Proceedings of ICRC 2001: 2742 © Copernicus Gesellschaft 2001

ICRC 2001

Spectra and time scales for particle acceleration in ultra-relativistic flows applicable to gamma-ray bursters

A. Meli and J. J. Quenby

Imperial College, Blackett Laboratory, Astrophysics Group, London SW7 2BW

Abstract. Monte-Carlo computations of ultra-relativistic parallel and oblique shock acceleration is presentedfor upstream flow gamma factors, $\Gamma = (1 - V_{up}^2/c^2)^{-0.5}$ up to $\Gamma \sim 1000$, relevant to Gamma-ray burst (GRB) fireballs. For oblique sub-luminal shocks, the spectra depend on whether or not the upstream scattering is small angle with $\delta\theta < \Gamma^{-1}$ or isotropic, which is possible if $\lambda > r_g\Gamma$ where λ is the scattering mean free path along the field line and r_g the gyroradius. The large angle case exhibits distinctive structure in the basic power-law spectrum not nearly so obvious for small angle scattering but both cases yield a significant speed-up of acceleration rate when compared with the conventional, non-relativistic expression, $t_{acc} = [c/V_{up} - V_{down}][\lambda_{up}/V_{up} + \lambda_{down}/V_{down}]$. The Γ^2 energisation factor per shock crossing, important in the Vietri work on GRB ultra-high energy neutrino, and possibly cosmic ray and gamma-ray output, is supported for the first crossing cycle but the factor is less subsequently. Super-luminal shock results are discussed in a companion paper (Meli and Quenby, 2001).

1 Introduction

Three distinct astrophysical situations where the bulk plasma flow is extremely relativistic, with Lorentz factors $\Gamma = (1 - v^2/c^2)^{-0.5} \geq 10$ are some AGN (Active Galactic Nuclei) jets, GRB (gamma-ray burst) fireballs and isolated pulsar polar winds. In each case, there is evidence for energetic particle acceleration to much higher Γ factors but the upper limit to the possible energy attained becomes less certain with increasing bulk flow velocity. For AGN jets, $\Gamma \sim 10$ plausibly results in 10^{19} eV particles via diffusive shock acceleration at the termination shocks. For GRB fireballs, $\Gamma \sim 1000$ appears to eventually produce gamma-rays at least up to 100 MeV but at such flow speeds, it is not clear whether repeated shock crossings are possible and the predictions of 10^{19} eV neutrinos (Vietri, 1998) which would be direct evidence of a diffusive like process have yet to be verified. Similarly, pulsar winds could result in a $\Gamma \sim 10^4$ flow encountering the nebula envelope, but there is no evidence for more than TeV acceleration anywhere in the system. Past work has shown that if the shocks are close to parallel so that an $\boldsymbol{E} = 0$ frame can be defined, diffusive shock acceleration is in principle possible and each shock crossing cycle, upstream-downstream-upstream, yields a $\sim \Gamma^2$ energy increase (Quenby and Lieu, 1989) which has been exploited by Vietri (1998) for fireball acceleration. With $\Gamma >> 1$, the range of shock inclinations yielding the subluminal, where $\boldsymbol{E} = 0$ frame is possible, is much reduced and single crossings become likely unless special trajectories are invoked with field nulls (Lucek and Bell, 1994). It is the purpose of this paper and the accompanying paper, Meli and Quenby (2001), to explore acceleration models with Γ values up to ~ 1000. Here we concentrate on the parallel and oblique subluminal situations but in Meli and Quenby (these proceedings) the superluminal (no $\boldsymbol{E} = 0$ is possible) situation will be considered, where the particles helix-trajectory is followed explicitly in the suitable frames of reference. Our results will be compared with the standard spectral prediction of non-relativistic theory where the momentum spectrum is $n(p) \propto p^{-\alpha}$ with $\alpha = (r+2)/(r-1)$ for shock compression ratio $r = V_1/V_2$ independent of scattering details and $\alpha = 2$ for strong unmodified shocks with r=4. Analytically it has shown tha over a limited momentum range, the non-relativistic time scale for acceleration is $T = [c/(V_1 - V_2)][\lambda_1/V_1) + (\lambda_2/V_2)]$ where "1" and "2" refer to upstream and downstream and λ is the scattering mean free path. The first noticed relativistic effect correcting non-relativistic theory was the spectral flattening seen in parallel shocks (Kirk and Schneider, 1987), followed by the discovery of a speed-up in acceleration rate (Quenby and Lieu, 1989; Ellison et al, 1990). These results have subsequently been extended to non-parallel and non-linear subluminal shocks (eg Bednarz and Ostrowski, 1996) where scattering is large-angle. More recently, interest has focussed on differences occuring between small-angle and large angle scattering models for the fluid frame test particle-turbulence

