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MUM: Flexible precise algorithm for the muon propagation

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Abstract. We present a new muon propagation Monte Carlo FORTRAN code MUM (MUons+Medium) which possesses some advantages over analogous codes presently in use. The most important features of the algorithm are described. Data on the test for algorithm accuracy are presented. Contributions of different sources to the resulting error of simulation are considered. Selected results obtained with MUM are given and compared with ones from other codes.

1 Introduction

Propagation of muons in medium plays an important role for underground (-water, -ice) experiments with natural fluxes of high energy (HE) neutrinos and muons. Firstly, neutrinos are detected by muons which are born in νN interactions and propagate a distance in medium from the point of interaction to a detector. Secondly, muons which are produced in atmospheric showers generated by cosmic rays represent the principal background for neutrino signal and therefore their flux at large depths should be well known. Besides, the atmospheric muons deep under sea or earth surface are the only natural calibration source which allows to confirm correctness of the detector model by comparison experimental and expected detector response. For the muon propagation along with analytical methods one uses the Monte Carlo (MC) technique which directly accounts for stochastic nature of the muon energy losses. There are several MC muon transportation algorithms currently in use (see, e.g. review in (Rhode and Cârloganu, 1998)) but theoretical and experimental progress makes to create new ones.

Here we present a new MC muon propagation code MUM (MUons+Medium) written in FORTRAN. When working on MUM we aimed at creation of an algorithm which would: *(a)* account for the most recent corrections for the muon cross-sections; *(b)* be of adequate and known accuracy, i.e., does not contribute an additional systematic error which would

exceed one from "insuperable" uncertainties (i.e. muon and neutrino spectra and cross-sections); *(c)* be flexible enough, i.e. could be easily optimized for concrete purpose to desirable and well understood equilibrium between CPU time and accuracy and easily extended for any medium and any correction for the cross-sections of the processes in which HE muon looses its energy; *(d)* be "transparent", i.e. provide user with the whole set of data related to used models for the muon cross-sections, energy losses, etc.; *(e)* be as fast as possible. The MUM code has been developed for the Baikal experiment (Sokalski and Spiering, 1992; Balkanov et al., 2000) but we believe it to be useful also for other experiments with natural fluxes of HE muons and neutrinos.

2 The basic features of the MUM algorithm [1]

To get finite CPU time T_{CPU}, the energy losses in any MC muon propagation algorithm have to be decomposed into two parts: muon interactions with fraction of energy lost v which exceeds some value v_{cut}, are simulated directly while the part of interaction with $v < v_{cut}$ is treated by the approximate concept of "continuum" energy loss. Setting v_{cut} too low one looses the speed (roughly, $T_{CPU} \propto v_{cut}^{-1}$) but setting it too high, one looses the accuracy. We did not fix v_{cut} in MUM, it may be set optionally within a range of $10^{-4} \leq v_{cut} \leq 0.2$, since the optimum value depends on the concrete case (Bugaev et al., 2000).

An "absolute" energy transfer threshold ΔE_{cut} in a range of $10\,\text{MeV} \leq \Delta E_{cut} \leq 500\,\text{MeV}$ can be used in MUM along with "relative" threshold v_{cut} to simulate the muon interactions within detector sensitive volume. It is important for deep underwater (-ice) Cherenkov neutrino telescopes (Sokalski and Spiering, 1992; Balkanov et al., 2000; Barwick et al., 1991; Andres et al., 2000; Aslanides et al., 1999; Amram et al., 1999; Resvanis et al., 1994; Anassontzis et al., 2000), where the water or ice are used both as a shield which absorbs atmospheric muons and as a detecting medium.

