Proceedings of ICRC 2001: 1158 © Copernicus Gesellschaft 2001

ICRC 2001

Extragalactic neutrino background from PBH evaporations

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Abstract. Energy spectra and fluxes of electron neutrinos from evaporation of primordial black holes (PBHs) in the early universe are calculated. The constraints on the spectral index of primary density perturbations following from the analysis of solar and atmospheric neutrino experiments are obtained.

1 Introduction

Some recent inflation models (e.g., the hybrid inflationary scenario) predict the ”blue” power-spectrum of primordial density fluctuations. In turn, as is well known, the significant abundance of primordial black holes (PBHs) is possible just in the case when the density fluctuations have an $n > 1$ spectrum (n is the spectral index of the initial density fluctuations, $n > 1$ spectrum is, by definition, the ”blue perturbation spectrum”).

Particle emission from PBHs due to the evaporation process predicted by Hawking may lead to observable effects. Up to now, PBHs have not been detected, so the observations have set limits on the initial PBH abundance or on characteristics of a spectrum of the primordial density fluctuations. In particular, PBH evaporations contribute to the extragalactic neutrino background. The constraints on an intensity of this background (and, correspondingly, on an PBH abundance) can be obtained from the existing experiments with atmospheric and solar neutrinos. The obtaining of such constraints is a main task of the present paper.

The spectrum and the intensity of the evaporated neutrinos depend heavily on the PBH's mass. Therefore, the great attention should be paid to the calculation of the initial mass spectrum of PBHs. We use in this paper the following assumptions leading to a prediction of the PBH's mass spectrum.

1. The formation of PBHs begins only after an inflation phase when the universe returns to the ordinary radiation-dominated era. The reheating process is such that an equation of state of the universe changes almost instantaneously into the radiation type (e.g., due to the parametric resonance) after the inflation.

2. It is assumed, in accordance with analytic calculations (Carr , 1975), that a critical size of the density contrast needed for the PBH formation, δ_c, is about $1/3$. Further, it is assumed that all PBHs have mass roughly equal to the horizon mass at a moment of the formation, independently of the perturbation size.

3. Summation over all epochs of the PBH formation can be done using the Press-Schechter formalism. This formalism is widely used in the standard hierarchial model of the structure formation for calculations of the mass distribution functions.

2 Formula for neutrino diffuse background from PBHs

The expression for the neutrino background is

$$S(E) = n_i \int dt \frac{a_0}{a} \left(\frac{a_i}{a_0}\right)^3 f\left(E(1+z)\right) \quad . \tag{1}$$

Here, n_i is the initial density of the sources (in our case the source is an evaporating PBH of the definite mass m), a_0 is the scale factor at present time, $t = t_0$, $f(E)$ is a differential energy spectrum of the source radiation. The source of the radiation is a Hawking evaporation:

$$n_i f\left(E(1+z)\right) = \int dm \, n_{BH}(m,t) f_H\left(E(1+z), m\right) . \tag{2}$$

Here , $n_{BH}(m,t)$ is the PBH mass spectrum at any moment of time, $f_H(E,m)$ is the Hawking function,

$$f_H\left(E,m\right) = \frac{1}{2\pi\hbar} \frac{\Gamma_s(E,m)}{exp\left(\frac{8\pi GEm}{\hbar c^3}\right) - (-1)^{2s}} \quad . \tag{3}$$

E is the energy of an evaporated particle, $\Gamma_s(E,m)$ is the coefficient of the absorption by a black hole of a mass m, for an particle having spin s and energy E.

