind shells composed of carbon and oxygen, and their envelope consists also mainly of carbon and oxygen; while lower mass stars like Red Super Giants have thin shells of mostly H and He (Biermann et al., 2001). It is a natural consequence of this model that light and heavy nuclei have different propagation histories.

In a simplified version of this model, all light nuclei, such as H and He, originate from explosions of Red Super Giant ($25\,M_\odot \gtrsim M \gtrsim 15\,M_\odot$) supernova; while all heavier elements originate from supernova explosions of Wolf-Rayet ($M \gtrsim 25\,M_\odot$) stars. In the full theory some elements may originate from both progenitors, and some cosmic-ray protons originate from explosion of very low mass ($15\,M_\odot \gtrsim M \gtrsim 8\,M_\odot$) supernovae in the ISM. In the simplest model two sources are considered.

3.1 Red Super Giants

Red Super Giants are assumed to have a thin non-diffusive shell and the grammage in the shell is given by (Biermann, 1993)

$$\Lambda_{\mathrm{sh}}(E) = \begin{cases} C_1\,\beta, & \beta < \beta_{1*}; \\ C_1\,\beta_{1*}, & \beta > \beta_{1*}. \end{cases} \tag{7}$$

Here $\beta \equiv v/c$ is the velocity of the particle and β_* is the critical velocity below which convection dominates. We shall assume that the composition in the shell is 50% H and 50% He by mass

3.2 Wolf Rayet Stars

Wolf Rayet Stars have extensive diffusive shell, which to the first approximation, are composed of 50% C and 50% O, with

$$\Lambda_{\rm sh}(E) = \begin{cases} C_2\,\beta, & \beta < \beta_{2*}; \\ C_2\,\beta_{2*}\left(\dfrac{E}{E_{2*}}\right)^{-5/9}, & \beta > \beta_{2*}. \end{cases} \quad (8)$$

4 Comparison With SLB

It is essential that one distinguishes between the flux in the wind shell and the flux in the ISM. The flux of an element i in the shell of a star of type k, is given by

$$I_{\rm sh\,i,\,k}(E) = \frac{Q_{\rm sh\,i,\,k}(E)\,\Lambda_{\rm sh\,i,\,k}(E)}{4\,\pi}, \quad (9)$$

where Q is again the production rate, and the ISM flux is summed over the stellar types, k, which contribute to the flux.

$$I_{\rm ism,\,i}(E) = \sum_k \alpha_{i,k}\,\frac{\Lambda_{\rm ism}(E)}{\Lambda_{\rm sh\,i,\,k}(E)}\,I_{\rm sh\,i,\,k}(E), \quad (10)$$

where $\alpha_{i,k}$ is the fraction of stars of type k, which are sources of element i, with $\sum_k \alpha_{i,k} = 1$, and is related to the initial mass function (IMF) used. The Salpeter IMF (Page, 1997) with $\xi(m) \propto m^{-2.35}$ indicates that there are equal numbers of Wolf-Rayet and Red Super Giants. Other IMF, such as the Kroup et al. (1991) IMF with $\xi(m) \propto m^{-2.7}$ may give a slightly different ratio. For the present discussion, however, such a difference is immaterial.

In analogy with the SLB case, the grammage for element i, in shell k, for this model is given by

$$\Lambda_{\rm sh\,i,\,k} = \left(\frac{1}{\lambda_{\rm sh\,k}(E)} + \frac{\sum_j n_{jk}\,\sigma_{ij}}{\sum_j n_{jk}\,m_{jk}}\right)^{-1}, \quad (11)$$

with straight forward generalization of the notation. After simple algebra the ISM flux of secondary particles become

$$I_{\rm ism\,S}(E) = E^{-1/3}\sum_k F_k(E) \quad (12)$$

where

$$F_k(E) = \frac{\sum_i \sum_j \int I_{\rm ism\,i,\,k}(E')\,\frac{\Lambda_{\rm sh\,i,\,k}}{E'^{-1/3}}\,n_{jk}\,\frac{d\sigma_{ij\,\rightarrow\,S}}{dE}\,dE'}{\sum_j n_{jk}\,m_{jk}}. \quad (13)$$

When the cross section is a delta function, the ISM flux of secondary particles simplifies to

$$I_{\rm ism\,S}(E) = \sum_k \frac{\sum_{i,j}\ I_{\rm ism\,i,\,k}(E)\,\Lambda_{\rm sh\,i,\,k}\,n_{jk}\,\sigma_{ij\,\rightarrow\,S}}{\sum_j n_{jk}\,m_{jk}}. \quad (14)$$

Assuming that carbon is the sole source of boron, and further assuming that all carbon is initiated in Wolf-Rayet stars, we obtain from Eqs. (14) and (8) that $I_{\rm ism\,B}/I_{\rm ism\,C} \propto E^{-5/9}$ which is correct high energy ratio of observed B/C. Thus the result of the SLB model for B/C ratio is reproduced by the present model.

It is nevertheless important to note that this model does have predictions that differ from the SLB model that can easily be tested. In particular, when kinetic energy per nucleon is not constant in the interactions, as in antiproton production $N + N' = \bar{p} + X$, then the approximations in Eqs. (5) and (14) are invalid, and the full Eqs. of (2) and (13) need to be used to calculate the secondary spectrum. There are two important differences between the two set of equations. In SLB, the interaction takes place after propagation in ISM and the target particles are mostly protons. In the present model, the interaction takes place in the shells of stars and therefore before propagation in the ISM (hence the factor $E^{-1/3}$ in Eq. 13), and also the target particles can be heavy nuclei (since the shells of Wolf Rayet stars are made of mainly C and O). For the latter the cross section of interaction is enhanced by an approximate factor

$$\sigma_{A_i\,A_T} \approx \left(A_i^{1/3} + A_T^{1/3}\right)^2\,\frac{\sigma_{pp}}{4}, \quad (15)$$

where A_i and A_T are the mass numbers of the incident and target particles. A particular advantage of using the secondary antiproton flux to test the model under discussion is that the production cross section peaks at about a few GeV, and the $\bar{p}$ flux, therefore, is not a power law. The low energy turnover should amplify any difference in the prediction of the two models. A full comparison of the antiproton production in the two models will be presented elsewhere (Sina et al., 2001).

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