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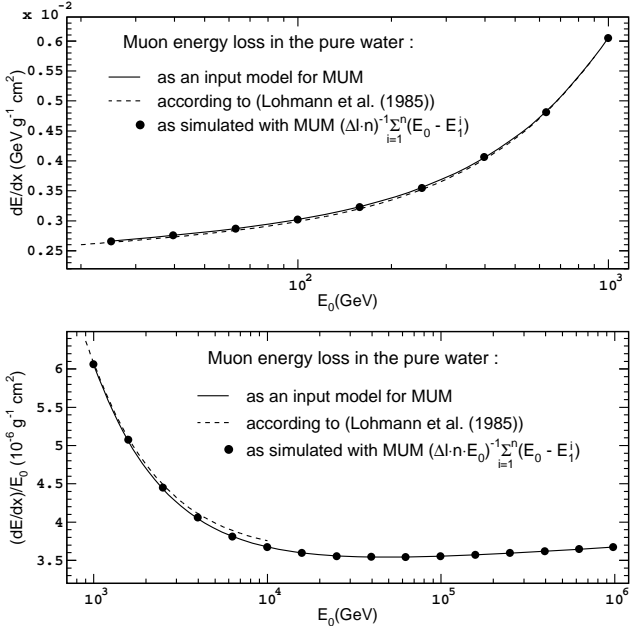[1]Detailed description can be found in (Sokalski et al., 2000)

Fig. 1. The *simulated* with MUM muon energy losses (markers) and *model of energy losses* as used by MUM (solid lines). Also results on the muon energy losses from (Lohmann et al., 1985) are presented (dashed lines). The plot corresponds to simulation in pure water with $v_{cut} = 10^{-2}$.

Fig. 2. Simulated with MUM distributions for the fraction of energy lost in a single interaction v for muons with $E = 100$ GeV (triangles) and $E = 1$ PeV (circles) in comparison with corresponding differential cross-sections (lines). The case for pure water and $v_{cut} = 10^{-4}$ is presented. The scales on Y-axis for $E = 100$ GeV and $E = 1$ PeV are different.

The formulae for the cross-sections of muon interactions (bremsstrahlung, e^+e^--pair production, photo-nuclear interaction, knock-on electron production) are given in (Sokalski et al., 2000). The code does not use any preliminary computed files, all necessary data are prepared at the initiation on the base of several relatively short routines, which give cross-sections for the muon interactions and stopping-power formula for ionization. It allows user to correct or even entirely change the model for any type of the muon interaction.

At each step we tried to avoid when possible any simplifications when computing/simulating the free path between two interactions, energy transfers, etc., or, at least, to track the error which comes from this or that kind of simplification. In most cases, to keep T_{CPU} at low level, roots for equations and values for functions are found at initiation, tabulated and then referenced when necessary by a interpolation algorithm which was carefully checked for each case to guarantee the high enough level of accuracy.

Formally, MUM simulates the propagation of the muons with the energies up to 1 EeV but one should keep in mind that above 1 PeV uncertainties with muon cross-sections grow remarkably, some effects which expose at UHE (e.g., LPM effect) are not accounted in MUM. Three media are available instantly for the muon propagation with MUM, namely pure water, ice and standard rock. But any medium can be easily composed by user following examples which are given in the initiation routine. At its current version MUM represents an 1D–algorithm which does not track the angular and lateral deviations of muons, but it is planned to be 3D–extended.
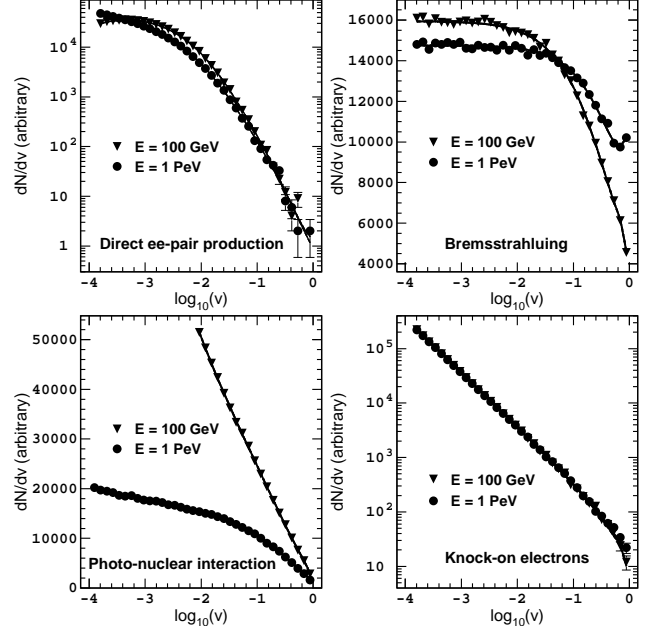
3 The accuracy of the algorithm and an optimum setting of simulation parameters