The initial spectrum of PBHs is given by the formula (Kim and Lee , 1996)

$$n_{BH}(M_{BH}) = \frac{n+3}{4}\sqrt{\frac{2}{\pi}}\gamma^{7/4}\rho_i M_i^{1/2}$$
$$\times M_{BH}^{-5/2}\sigma_H^{-1}\exp\left(-\frac{\gamma^2}{2\sigma_H^2}\right) \ . \quad (4)$$

Here, $\gamma = 1/3$, M_i is the horizon mass at the moment of a beginning of the growth of density fluctuations, σ_H is the horizon crossing amplitude. The minimum value of a PBH mass in this PBH mass spectrum is given by the simple expression (Bugaev and Konishchev , 2001)

$$M_{BH}^{min} = \gamma^{1/2} M_i \ . \quad (5)$$

To take into account the existence of the minimum we must add to the initial spectrum expression the step factor $\Theta(M_{BH} - \gamma^{1/2}M_i)$. The connection of the initial mass value M_{BH} and the value at any moment t is determined by the solution of the equation

$$\frac{dm}{dt} = -\frac{\alpha(m)}{m^2} \ . \quad (6)$$

The function $\alpha(m)$ accounts for the degrees of freedom of evaporated particles and determines the lifetime of a black hole. In the approximation $\alpha = const$ the solution of Eq.(6) is

$$M_{BH} \cong \left(3\alpha t + m^3\right)^{1/3} \ . \quad (7)$$

This decrease of PBH mass leads to the corresponding evolution of a form of the PBH mass spectrum. At any moment one has

$$n_{BH}(m,t)dm = n_{BH}\left((3\alpha t + m^3)^{1/3}\right)$$
$$\times\frac{m^2}{(3\alpha t + m^3)^{2/3}}\Theta\left[m - \left((M_{BH}^{min})^3 - 3\alpha t\right)^{1/3}\right]dm. \quad (8)$$

Substituting Eqs. (3), (8) in the integral in Eq.(2) , we obtain the final expression for the spectrum of the background radiation.

It is convenient to use the variable z instead of t. In the case of flat models with nonzero cosmological constant one has

$$\frac{dt}{dz} = -\frac{1}{H_0}\frac{a}{a_0}\left(\Omega_m(\frac{a_0}{a})^3 + \Omega_r(\frac{a_0}{a})^4 + \Omega_\Lambda\right)^{-1/2} \quad , \quad (9)$$

$$\Omega_r = (2.4\cdot 10^4 h^2)^{-1} \quad , \quad \Omega_m = 1 - \Omega_r - \Omega_\Lambda \ . \quad (10)$$

The factor $(\frac{a_i}{a_0})^3$ can be expressed through the value of t_{eq}, the moment of matter-radiation density equality:

$$\left(\frac{a_i}{a_0}\right)^3 \simeq (1+z_{eq})^{-3}\left(\frac{t_i}{t_{eq}}\right)^{3/2}$$
$$= \left(\frac{2}{3}(2-\sqrt{2})\right)^{-3/2} H_0^{3/2}\left(2.4\cdot 10^4 h^2\right)^{-3/4} t_i^{3/2} \ . \quad (11)$$

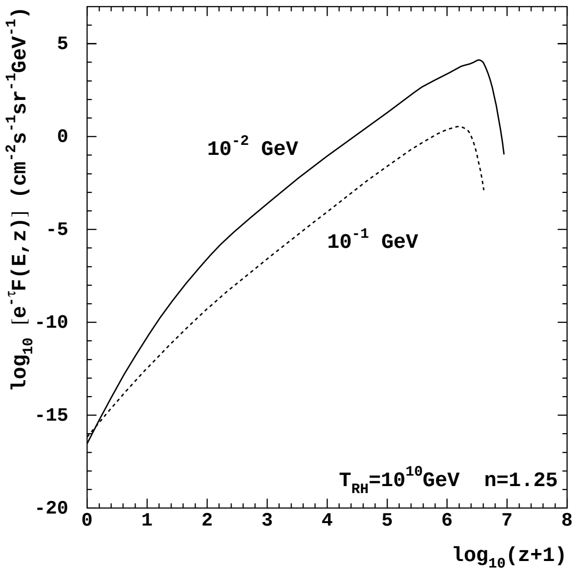

Fig. 1. Redshift dependence of the integrand of the expression (12) for a neutrino background spectrum (with $\Omega_\Lambda = 0$), for two values of the neutrino energy.

