Proceedings of ICRC 2001: 1795 © Copernicus Gesellschaft 2001

ICRC 2001

# Complex electron energy distributions in supernova remnants with non-thermal X-rays

**A.-C. Donea**[1] **and P. L. Biermann**[2]

[1]Department of Physics and Mathematical Physics, University of Adelaide, SA 5001, Australia
[2]Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, D-53121, Bonn, Germany

**Abstract.** We address the problem of the diffusive acceleration of electrons in shocks of supernova remnants with non-thermal X-ray emission. A complex electron energy distribution develops, with energies within the range of thermal to highly relativistic energies.

Starting from a Maxwellian distribution, drift acceleration produces, within the finite-size layer of the shock, a steep power-law supra-thermal electron energy distribution. Diffusive shock acceleration then produces an $\sim E^{-2.42\pm0.04}$ spectrum (Biermann, 1993). We find that at higher energies the spectrum steepens, due to the existence of the individual blob shocks and the substructure of the shock region (observed in radio emission).

We discuss the consequences of this for X-ray emission in SNRs showing non-thermal emission. This may be a paradigm for acceleration of energetic electrons also in other astrophysical sites, such as clusters of galaxies.

## 1 Introduction

Several individual supernova remnants (SNRs) show power-law X-ray spectra suggesting that the source of the radiation should be of non-thermal origin. The SNRs which have drawn our attention are: SNR1006 (Koyama et al. 1995), IC443 (Keohane et al. 1997), RX J1713.7-3946 (Koyama et al. 1997) and Cas A (Allen et al. 1997), RCW 86 (Vink et al., 1997).

Observations of the X-ray continua from SNRs are generally interpreted as non-thermal synchrotron emission from very high energetic electrons with energies up to 100 TeV (Reynolds 1996; Dyer et al., 2001). Despite the large acceptance of this model, there may be other models which may produce X-ray non-thermal spectra. First, nonMaxwellian distribution of electrons with energies of several keV can produce bremsstrahlung emission in X-rays. Second, a spectral break to a steeper power-law at the high energy end of the relativistic electron distribution could replace the exponential cut-off, and this modifies the shape of the synchrotron radiation, allowing for a power-law X-ray emission as observed in most of non-thermal SNRs.

In this paper, we discuss these mechanisms showing that power-law distributions of electrons are allowed in supernova shells as long as the acceleration is an efficient mechanism.

Unlike protons, which resonate with Alfvén waves at all energies, the injection of electrons into the Fermi acceleration process is not well understood; electrons resonate with Alfvén waves only for energies above 20 MeV. Below this energy supra-thermal electrons (STH) interact with whistler waves (Levinson, 1996; Melrose, 1974) which for high temperatures can be quickly damped decreasing the efficiency in scattering. The existence of the non-thermal radiation in SNRs (radio, X-ray, $\gamma$-ray observations) is direct proof that electrons do accelerate from thermal to higher energies. Shock acceleration is the most efficient mechanism in accelerating electrons.

We briefly address the problem of the injection of STH electrons, and calculate their spectral index assuming that the shocked layer is finite and has a self-defined structure. We then obtain a power-law distribution of STH electrons which covers the gap in the phase space between the thermal and highly relativistic components of the distribution. One expects that bremsstrahlung emission from STH electrons making up this bridge may also explain the SNR X-ray data.

On the other hand, the existence of many small bow-shocks along the supernova shells (see Cas A, Anderson et al., 1991) may contribute to particle acceleration. In Section 3 we analyze the importance of the free expansion blobs in modifying the cut-off end of the distribution of highly relativistic electrons.

*Correspondence to:* adonea@physics.adelaide.edu.au
on temporary leave from the Astronomical Institute of the Romanian Academy, Bucharest, Romania

## 2 Injection and Power-law distribution of STH electrons at the shock

A complex description of injection of STH electrons is given by Levinson (1996) and Mc Clements et al.(1997), suggesting that the geometry of the magnetic field is important for the efficiency in injection and acceleration of electrons. For quasi-perpendicular shocks they found that the velocity of the blast wave should be $\gg (m_e/m_p)^{1/2}c = 0.023c$. On the other hand, quasiparallel MHD shocks with $M_A \leq (m_e/m_p)^{1/2}$ could provide efficient acceleration of STH electrons.