Fig.1 shows results of an accuracy test which consisted of following. For a muon energy E_0 the short distance Δl was chosen and n muons were propagated through this distance with MUM [2]. Then, $(E_0 - \frac{1}{n} \sum_{i=1}^{n} E_1^i)/\Delta l$ (where E_1^i is energy of i-th muon at the end of distance Δl) represents energy losses *simulated* by the algorithm. In an ideal case they should be equal to ones which can be directly calculated by integration of differential cross-sections which represents the *incoming model* for given code but, since algorithm itself necessarily contributes an error which originates from application of numerical procedures which the code consists of, the simulated energy losses and incoming model for energy losses are not the same for any real code. The only case is presented in Fig.1 but actually such test was performed both for water and standard rock with v_{cut} in a range from 10^{-4} to 0.05 (Sokalski et al., 2000). The difference between simulated energy losses and incoming model for energy losses for MUM does not exceed 1% except for the case when $v_{cut} \geq 0.01$ and ionization is treated as completely "continuous" process. It means that inner inaccuracy of MUM contributes to the resulting error much less than principal uncertainties with muons and neutrinos fluxes, cross-sections, etc.

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[2]The value $l = n \cdot \Delta l$ must be much greater than mean free path $\bar{L}(E_0)$ between two interactions with $v > v_{cut}$ to obtain statistically significant result but also must be small enough since there should be no stopped muons.
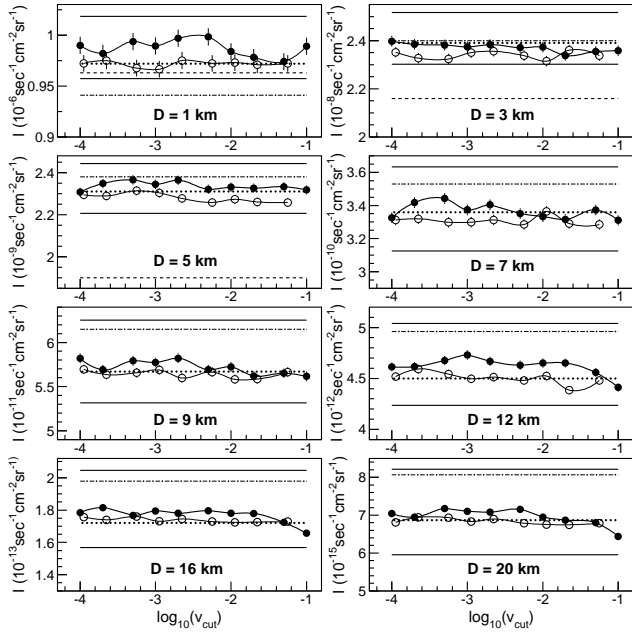
Fig. 3. Intensity of vertical atmospheric muon flux I at different depths D of pure water vs. v_{cut} as obtained by simulation with MUM. Muons were sampled according to sea-level spectrum from (Klimushin et al., 2000). Closed circles: knock-on electron production with fraction of energy lost $v \geq v_{cut}$ is simulated; open circles: ionization is completely "continuous". Two horizontal solid lines on each plot show the flux intensity simulated with all muon cross-sections multiplied by a factor 1.01 (lower line) and 0.99 (upper line) for $v_{cut} = 10^{-4}$. Dashed lines on plots for $D \leq 5$ km correspond to intensity which was calculated for all energy loss treated as "continuous". Dash-dotted lines show the flux intensity simulated with muons sampled according to sea level spectrum (Gaisser, 1990) and $v_{cut} = 10^{-4}$. Dotted lines correspond to $v_{cut} = 10^{-4}$ and cross-section for absorption of a real photon at photo-nuclear interaction parameterized according to (Breitweg et al., 1999) instead of parameterization (Bezrukov and Bugaev, 1980,1981) which is the basic in MUM.

