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Propagation of cosmic rays, an analytical model

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Abstract. Supernovae are considered as the most probable sources of Galactic cosmic rays (CR). In this analytical approach, we investigate the influence of the discrete nature of this kind of CR-sources on the CR-propagation and spectrum measured at earth. We use a diffusion model with three independent spatial coordinates. As this is an analytical approach, we only consider the most important interactions with the interstellar medium: continuous and catastrophic losses. We assume a geometry of a thin disk, filled with gas and containing the sources, embedded in a halo, which is assumed to contain little or no gas.

We present the solution for the Galactic disk and in the halo for arbitrary CR source distributions in the Galactic disk, both for the steady state and the time-dependent case. The aim is an analytical solution for the full system in both cases.

1 Introduction

Supernovae and supernova remnants (SNR) are the most probable sources of Galactic cosmic rays with energies up to some 10^5GeV, and recent observations (Koyama et al., 1995, 1997; Keohane et al., 1997; Slane et al., 1999; Allen et al., 1997; Borkowski et al., 2001) show that at least electrons are accelerated at these sites. To gain a better understanding of the processes involved in the Galactic CR propagation and the influences of point-like sources, as are SNR, on the observed CR spectra, we use an analytical approach of investigating the propagation of CR in our Galaxy with three independent spatial coordinates. These calculations are also useful as a test for numerical simulations which consider more processes during propagation, e.g. reacceleration and adiabatic losses, to get a more realistic setup, as it is difficult to develop numerical codes for five or more independent variables.

Although calculations on the propagation of cosmic rays (CR) have been done for more than 40 years both analytically (e.g. Jokipii (1966); Lerche and Schlickeiser (1982)) and numerically (e.g. (Strong and Moskalenko, 1998)), none of the analytical solutions consider three independent spatial coordinates and the geometry of the Galaxy plus halo. Also, the work presented here is still in progress.

As this is an analytic approach, we consider only the most important processes in CR propagation for energies between 1 GeV and some 10^5 GeV: spatial diffusion, continuous, and catastrophic losses. The continuity equation for the CR differential particle density N for particles of momentum p then reads

$$\frac{\partial N}{\partial t} - S(\boldsymbol{r}, p, t) \quad = \quad \nabla(k_x \nabla N) - \frac{\partial}{\partial p}(\dot{p}N)\,\frac{N}{T} \qquad (1)$$

with S being the sources, k_x the coefficient of spatial diffusion, $\dot{p}$ the rate of energy loss and T the timescale for catastrophic losses.

To solve Eq. (1) for three spatial dimensions we further have to assume that in the region of interest the spatial diffusion coefficient is independent of the spacial coordinates, but still may depend on the particle momentum p. The propagation equation (1) for this case has been solved by Syrovatskii (1959) for an infinite medium. This solution may be used if losses are dominant and sources are nearby, e.g. for electrons. Considering sources in the whole Galaxy the geometry of our Galaxy is important. As mentioned above, there are a lot of papers dealing with cosmic ray propagation in the Galactic disc, surrounded by a halo (e.g. LeGuet & Stanton (1974)), but all these papers assume cylindrical symmetry of the source distribution. To consider cosmic ray point sources like SNR, all spatial coordinates have to be taken into account.

2 The Galactic model

To solve Eq. (1) for three spatial dimensions we have to use a spatial diffusion coefficient not varying in space. Approximately this is true on large scales for the Galactic disk and the halo separately. We therefore assume the Galaxy to be a

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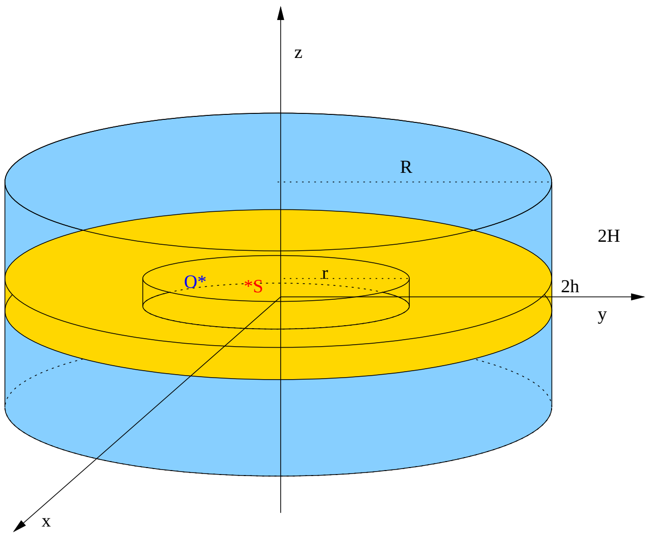