Integrating over PBH's mass in Eq.(1), one obtains finally, after the change of the variable t on z, the integral over z:

$$S(E) = \int F(E,z)d\log_{10}(z+1) \ . \quad (12)$$

The evaporation process of a black hole with not too small initial mass is almost an explosion. So, for a calculation of spectra of evaporated particles with acceptable accuracy it is enough to know the value of α for an initial value of the PBH mass only. Taking this into account and having in mind the steepness of the PBH mass spectrum, we use the approximation

$$\alpha(m) = \alpha(M_{BH}^{min}) = \alpha(\gamma^{1/2}M_i) \ , \quad (13)$$

and just this value of α is meant in the expressions (7)-(8).

3 Normalization of the perturbation amplitude

Our normalization of perturbation amplitude on COBE data is based on two inputs.

1. Connection between CMB fluctuations and scalar density fluctuations is described (Turner and White , 1996) by the following relations (derived for a lowest-order reconstruction of the inflationary potential):

$$S \equiv \frac{5C_S^2}{4\pi} = 0.104 f_S^{(0)}(\Omega_\Lambda)A_S^2(k_*) \ , \quad (14)$$

$$f_S^{(0)}(\Omega_\Lambda) = 1.04 - 0.82\Omega_\Lambda + 2\Omega_\Lambda^2 \ , \quad (15)$$

$$0.0 \le \Omega_\Lambda \le 0.8 \ . \quad (16)$$

Here, C_2^S is the scalar contribution to the angular power spectrum of CMB temperature fluctuations for $l = 2$, $k_* \sim a_0 H_0$ is the comoving wave number at the present horizon scale. The dependence $f_S^{(0)}$ on Ω_Λ is due to the integrated Sachs-Wolfe effect, i.e., due to the evolution of the potentials from last scattering surface till the present time.

2. From 4-year COBE data one has (assuming that the scalar contribution dominates over the tensor on the large scales) the following results for small l values (Bond , 1996):

$$\left(\frac{l(l+1)C_l^S}{2\pi}\right)^{1/2} \simeq (1.03 \pm 0.07) \cdot 10^{-5}, \tag{17}$$

$$n = 1.02 \pm 0.24 \; . \tag{18}$$

From Eqs.(14-17) one obtains, finally, the normalization of the amplitude $A_S(k)$:

$$A_S(k_*) = 2 \cdot 10^{-5} \frac{1}{\sqrt{f_S^{(0)}(\Omega_\Lambda)}}. \tag{19}$$

The connection between $A_S(k)$ and the horizon crossing amplitude at present time is given by

$$\delta_H^2(k,t) = A_S^2(k) \left(\frac{g(\Omega_m)}{\Omega_m}\right)^2 \quad . \tag{20}$$

Here, the factor $g(a, \Omega_m)$ accounts for the growth of density perturbations.

All the dependence of $\delta_H(k,t)$ on comoving wave number k is contained in an amplitude $A_S(k)$ and, assuming the power law of primordial density perturbations, is very simple:

$$A_S(k) \sim k^{\frac{n-1}{2}} \; . \tag{21}$$

Expressing the variable k through M_h, horizon mass at the moment when the scale $\lambda \sim \frac{1}{k}$ crosses the Hubble radius, one obtains

$$A_S(M_h) \sim \begin{cases} M_h^{\frac{1-n}{4}} & \text{, radiation era ,} \\ M_h^{\frac{1-n}{6}} & \text{, matter era .} \end{cases} \tag{22}$$

The final expression for horizon crossing amplitude, as a function of horizon mass (for $M_h < M_{eq}$) is (Bugaev and Konishchev , 2001)

$$\delta_H(M_h) = \frac{2 \cdot 10^{-5}}{\sqrt{f_S^{(0)}(\Omega_\Lambda)}} \left(\frac{M_{eq}}{M_{h0}}\right)^{\frac{1-n}{6}} \left(\frac{M_h}{M_{eq}}\right)^{\frac{1-n}{4}} \quad . \tag{23}$$