If the upstream and downstream gas moves with velocity $U_1$ and $U_2$, respectively, in the shock frame, the condition for resonant scattering of electrons with whistlers is given by (Melrose, 1992):

$$\sqrt{\frac{m_p}{m_e}}c_s f_e \geq \sqrt{\frac{m_p}{m_e}}v_A \tag{1}$$

where the left-hand side represents the speed of STH electrons, which is considered to be a fraction $f_e$ of the averaged speed when electrons are thermalized with protons ($m_e$ and $m_p$ are the electron and proton mass, respectively). Since we do not expect that $T_e = T_p$ in SNRs, we take $f \leq 1$. Downstream the local sound speed is $c_{s2} = \sqrt{2\gamma_{ad}/(\gamma_{ad}-1)}U_2$. The right-hand side defines the threshold speed of STH electrons above which electron-whistler resonance occurs. The Alfvén speed is: $v_A = B/\sqrt{4\pi n_{\rm ISM} m_p}$ ($n_{ISM}$ and $B$ are number density and magnetic field strengths in the upstream plasma, respectively). From (1) one obtains the injection condition of STH electrons: $U_1/v_{\rm A1} \geq 2f_e/\sqrt{2\gamma_{ad}/(\gamma_{ad}-1)} = 3.57 f_e$, for an adiabatic index of $\gamma_{ad} = 5/3$. The most extreme possibility is when $f = \sqrt{m_e/m_p}$ and, for an upstream $B = 9\mu$G and $n_{\rm ISM} = 0.1$ cm$^{-3}$ the condition for injection becomes $U_1 \geq 0.03c$. This result is qualitatively different, but quantitatively similar to the criterion obtained by McClements et al.(1997).

We consider the energy gain of the STH electrons from drift acceleration at the shock. Jokipii (1987) has shown that quasi-perpendicular geometry of the shock allows effective shock drift acceleration mechanisms where electric fields accelerate particles. In the moving frame of the shock there is an electric field $\mathbf{E} = -\mathbf{U} \times \mathbf{B}/c$, from any magnetic field component perpendicular to the shock normal.

The key point in our work is based on two fundamental arguments extensively discussed in Biermann (1993, hereafter paper CRI) and Biermann and Strom (1993, hereafter paper CRIII). Argument number one is based on observational evidence that, for a cosmic ray mediated shock, the convective random walk of particles perpendicular to the unperturbed magnetic field can be described by a diffusive process. Consequently, a diffusion coefficient may be defined inside the shock layer. Argument number two is based on the concept of the smallest dominant scale of the system, in geometric length (the thickness of the shocked layer in this model) and in velocity space (the difference of the two velocities $U_1$ and $U_2$).

The thickness of the shock $z_{\rm sh}$ is very small compared to the current radius of the supernova shell and we consider that locally, the shock front is plane. The diffusion coefficient $\kappa$ is taken to be:

$$\kappa = \frac{1}{3}z_{\rm sh}(U_1 - U_2) \tag{2}$$

under the assumption that the random walk of STH electrons is described by a diffusive process with a diffusion coefficient $\kappa$ independent of the energy of the particle (CRI, CRIII) but dependent on $z_{\rm sh}$ and $r$.

STH electrons enter the shock from the upstream region with a compressed, pointed distribution. Scattering of particles at the shock isotropizes any distribution; smearing it out in energy or momentum will take much longer. We can then solve the simplest diffusion equation of propagation of STH trough the shock analytically (Donea et al., 2001).

The flow speed inside the shock is approximated by a linear dependence with $z$, in this case. The fluid is moving in the direction of increasing $z$, the negative $z$-direction corresponding to the upstream medium. Outside the finite layer of the shock the flow velocity is given by constant velocities $U_1$ and $U_2$, corresponding to the upstream and downstream media. As in Ellison and Reynolds (1991) and Baring et al.(1999) we use a finite width for the shock profile. Following similar steps as Drury (1983) we calculate the mean residence time of STH electrons inside the shock and then we find the index of the power-law distribution of STH electrons. The drift velocity in terms of the magnetic field values on the two sides of the shock inside the shock (Jokipii, 1982) is:

$$\mathbf{v}_d = \mathbf{e}_y \frac{pcv}{3qB}\frac{(r-1)[1-(r+1)\sin^2\theta]}{(\cos^2\theta + r^2\sin^2\theta)}\frac{\sin\theta}{z_{\rm sh}} \tag{3}$$

where $p$ is the momentum of electron and $q$ is the electric charge of the electron. At large $\theta$ (the angle between the upstream magnetic field lines and normal to the shock) the drift velocity is given mainly by the contribution of the gradient drift.