Fig. 1. We assume a simple model for our Galaxy and its souround-ing halo. The Galactic disc is assumed to be filled with gas and containing the sources. It is assumed the halo contains no gas, but a weak magnetic field which means there is only diffusion in the halo. The cosmic ray density-distribution is assumed to vanish at the boundaries at $r = R$ and $z = \pm H$.

disk with radius R and height $2h$ containing gas, the Galactic magnetic field and the CR sources; a halo of the same radius and height H on each side of it, containing no gas but a magnetic field. This means there are virtually no losses in the halo (see Fig.1). At the boundaries at $r = R$ and at $z = H$ the CR differential particle density N is assumed to vanish. Also, as there are different spatial diffusion coefficients in the disk and the halo, we get at the boundary between disk and halo from the condition that the particle density is continuous

$$N_D(z = h_D) = N_H(z = h_D) \tag{2}$$

and from continuity of the flux over the surface

$$k_H \frac{\partial N_H}{\partial n} = k_D \frac{\partial N_D}{\partial n} \tag{3}$$

$\frac{\partial}{\partial n}$ denoting differentiation along the normal of the surface of separation.

3 Calculation

Assuming $k = k(p)$, $\dot{p} = B(p)$, $T = T(p)$ and, as we are dealing with losses only, $\dot{p} = B(p) < 0$, we get for the propagation equation (1):

$$\frac{\partial N}{\partial t} - S = k(p)\Delta N - \frac{\partial}{\partial p}\left(B(p)N\right) - \frac{N}{T(p)} \tag{4}$$

We solve (4) by transforming it into a form similar to the heat equation.

3.1 Steady state case

We transform

$$-S = k(p)\Delta N - \frac{\partial}{\partial p}\left(B(p)N\right) - \frac{N}{T(p)} \tag{5}$$

with the ansatz

$$N = \frac{\exp\left(-\int_{p_0}^{p} \frac{dp'}{B(p')T(p')}\right)\phi}{B(p)} \tag{6}$$

and the change of coordinates

$$p \to \tilde{p} = \int^{p} \frac{k(\bar{p})}{B(\bar{p})} d\bar{p} \tag{7}$$

into

$$-\exp\left(\int^{p} \frac{dp'}{T(p')B(p')}\right) \frac{B(\tilde{p})}{k(\tilde{p})} S = \Delta\phi - \frac{\partial \phi}{\partial \tilde{p}} \tag{8}$$

which is the same as a heat equation.

Using the solutions for the heat equation (Carslaw and Jaeger, 1959), we get as a homogenous solution in the disk

$$\begin{aligned} N \quad = \quad & \frac{e^{-\int^{p} \frac{dp'}{B(p)T(p)}}}{B(p)} \sum_{n=0}^{\infty} \sum_{\alpha_n} \int_{\xi} d\xi e^{-(\alpha_n^2 + \xi^2)\int^{p} \frac{k(p')}{B(p')} dp'} \\ & \times \jmath_n(\alpha_n r) \cos n\left(\varphi - C_{\alpha_n}\right) \sin\left(\xi z - E_{\alpha_n}\right) \end{aligned} \tag{9}$$

with α_n being a solution of $\jmath_n(\alpha_n R) = 0$. The constants are determined by the boundary conditions between disk and halo (2), (3) at $z = z_D$ and $z = -z_D$.

As a Greens' function, assuming the disk being an infinite cylinder, we get :

$$\begin{aligned} G \quad = \quad & \frac{1}{B(p)} e^{\int_{p}^{p_0} \frac{dp'}{B(p') \cdot T(p')}} \cdot \frac{e^{-\frac{(z-z_0)^2}{4\left(\int_{p_0}^{p} \frac{k(p')}{B(p')} dp'\right)}}}{2\pi R^2 \sqrt{\pi} \sqrt{\int_{p_0}^{p} \frac{k(p')}{B(p')} dp'}} \\ & \times \sum_{n=-\infty}^{\infty} \cos(n(\varphi - \varphi_0)) \cdot \Theta\left(p_0 - p\right) \\ & \times \sum_{\alpha_n} e^{-\alpha_n^2 \left(\int_{p_0}^{p} \frac{k(p')}{B(p')} dp'\right)} \frac{\jmath_n\left(\alpha_n r\right) \jmath_n\left(\alpha_n r_0\right)}{\left(\jmath_n'\left(\alpha_n R\right)\right)^2} \end{aligned} \tag{10}$$

In the halo (no losses) we are left in the steady state case with Laplace's equation. A solution fulfilling the boundary conditions is given by

$$\begin{aligned} N \quad = \quad & \sum_{n=0}^{\infty} \sum_{\alpha_n} A_{n\alpha_n} \sinh\left(\alpha_n \left(z - h_H\right)\right) \jmath_n\left(\alpha_n \cdot r\right) \\ & \times \cos m\left(\varphi - F_{\alpha_n}\right) \end{aligned} \tag{11}$$

The coefficients A_{α_n}, F_{α_n} and C_{α_n}, E_{α_n} in (9) are determined by the boundary conditions between halo and disk.