The connection of this amplitude with the smoothed amplitude σ_H, entering the expression for PBH mass spectrum (4) is given by

$$\sigma_H(M_h) \approx 5\delta_H(M_h) \quad . \tag{24}$$

4 Constraints on the spectral index

A precise calculation of the PBH neutrino background must include also a taking into account the neutrino absorption during a travelling in the space. The analog of an optical depth of the universe for the neutrino emitted at a redshift z and having today an energy E is given by the integral

$$\tau(z,E) = \int_0^z \sigma\left(E(1+z')\right) \cdot n(z') \frac{dt}{dz'} dz' \quad . \tag{25}$$

Here, $\sigma(E)$ is the neutrino interaction cross section, $n(z)$ is a number density of the target particles.

Two processes are potentially "dangerous": neutrino - nucleon inelastic scattering growing linearly with an energy, and annihilations with neutrinos of the relic background

$$\nu_e + N \to e^- + anything, \tag{26}$$

$$\nu_e + \tilde{\nu}_e(relic) \to \sum_i (f_i + \tilde{f}_i). \tag{27}$$

Calculations show that the contribution of νN channel to the total τ is negligibly small everywere, and for typical neutrino energy ~ 100 MeV the absorbtion through annihilation is essential beginning from $z \sim 3 \cdot 10^6$. The example of a calculation of redshift dependence with taking into account the absorption is given on Fig.1. Typical neutrino background spectra from PBHs are shown on Fig.2.

For an obtaining of the constraints on the spectral index we use three types of neutrino experiments.

1. *Radiochemical experiments for the detection of solar neutrinos.* There are data on solar neutrino fluxes from the famous Davis experiment and the $Ga - Ge$ experiment.

In general, the cross section for the neutrino absorption via a bound-bound transition can be calculated using the approximate formula

$$\sigma = \begin{cases} \frac{G_F^2}{\pi}\left(\langle 1\rangle^2 + \left(\frac{g_a}{g_v}\right)^2 \langle\sigma\rangle^2\right) p_e E_e & , E < 100 MeV \\ \textit{const.} & , E > 100 MeV. \end{cases}$$

2. *The experiment on a search of an antineutrino flux from the Sun.* In some theoretical schemes (e.g., in the model of a spin - flavor precession in a magnetic field) the Sun can emit rather large flux of antineutrinos. LSD experiment (Aglietta *et al.* , 1996) sets the upper limit on this flux, $\Phi_{\tilde{\nu}}/\Phi_\nu \le 1.7\%$. In this experiment the neutrino detection is carried out using the reaction

$$\tilde{\nu}_e + p \to n + e^+ \; . \tag{28}$$

The number of target protons is $\sim 8.6 \cdot 10^{28}$ per 1 ton of the scintillation detector, and the obtained upper limit is 0.28 antineutrino events per year per ton.

3. *The Kamiokande experiment on a detection of atmospheric electron neutrinos* (Hirata *et al.* , 1992). In this experiment the electrons arising in the reaction

$$\nu_e^{atm} + n \to p + e^- \tag{29}$$

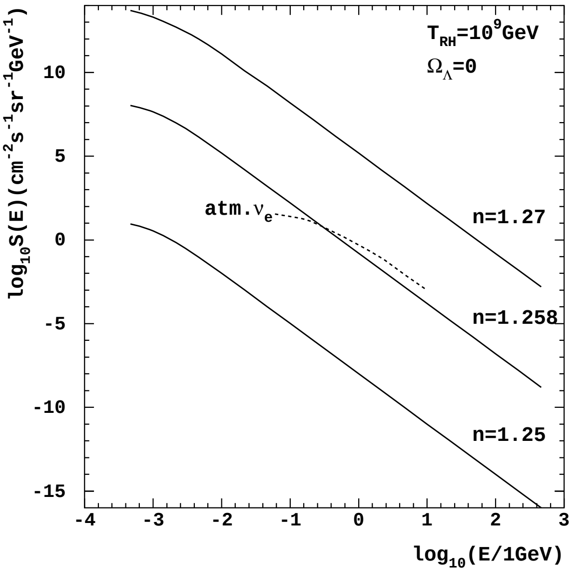

Fig. 2. Electron neutrino background spectra from PBHs, calculated for several values of the spectral index. Dashed curve shows the theoretical atmospheric neutrino spectrum at Kamiokande site (Bugaev an Naumov , 1989) (averaged over all directions).