From classical arguments, the expression for the energy gain of the electrons due to the drift is the product of the drift velocity, the residence time and the electric field at the shock. A detailed calculation will be given in a forthcoming paper (Donea et al., 2001). The momentum gained by electrons due to the shock drift mechanism at the shock is:

$$\frac{\Delta p}{p} = \frac{4}{3}\frac{U_1}{v}\left(1 - \frac{U_2}{U_1}\right)x \tag{4}$$

where

$$x = \frac{2}{3}\frac{U_1}{U_2}\xi\frac{(r-1)[1-(r+1)\sin^2\theta]}{(\cos^2\theta + r^2\sin^2\theta)}\sin^2\theta \tag{5}$$

with $\xi(r)$ dependent on the boundary conditions in the diffusion problem. The phase-space distribution of STH electrons is:

$$f(p) \sim p^{-\left(\frac{3U_1}{U_1-U_2} + \frac{3U_1}{U_1-U_2}\frac{U_2}{U_1}\left(\frac{1}{x}-1\right)\right)} \tag{6}$$

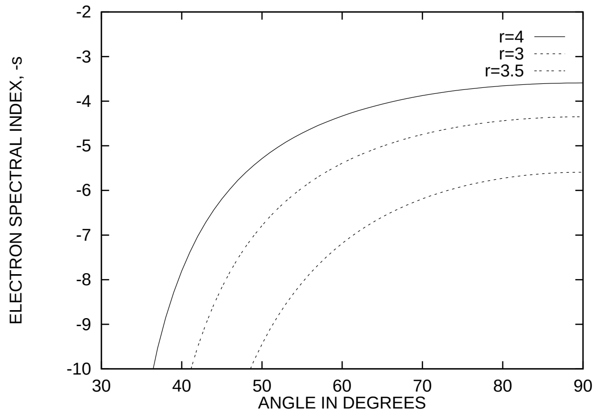


**Fig. 1.** Variation of spectral index of STH electrons with compression ratio and angle $\theta$ . At $r = 4$ and $\theta = 90^\circ$ the electron index is $3.59$.

The surprising result of our analysis is that the electron index does not depend on the thickness $z_{\rm sh}$ of the shock. It depends only on the angle $\theta$ and $r$ (see Fig. 1). We also calculate the contribution to the steepening of the spectrum from the adiabatic expansion of the shocked material (Drury, 1983) and find that this does not change the index significantly. The electron spectrum depends also on the history of the acceleration and the rate at which particles have been injected earlier. Inside the shock, these effects do not modify drastically the index of eq. (6). Our results show that for a strong perpendicular shock ($r = 4, \theta = 90^\circ$), the power-law distribution of electrons is $N(p) = 4\pi p^2 f(p) \approx p^{-A}$, where $A \approx 3.59$. The electron index (in momentum) is much steeper for weak and quasi-perpendicular shocks.

We can now construct a complex distribution of electron in SNRs: thermal electrons, then STH electrons which are accelerated into a power-law distribution up to energies where relativistic electrons take over the spectrum with $\sim E^{-2.42\pm0.04}$ (CRIII). The number of STH electrons inside the shock is normalized to the compressed number density of the upstream freshly injected electrons. The injection energy of STH electrons and the critical energy of relativistic electrons above which the Fermi acceleration occurs, depends on the local conditions in each SNR.

The STH electrons accelerated in the shell of SN1006 would emit bremsstrahlung photons with keV energies and with a X-ray spectral index of $A/2 = 1.8$ (Brown 1971) which is within the observed spectral limits $\alpha = 1.95 \pm 0.20$ (Koyama et al., 1995). For the case of Cas A, a thermal component overlays the non-thermal X-ray spectra, eliminating the problem of the pollution by emission lines. A STH distribution of electrons with $A \approx 4$ (for $\cos\theta < 90$, see Fig.1) gives an X-ray spectral index of $\alpha \approx 2$ which is also in agreement with the data.

## 3 Power-laws at the end of the energy spectra of relativistic electrons

We note that a pure synchrotron radiation model which models the X-ray flux of SNR 1006 from shock-accelerated electrons with energies of several tens of TeV would give curved X-ray spectra (Reynolds, 1996), while X-ray data show power-law continua sometimes polluted by emission lines from a thermal component. An additional contribution from different thermal components of the shocked plasma makes the ASCA observations of SN1006 well described by a synchrotron radiation model plus a more complex thermal shock model (Dyer et al., 2001).