3.2 Time dependent case

To transform

$$\frac{\partial N}{\partial t} - S = k(p)\Delta N - \frac{\partial}{\partial p}\left(B(p)N\right) - \frac{N}{T(p)} \tag{12}$$

we use the ansatz

$$N = \frac{\exp\left(-\int^p \frac{dp'}{B(p')T(p')}\right)\phi}{B(p)} \tag{13}$$

and change the coordinates $(t, p) \to (t', \lambda)$

$$t'(t,p) = t - \int^p \frac{dp'}{B(p')} \tag{14}$$

$$\lambda(p) = \int^p \frac{k(p')}{B(p')} dp' \tag{15}$$

so for $k \neq 0$ we get:

$$-\exp\left(\int^p \frac{dp'}{B(p')T(p')}\right)\frac{B(p)}{k(p)}S = \Delta\varphi - \frac{\partial\varphi}{\partial\lambda} \tag{16}$$

So in the Galactic disk, we get as a homogenous solution:

$$\begin{aligned} N &= \sum_{n=0}^{\infty}\sum_{\alpha_n}\int_{\xi} d\xi e^{-\left(\left(\alpha_n^2+\xi^2\right)\left(\int^p \frac{k(p')}{B(p')}dp'\right)\right)} \\ &\times \frac{1}{B(p)} \jmath_n\left(\alpha_n r\right)\cos\left(n\varphi - A_{\alpha_n}\right) \\ &\times \left(C_{\alpha_n}\sin\left(\xi z\right) + D_{\alpha_n}\cos\left(\xi z\right)\right) \\ &\times \exp\left(\int^p \frac{dp'}{B(p')\cdot T(p')}\right)\cdot f\left(t - \int_{\bar{p}}^p \frac{dp'}{B(p')}\right) \end{aligned} \tag{17}$$

f being an arbitrary function of $\left(t - \int_{\bar{p}}^p \frac{dp'}{B(p')}\right)$.

As a Greensfunction, assuming the disk being an infinite cylinder, one gets:

$$\begin{aligned} G &= \frac{1}{B(p)}\delta\left(t - t_0 + \int_p^{p_0}\frac{dp'}{B(p')}\right) \\ &\times \frac{e^{-\frac{(z-z_0)^2}{4\left(\int_{p_0}^p \frac{k(p')}{B(p')}dp'\right)}}}{2\pi R^2\sqrt{\pi}\sqrt{\int_{p_0}^p \frac{k(p')}{B(p')}dp'}} \\ &\times \sum_{n=-\infty}^{\infty}\cos(n(\varphi-\varphi_0)) \\ &\times \sum_{\alpha_n} e^{-\alpha_n^2\left(\int_{p_0}^p \frac{k(p')}{B(p')}dp'\right)}\frac{\jmath_n\left(\alpha_n r\right)\jmath_n\left(\alpha_n r_0\right)}{\left(\jmath_n'\left(\alpha_n R\right)\right)^2} \\ &\times \Theta\left(p_0 - p\right)e^{\int_p^{p_0}\frac{dp'}{B(p')\cdot T(p')}} \end{aligned} \tag{18}$$

In the halo we get the ordinary heat equation. A solution fulfilling the boundary conditions is given by

$$\begin{aligned} N &= \sum_{n=0}^{\infty}\sum_{\alpha_n}\int \xi d\xi e^{-\kappa\left(\alpha_n^2+\xi^2\right)t}\jmath_n\left(\alpha_n r\right) \\ &\times \cos\left(n\varphi - F_{\alpha_n}\right)\sin\left(\xi\left(z-h_H\right)\right) \end{aligned} \tag{19}$$

As in the steady state case, the coefficients F_{α_n} and in (17) $A_{\alpha_n}, C_{\alpha_n}, D_{\alpha_n}$ are determined by the boundary conditions between halo and disk.

4 Summary

We here presented solutions for the CR propagation equation in three spatial dimensions, taking into acount diffusion, continuous and catastrophic losses in the context of a Galactic model. In this model, we assume the Galactic disk to be a flat cylinder, containing the CR sources, the Galactic magnetic field and gas; a halo of the same radius and height H on each side of it, containing no gas. This means there are no losses in the halo. As this calculations are still in progress, it remains the task to fulfill the boundary conditions at the boundary between disk and halo.

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