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Propagation of cosmic ray electrons in the Galaxy

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Abstract. We have made a new calculation of the cosmic ray electron spectrum using an anomalous diffusion model to describe the propagation of electrons in the Galaxy. The parameters defining the anomalous diffusion have been recently determined from the study of nuclei propagation. The predicted electron spectrum is in a good agreement with the measurements. The source spectral index, found from experimental data, in this approach turns out to be equal to 2.95.

1 Introduction

Observations of non-thermal radiation of the Galaxy stimulated investigations of propagation of cosmic ray electrons through the interstellar medium. Since basic paper of Syrovatskii (1959), the problem of calculation of electron spectrum was considered in series of papers (see, for example, Ginzburg and Syrovatskii (1964); Berkey and Shen (1969); Shen (1970); Bulanov et al. (1972); Cowsik and Lee (1979); Nishimura et al. (1979); Berezinski et al. (1990); Atoyan et al. (1995); Moskalenko and Strong (1998)). The normal diffusion equation for concentration of the electrons with energy E, $N(\boldsymbol{r}, t, E)$, generated by sources distribution with density function $S(\boldsymbol{r}, t, E)$,

$$\frac{\partial N}{\partial t} = D\Delta N(\boldsymbol{r}, t, E) + \frac{\partial}{\partial E}\big(b(E)N(\boldsymbol{r}, t, E)\big) + S(\boldsymbol{r}, t, E), \quad (1)$$

has been used to study the electron energy spectrum modifications in the interstellar medium (ISM). In the equation (1) D is the diffusivity, $b(E)$ — the energy-loss rate of electrons.

Recently, in the papers (Lagutin et al. (2001a,b); Lagutin and Uchaikin (2001)), new view of the cosmic ray propagation problem was presented. It has been shown that the

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"knee" in the primary cosmic ray spectrum is due to large free paths ("Lévy flights") of cosmic rays particles between magnetic domains — traps of the returned type. As the "Lévy flights" distributed accordingly to inverse power law $\propto Ar^{-3-\alpha}$, $r \to \infty$, $\alpha < 2$, is an intrinsic property of fractal structures, in the fractal-like medium the normal diffusion equation (1) certainly does not hold.

Based on these arguments in (Lagutin et al., 2001c) an anomalous diffusion (superdiffusion) model for describing of electrons transport in the fractal-like ISM was proposed. This superdiffusion equation for concentration of the electrons without convection has been presented in the form

$$\frac{\partial N}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2}N(\boldsymbol{r}, t, E)$$
$$+ \frac{\partial}{\partial E}\big(b(E)N(\boldsymbol{r}, t, E)\big) + S(\boldsymbol{r}, t, E), \quad (2)$$

where $D(E, \alpha)$ is the anomalous diffusivity and $(-\Delta)^{\alpha/2}$ is the fractional Laplacian (called "Riss" operator, Samko et al. (1987)).

The solution of superdiffusion equation (2) in the case of point impulse source with inverse power spectrum and the behaviour of energy spectrum of electrons in high energy region were found.

The main goal of this paper is to calculate the spectrum of electrons from sub-GeV to TeV energies in the framework of anomalous diffusion model . We don't use the assumption that the mean time of particle staying in a trap is finite. In this paper, similarly to (Lagutin and Uchaikin, 2001), we suppose that a particle can spend a long time in a trap, that is $q(t) \propto Bt^{-\beta-1}$, $t \to \infty$, $\beta < 1$ ("Lévy trapping time"). The spectrum of electrons both in ISM and in the solar system is found too.

2 Flux of electrons from point source

The flux of electrons, $J(\boldsymbol{r}, t, E)$, is related to the source $S(\boldsymbol{r_0}, t_0, E_0)$ by the propagator $G(\boldsymbol{r}, t, E; \boldsymbol{r_0}, t_0)$:

$$J(\boldsymbol{r}, t, E) = \frac{c}{4\pi} \int d\boldsymbol{r_0} \int\limits_{E}^{\infty} dE_0 \int\limits_{-\infty}^{t} dt_0$$

$$\times \, G(\boldsymbol{r}, t, E; \boldsymbol{r_0}, t_0) S(\boldsymbol{r_0}, t_0, E_0) \delta(t - t_0 - \tau). \quad (3)$$

Here

$$\tau = \int\limits_{E}^{E_0} \frac{dE'}{b(E')}, \quad (4)$$

$\delta(t - t_0 - \tau)$ reflects the law of energy conservation in the continuous losses approach.