Using the simple spherical model of CRIII the particle spectrum for cosmic ray electrons is predicted to be $N(p) \sim p^{-2.42\pm0.04}$. This spectrum gives an X-ray synchrotron emission far above the observed fluxes, unless the spectrum starts decreasing above a rolloff frequency (Dyer et al., 2001). We have seen that non-thermal X-ray fluxes in all shell-like SNRs are generally described as power-laws in SNRs mentioned in the Introduction. We notice that power laws may also be an important feature of the upper cut-off of relativistic distributions of electrons in SNR with strong radio fluxes. This type of power-law energy spectrum may characterize the emission in clusters of galaxies (Ensslin and Biermann, 1998). We propose that the spectrum of relativistic electrons goes up to a cut-off energy ($E_{\rm e,max}$) above which the spectrum steepens.

The maximum energy $E_{\rm e,max}$ of relativistic electrons is calculated considering the energy losses from synchrotron inverse Compton emission of electrons. The adiabatic loss of energy is taken from Völk and Biermann (1988). The acceleration time of relativistic electrons is derived in CRIII:

$$\frac{d\gamma_e}{dt} \;=\; \frac{1}{6}\,\gamma_e\,\frac{U_1^2}{\kappa_1}\,(1 - \frac{U_2}{U_1}) \tag{7}$$

Balancing energy losses and gains in the downstream region we obtain $\gamma_{e,max} \sim U_1/RB^2$. The corresponding maximum photon energy of synchrotron emission is $h\nu_{max} = 3heB\gamma_{e,max}^2/4\pi m_e c$ where we simplify the calculations by setting: $U_1/U_2 = 4$, and $U_{ph} \ll U_B$.

For the case of SNR 1006, $U_1 = 4500$ km/s, $R = 7.4$ pc and $h\nu_{max} \approx (29.1, 3.63, 0.35, 0.14)$ keV when $B \approx (1.5, 3, 6.5, 9)\mu$G. The cut-off energy is $E_{max,e} \approx (269, 67, 14, 7.7)$ TeV. Therefore, a break in the relativistic electron distribution gives synchrotron photons close to keV energies, quite possibly also below keV energies, as would be the case for higher, more normal, magnetic field strengths.

The blob shock has a radius $R$ in a medium with a speed of sound $c_{s2} = 0.559\,U_1$ (section 2.1) for $\gamma_{ad} = 5/3$. The blob shock velocity is $U_{b1} = U_1$. Upstream the Mach number $\mathcal{M}$ of one blob is $U_{b1}/c_{s2} = 1.79$, and from the standard shock conditions, the associated compression ratio is $r = (\gamma_{ad}+1)\mathcal{M}^2/[(\gamma_{ad}-1)\mathcal{M}^2+2] = 2.06$. The shock velocity of the blob downstream is $U_{b2} = U_{b1}/2.06$. The resulting spectral index (details calculations in Donea et al., 2001) is given by:

$$s = \frac{3U_{b1}}{U_{b1}-U_{b2}} + \frac{3U_{b1}}{U_{b1}-U_{b2}} \left( \frac{U_{b2}}{U_{b1}} \left(\frac{1}{x}-1\right) + \frac{6}{x}\frac{\kappa_2}{RU_{b2}} \right) \quad (8)$$

where the energy distribution of very relativistic electrons is $N(E) \approx E^{-s}$ ($\kappa_2 = 1/9R \cdot U_{b2}/U_{b1} \cdot (U_{b1} - U_{b2})$, as in CRIII). Therefore, the relativistic spectrum of electrons goes up to a cut-off energy $E_{e,max}$, from where the spectrum steepens into a power law with $E^{-5.13}$ for $x = 7/6$ for any distribution. The third part of eq. (8) shows the contribution from the history of acceleration of particles (Drury, 1983; CRIII).

The last question is then, to what maximum energy this steep tail-spectrum can extend? For smaller blobs the losses per cycle are smaller, and so the smallest limit is given by the Jokipii criterion (1987) $\kappa_1 > U_{b1} r_g$, where $r_g$ is the Larmor radius of the particle. The combination of eq. (7) and relations for energy losses and gains gives, for the case of SN1006, a maximum electron energy upstream of $\approx 170$ TeV, and a maximum synchrotron emission photon energy downstream of $h\nu_{max,*} \approx 233$ keV. On the basis of these calculation the radio through optical spectrum in SNRs may have a synchrotron spectrum $\nu^{-0.72\pm0.02}$ and from a point somewhere in the low keV region it has $\nu^{-2.06}$ up to a point in the high keV region corresponding to the end of the highly relativistic electrons.