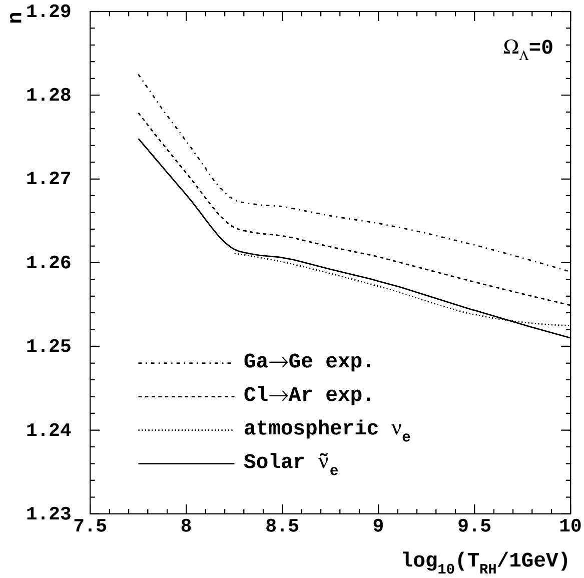

Fig. 3. Constraints on the spectral index n as a function of the reheating temperature T_{RH}, from three types of the neutrino experiments.

in the large water Cherenkov detector were detected and, moreover, their energy spectrum was measured. This spectrum has a maximum at the energy about $300MeV$. The spectrum of the atmospheric electron neutrinos is calculated with a very large accuracy (assuming an absence of the neutrino oscillations) and the experimentally measured electron spectrum coincides, more or less, with the theoretical prediction. The observed electron excess at $E \sim 100MeV$ (which is a possible consequence of the oscillations) is not too large. We use the following condition for an obtaining the our constraint : the absolute differential intensity of the PBH neutrino background at the neutrino energy $E \sim 0.3GeV$ cannot exceed the theoretical differential intensity of the atmospheric electron neutrinos at the same energy.

5 Results and discussions

Fig.3 shows our results for the spectral index constraints. It is seen that the best constraints are obtained using the Kamiokande atmospheric neutrino data and the LSD upper limit on an antineutrino flux from the Sun.

The slight bend at $T_{RH} \sim 10^{10}$GeV of the constraint curve based on the atmospheric neutrino experiment (Fig.3) is an effect of the neutrino absorption in the space. Other constraints are less sensitive to the absorption because neutrinos of smaller energies (in average) are responsible for them.

Flattening out of the constraint curve $n(T_{RH})$ with increase of T_{RH} is connected with the dependence $M_{BH}^{min}(T_{RH})$. At small T_{RH} the minimum value of PBH mass in the initial PBH mass spectrum is large and the neutrino background is dominated by the evaporations at recent epochs. In opposite, at large T_{RH}, when M_{BH}^{min} is small, the neutrino background is dominated by the evaporations at earlier times.

One should note that usually the calculations of these constraints are accompanied by the calculation of the bounds based on requirement that the energy density in PBHs does not overclose the universe at any epoch ($\Omega_{BH} < 1$). For a setting of such bounds one must consider the cosmological evolution of the system PBHs + radiation. We intend to carry out these calculations in a separate paper. Estimates show that at large T_{RH} values ($T_{RH} \geq 10^{10}$GeV) the constraints based on $\Omega_{BH} < 1$ are stronger than those based on the neutrino experiments (it is the reason why we did not explore the region $T_{RH} > 10^{10}$GeV in the present paper).

Acknowledgements. We wish to thank G. V. Domogatsky for valuable discussions and comments.

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