The propagator in the anomalous diffusion model under consideration has the form (see, Lagutin and Uchaikin (2001))

$$G(\boldsymbol{r}, t, E; \boldsymbol{r_0}, t_0) = \left(D(E, \alpha, \beta) t^\beta\right)^{-3/\alpha}$$

$$\times \, \Psi_3^{(\alpha,\beta)} \left(|\boldsymbol{r}| \left(D(E, \alpha, \beta) t^\beta\right)^{-1/\alpha}\right), \quad (5)$$

where

$$\Psi_3^{(\alpha,\beta)}(r) = \int\limits_{0}^{\infty} q_3^{(\alpha)}\left(r\tau^\beta\right) q_1^{(\beta,1)}(\tau) \tau^{3\beta/\alpha} \, d\tau. \quad (6)$$

Here $q_3^{(\alpha)}(r)$ is the density of three-dimensional spherically-symmetrical stable distribution with characteristic exponent $\alpha \leq 2$ (Zolotarev et al. (1999)) and $q_1^{(\beta,1)}(t)$ is one-sided stable distribution with characteristic exponent β (Uchaikin and Zolotarev, 1999). The parameters α, β are determined by the fractional structure of ISM and by trapping mechanism correspondingly, the anomalous diffusivity $D(E, \alpha, \beta)$ — by the constants A and B in the asymptotic behaviour for "Lévy flights" (A) and "Lévy waiting time" (B) distributions:

$$D(E, \alpha, \beta) \propto A(E, \alpha)/B(E, \beta).$$

The energy-loss rate of relativistic electrons is described by the equation (see Atoyan et al. (1995))

$$-\frac{dE}{dt} = b(E) = b_0 + b_1 E + b_2 E^2$$

$$= b_2(E + E_1)(E + E_2), \quad (7)$$

where $b_0 = 3.06 \cdot 10^{-16} \cdot n \, (GeV \cdot s^{-1})$ is for ionization losses of electrons in ISM with number density $n \, (cm^{-3})$, $b_1 E$ with $b_1 = 10^{-15} \cdot n \, (s^{-1})$ corresponds to the bremsstrahlung energy losses, and $b_2 E^2$ with $b_2 = 1.38 \cdot 10^{-16} \, (GeV \cdot s)^{-1}$ — synchrotron and inverse Compton losses, for $B \approx 5\mu G$ and $\omega \approx 1 (eV/cm^3)$, $E_1 \approx b_1/b_0$, $E_2 \approx b_1/b_2$. Using (7), the

solution of the equaion (4) relative to E can be presented in the form (Atoyan et al., 1995)

$$E_0(\tau) = \frac{E + E_1}{1 - (1 - e^{-b_1 \tau})(E + E_2)/(E_2 - E_1)} - E_1.$$

Taking into account that $b_1 \tau \leq 3.15 \cdot 10^{-8} \cdot n \cdot 10^5 = 3.15 \cdot n \cdot 10^{-3} \ll 1$, we derive from the later equation

$$E_0(\tau) = \frac{E + E_1}{1 - b_1\tau(E + E_2)/(E_2 - E_1)} - E_1. \quad (8)$$

With help of the equations (3, 5, 8) it's easy to calculate the flux of electrons for the sources interesting for astrophysics. For example, for point impulse source

$$S(\boldsymbol{r}, t, E) = S_0 E^{-p} \delta(\boldsymbol{r}) \Theta(T - t) \Theta(t),$$

$$\Theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases}$$

we have

$$J_T(\boldsymbol{r}, t, E) = \frac{c}{4\pi} S_0 \int\limits_{\max[0, t-T]}^{\min(t, 1/b_2(E+E_2))} d\tau \, E_0(\tau)^{-p}$$

$$\times \, (\lambda(E, \tau^\beta)\tau^{\beta-1})^{-3/\alpha} (1 - b_2\tau(E + E_2)^{-2}$$

$$\times \, \Psi_3^{(\alpha,\beta)} \left(r(\lambda(E, \tau^\beta)\tau^{\beta-1})^{-1/\alpha}\right), \quad (9)$$

where

$$\lambda(E, \tau) = \int\limits_{E}^{E_0(\tau)} \frac{D(E', \alpha, \beta)}{b(E')} dE'.$$

It should be noted that in the case $\beta = 1$, the equation (9) comes to the solution, obtained earlier by (Lagutin et al., 2001c). If $\alpha = 2$, $\beta = 1$, we have the standard solution.