Fig.2 demonstrates the accuracy of simulation for fraction of energy lost v for different kinds of muon interaction. Simulated distributions are plotted along with functions for differential cross-sections $d\sigma/dv$. Again, only small part of the data is shown in the plot but agreement between simulated distributions and predictions is not worse for other media and other muon energies which has been tested carefully.

Results on simulation of atmospheric muon vertical flux at different depths in the pure water vs. v_{cut} are shown in Fig.3. Simulations were performed for 2 atmospheric muon surface spectra; with knock-on electron production included in simulation of energy losses and treated as totally "continuous"; with 2 different parameterizations for photo-nuclear interaction; with all muon energy losses multiplied by 0.99, 1.00 and 1.01 (which corresponds to the most optimistic evaluation for uncertainties with the muon cross-sections of 1% (Rhode and Cârloganu, 1998; Kokoulin and Petrukhin, 1991,1999)). The general conclusion is as follows: the principal uncertainties when computing the atmospheric muon

flux at large depth are ones for the muon cross-sections. Influence of set value for v_{cut} in a range 10^{-4}–10^{-1} is much less and, in principal, in a "ideal muon propagation code" one could set $v_{cut} = 10^{-1}$ which allows calculations to be rather fast without remarkable influence on the result. Also the ionization energy losses can be treated as completely "continuous" (which saves T_{CPU} with a factor of $\sim$2). But, as MUM's own accuracy (in the sense of reproducing the muon energy losses) becomes worse than 1% for $v_{cut} \geq 10^{-2}$ if ionization is excluded out of simulation and for $v_{cut} \geq 5 \cdot 10^{-2}$ if knock-on electrons are simulated (Sokalski et al., 2000) we conservatively affirm $v_{cut} = 5 \cdot 10^{-2}$ and knock-on electron production included in simulation as a optimum setting of parameters for simulation the atmospheric muon flux at large depths with MUM. With such setting the proportion of T_{CPU} which is necessary to get the same statistics with muon propagation algorithms MUM, PROPMU (Lipari and Stanev, 1991) and MUSIC (Antonioli et al., 1997) is approximately $1 : 10 : 600$ (note that MUM, in contrast both to PROPMU and MUSIC, is 1D algorithm).

We did not investigate specially the influence of simulation parameters on the results for the muon flux originated from neutrino. Generally, intensity of the muon flux I_μ^{AC} which accompanies the neutrino flux in a medium is proportional to the muon range, and, consequently, $I_\mu^{AC} \propto (dE/dx)^{-1}$ (in contrast to atmospheric muons whose flux at the large depths depends more sharply upon muon energy losses - see, e.g., Fig.3). That means that an error for simulated flux of muons produced by neutrino is proportional to an error in muon energy losses. So, the setting of parameters described above fits even better for propagation of muons originated from neutrino.

4 Comparison to other muon propagation algorithms

Fig. 4 shows survival probabilities (fractions of muons which have survived after propagation of distance D) vs. distance of propagation in pure water as computed with MUM, PROPMU and MUSIC for a set of muon energies from 500 GeV to 30 PeV. There are no statistically significant differences between MUM and MUSIC but survival probabilities computed with PROPMU are noticeably higher at energies $E \leq 30$ TeV (up to a factor of 6) and become less at $E \geq 30$ TeV.