The essential property of this break to a steep power law is as follows: In the case of synchrotron losses dominating over Compton losses, the break point in emission photon energy depends on the magnetic field strength as $1/B^3$. This break is likely to be observable only for regions of moderately low magnetic field strength. Otherwise the break-off point is at photon energies well below the normal X-ray range, and so the tail would not be visible above the other emission contributors (SN1006 case). *Observation*: Baring et al.(1999) have emphasized that for IC443 the bremsstrahlung from STH electrons may dominate the X-ray fluxes (see their fig. 9), being the alternative model to synchrotron emission for the non-thermal observed X-rays.

## 4 Conclusions

We have stressed the fact that the phase space gap between the thermal and the cosmic ray population is filled up by a power-law distribution of STH electrons. The oblique shock geometry plays an important role in accelerating STH electrons inside the shock. As the STH bremsstrahlung emission may not properly account for the hard X-ray tails of some SNRs, we have also searched the synchrotron emission from highly energetic electrons. We have proposed that the relativistic electron spectrum breaks to a steep power-law, instead of a exponential cut-off. This is understood if one thinks of the individual free expansion of blobs developing in shocks. In a forthcoming paper (Donea et al., 2001) we shall analyze in detail the matching conditions and the photon emission over the entire range of the electromagnetic spectrum. The electron to proton ratio above 1 GeV depends strongly on the matching conditions, and comparison to the multiwavelength observations of SNRs permits us to constrain the parameters of our model.

## References


Anderson, M., et al., Relativistic electron populations in Cassiopeia A, ApJ, 373, 146, 1991.

Allen, G.E., et al., Evidence of X-Ray Synchrotron Emission from Electrons Accelerated to 40 TeV in the Supernova Remnant Cassiopeia A, ApJ, 487, L97, 1997.

Baring, M. G., Ellison, D. C., Reynolds, S.P., Grenier, I. A., Goret, P., Radio to Gamma-Ray Emission from Shell-Type Supernova Remnants: Predictions from Nonlinear Shock Acceleration Models, ApJ, 513,311, 1999.

Biermann, P.L., 1 (CRI), Cosmic rays. I., AA, 271,649, 1993.

Biermann, P.L., Strom, R.G., (CRIII) Cosmic-Rays. III., AA, 275, 659, 1993.

Donea, A.-C., Biermann, P.L., Protheroe, R.J., in preparation.

Brown, J. C., The Deduction of Energy Spectra of Non-Thermal Electrons in Flares from the Observed Dynamic Spectra of Hard X-Ray Bursts, Solar Phys., 18, 489, 1971.

Drury, L.O'C., An introduction to the theory of diffusive shock acceleration, Rep. Prog. Phys., 46, 973, 1983.

Dyer, K.K., Reynolds, S.P., Borkowski, K.J., Separating thermal and nonthermal X-rays in supernova remnants I. Total fits to SN 1006 AD, ApJ, 551,439, 2001

Ellison, D., Reynolds, S.P., Electron acceleration in a nonlinear shock model with applications to supernova remnants, ApJ, 382, 242, 1991.

Ensslin, T.A., Biermann, P.L., Limits on magnetic fields and relativistic electrons in the Coma cluster from multifrequency observations, AA, 330,90, 1998.

Jokipii J.R., Rate of energy gain and maximum energy in diffusive shock acceleration, ApJ, 313, 842, 1987.

Jokipii, J.R., Particle drift, diffusion, and acceleration at shocks, ApJ, 255, 716, 1982.

Koyama, K., et al., Evidence for Shock Acceleration of High-Energy Electrons in the Supernova Remnant SN1006, Nature, 378, 255, 1995.

Koyama, K., et al., Discovery of Non-Thermal X-Rays from the Northwest Shell of the New SNR RX J1713.7-3946, PASJ, 49, 7, 1997.

Keohane, J.W. et al., A Possible Site of Cosmic Ray Acceleration in the Supernova Remnant IC 443, ApJ, 484, 350, 1997.

Levinson A., Electron injection and acceleration at nonlinear shocks, ApJ, 426, 327, 1994.

Melrose D.B.,Resonant scattering of particles and second phase acceleration in the solar corona, Solar Phys., 37, 353, 1974.

McClements, K.G., et al., Acceleration of cosmic ray electrons by ion-excited waves at quasi-perpendicular shocks, MNRAS, 291, 241, 1997.

Reynolds S.P., Synchrotron Models for X-Rays from the Supernova Remnant SN 1006 , ApJ, 459, L13, 1996.

Völk, H.J., Biermann, P.L., Maximum energy of cosmic-ray particles accelerated by supernova remnant shocks in stellar wind cavities, ApJ, 333, L65, 1998.

Vink, J., Kaastra, J.S., Bleeker, J.A., X-ray spectroscopy of the supernova remnant RCW 86 AA, 328, 628, 1997.