3 Energy spectrum of electrons

The flux J of electrons due to all sources of Galaxy can be separated into two components:

$$J = \int\limits_{0}^{1kpc} \ldots + \int\limits_{1kpc}^{R} \ldots =$$

$$J_L(r \leq 1kpc) + J_G(r > 1kpc). \quad (10)$$

The similar separation is frequently used in the studies of cosmic rays (see, for example, Atoyan et al. (1995) and references therein).

The first component (L) in (10) describes the contribution of the nearby sources (at distance $r \leq 1kpc$) to observed flux J. The second component (G) is the contribution of the distant sources ($r > 1kpc$) to J.
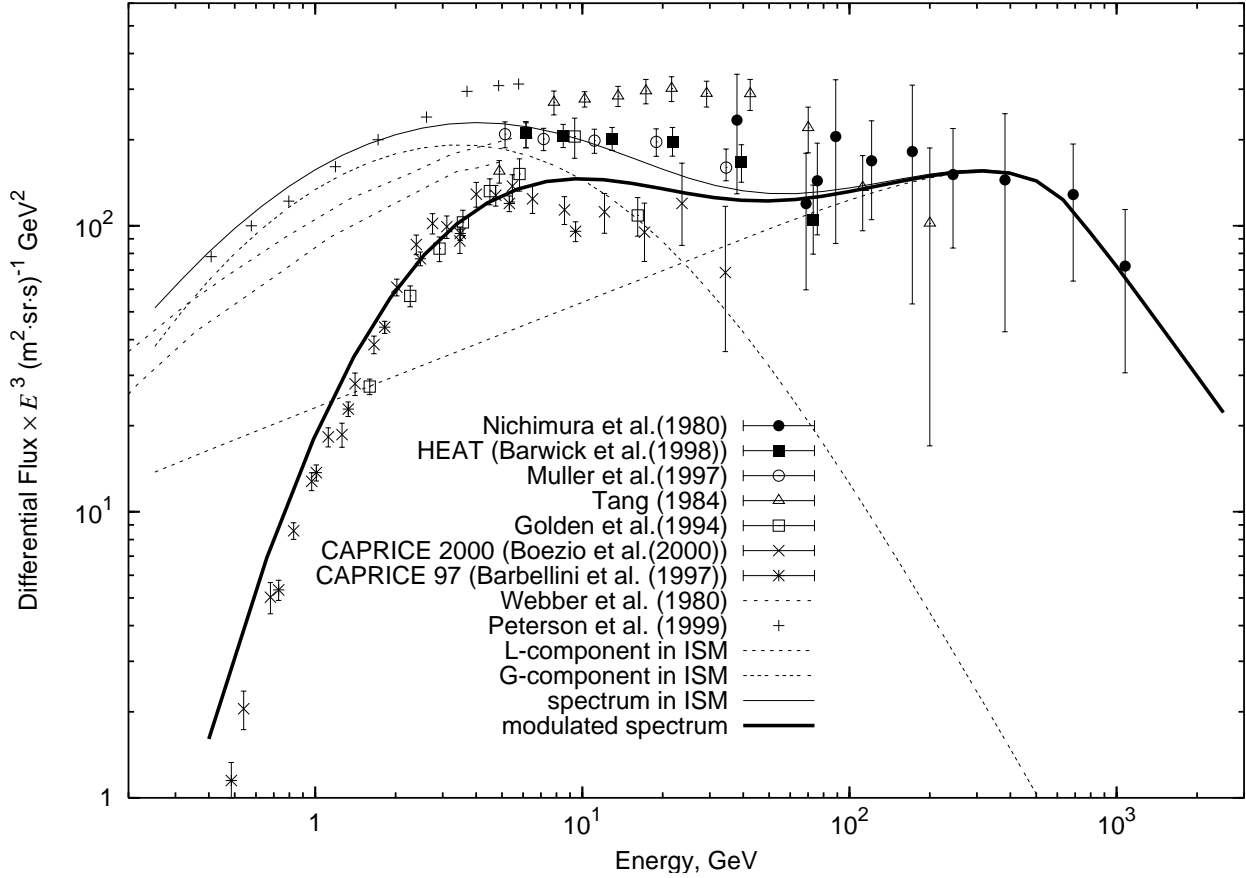
Fig. 1. Energy spectrum of electrons near the solar system.