In Fig. 5 differential spectra for vertical atmospheric muons at different depths in pure water are presented as simulated with MUM, PROPMU and MUSIC. Muons at the surface were sampled according to spectrum (Klimushin et al., 2000). Okada (Okada, 1994) and KBS (Klimushin et al., 2000) parameterizations for deep underwater muon spectra are shown, as well. MUSIC and MUM give almost the same results because survival probabilities for muons in pure water are the same when simulating with MUSIC and MUM. MUSIC's and MUM's spectra coincide with KBS parameterization which is based on the same sea-level muon spectrum as was used for simulation and on muon propagation with MUM. Okada parameterization is lower than KBS, MUM
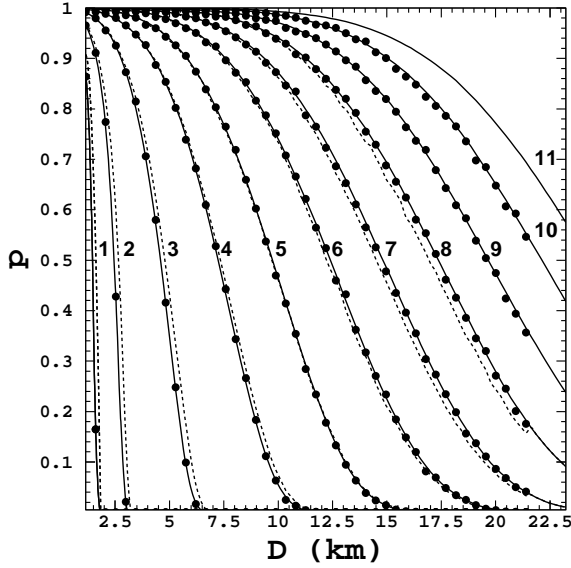
Fig. 4. Survival probabilities vs. distance of propagation in pure water as simulated with MUM (solid lines), PROPMU (dashed lines) and MUSIC (circles). Figures near curves indicate initial energy of mono-energetic muon beam as follows: 500 GeV (1), 1 TeV (2), 3 TeV (3), 10 TeV (4), 30 TeV (5), 100 TeV (6), 300 TeV (7), 1 PeV (8), 3 PeV (9), 10 PeV (10), 30 PeV (11).

Fig. 5. Differential spectra of vertical atmospheric muons at four depths in the pure water as simulated with MUM, PROPMU and MUSIC and parameterized by (Klimushin et al., 2000; Okada, 1994).

and MUSIC results (up to 18% in terms of integral muon flux at $D = 1$ km) at relatively shallow depths and becomes higher at $D \geq 5$ km because it is based on rather hard surface muon spectrum with index $\gamma = 2.57$ which leads to a deficit for low energy muons comparing to KBS parameterization. Simulation with PROPMU produces the muon spectra which *i)* are significantly higher ($31\%, 30\%, 27\%$ and 17% in terms of integral muon flux at the depths $D = 1$ km, 3 km, 6 km and 10 km, correspondingly) and *ii)* are expanded to the low energies. It is in qualitative agreement with results on survival probabilities presented in Fig. 4.

5 Conclusions

We have presented the muon transportation algorithm MUM and have given selected results obtained with it in comparison with ones obtained with analogous codes. We consider the current version of MUM as a basis for the further development. The code is available by request.

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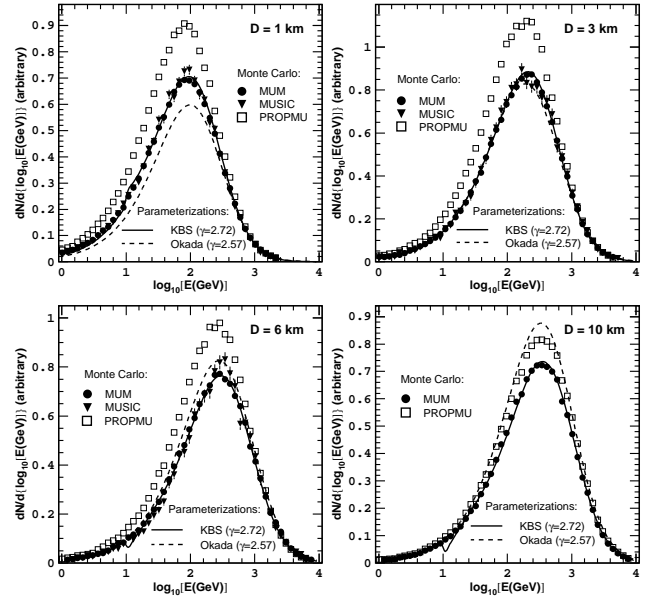Andres, E., et al., Astropart. Phys. **13**, 1 (2000) (astro-ph/9906203).
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