The nearby sources used in our calculations is presented in Table 1. Based on result (9) we suppose

$$J_L = \sum_{\substack{i \\ (r \leq 1kpc)}} J_T(\boldsymbol{r_i}, t_i, E), \qquad (11)$$

where injection time $T \approx 10^4 \div 10^5 y$.

The second component (G) is evaluated under assumption that the distant sources ($r > 1kpc$) are distributed uniformly both in space and time in the Galaxy.

The parameters defining the anomalous diffusion and used in our calculations have been recently derived from the study of nuclei propagation (Lagutin et al., 2001b): $\alpha = 1.7$; $\beta = 0.8$; $D(E, \alpha, \beta) = D_0(E/1GeV)^\delta$ with $D_0 \approx (1 \div 4) \cdot 10^{-3} pc^{1.7}y^{-0.8}$ and $\delta = 0.27$. Only one parameter p defining injection spectrum of electrons in the sources is found by fit. Extensive calculations show that the best fit of experimental data may be get at $p \approx 2.95$.

The spectra of L- and G- components and the total spectrum in ISM are demonstrated in Fig. 1.

To describe the influence of the solar modulation, the force model of Axford and Gleeson (1968) is used:

$$J_{mod}(E) = \frac{E^2 - (m_e c^2)^2}{[E + \Phi(t)]^2 - (m_e c^2)^2} J[E + \Phi(t)],$$

where $\Phi(t) = 600 MeV$. The modulated spectra are shown in Fig.1 too.

4   Conclusion

We have made a new calculation of the cosmic ray electron spectrum using an anomalous diffusion model to describe the propagation of electrons in the Galaxy. The parameters defining the anomalous diffusion have been recently determined from the study of nuclei propagation. The predicted electron spectrum is in a good agreement with the measurements. The source spectral index, found from experimental data, in this approach turns out to be equal to 2.95. The proximity of this exponent to one obtained earlier (Lagutin et al., 2001b) for nuclei components ($p \approx 2.9$) can indicate the same mechanism of particle acceleration.

We have shown that the sources of high-energy electrons ($E \geq 100$ GeV), observed in the solar system are relatively young local sources ($r \leq 200$ pc, $t \sim 10^5$ y), injecting particles during the time $T \sim 10^4 \div 10^5 y$. The behaviour of spectrum in the low-energy region is defined by distant ($r \geq 1kpc$) sources.

Table 1. List of the nearest SNR

Name	r, pc	t, 10^5 y	Source
Lopus Loop	400	0.38	(Nishimura et al., 1979)
Monoceros	600	0.46	(Nishimura et al., 1979)
Vela	400	0.11	(Nishimura et al., 1979)
Cyg. Loop	600	0.35	(Nishimura et al., 1979)
CTB 13	600	0.32	(Nishimura et al., 1979)
S 149	700	0.43	(Nishimura et al., 1979)
STB 72	700	0.32	(Nishimura et al., 1979)
CTB 1	900	0.47	(Nishimura et al., 1979)
HB 21	800	0.23	(Nishimura et al., 1979)
HB 9	800	0.27	(Nishimura et al., 1979)
Monogem	300	0.86	(Nishimura et al., 1997)
Geminga	400	3.4	(Nishimura et al., 1997)
Loop I	100	2.0	(Lozinskaya, 1986)
Loop II	175	4.0	(Lozinskaya, 1986)
Loop III	200	4.0	(Lozinskaya, 1986)
Loop IV	210	4.0	(Lozinskaya, 1986)

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