Correspondence to: a.meli@ic.ac.uk

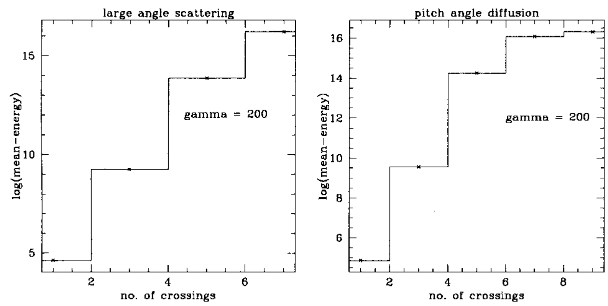
interactions. Gallant and Achterberg (1999) suggest that for a field model consisting either of randomly orientated uniform field cells or a uniform field, the scattering is limited to $\delta\theta < 1/\Gamma$ in the upstream fluid frame. Such a model yields a test particle differential spectral slope ~ -2.2 (eg Baring, 1999). Furthermore, on this model, Gallant and Achterberg, (1999) found that in the high Γ subluminal limit, only the first crossing cycle exhibited a Γ^2 energy increase and hence one may expect the relativistic speed-up to be limited to this crossing.

2 Numerical Method

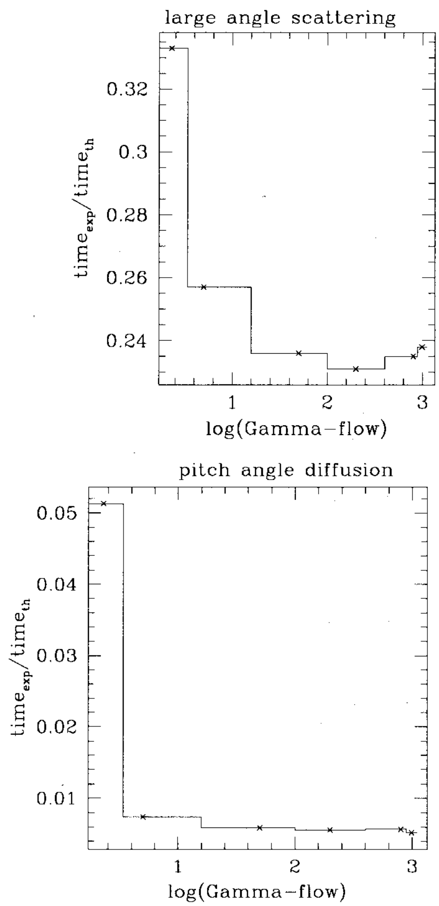
Because large anisotopies develop in the particle distribution function in the relativistic flow case, it is convienient to employ a numerical, Monte Carlo scheme to model the acceleration. We confine ourselves here to either parallel shocks or subluminal oblique shocks where a boost along the shock front in the shock rest frame by $V_1 tan\psi_1$ where ψ_1 is the upstream field-shock normal angle is at a speed less than c, allowing a de Hoffmann-Teller $\boldsymbol{E} = 0$ frame to exist. Particles are followed in the fluid frames under the guiding centre approximation along the field lines but scatter elastically according to a mean free path, $\lambda_{||} = \lambda_\circ p cos\theta$ where θ is pitch angle. In the small angle case with $\delta\theta < \Gamma^{-1}$, we define a mean $\delta\theta$ and find the $\lambda_\circ$ necessary to provide $\Sigma\delta\theta \sim \pi$ by random walk. The large angle case corresponds physically to motion where the main scattering is conceived as due to large scale rotational and tangential discontinuities separated by $\lambda > r_g\Gamma$ where r_g is the shock frame gyro-radius. This statement requires the particles in the upstream fluid frame to describe much of their helical period before encountering substantial scattering and since the scattering is hypothesised as being a discontinuity, the scattering can be large. Note we retain the momentum dependence of λ, essentially claiming that a particle mainly 'notices' field discontinuities of a scale $\geq r_g$. In the Monte Carlo, the scatter probability along a field line is given by $Prob(z) \sim exp(-z/\lambda|cos\theta|)$ where the cos factor takes into account the solid angle increment. Isotropic injection and z direction current is assured by a $sin\theta cos\theta$ weighting. The new pitch angle θ', for pitch angle diffusion, is calculated by the simple trigonometric formula: $cos\theta' = cos\theta\sqrt{1 - sin^2\delta\theta} + sin\delta\theta\sqrt{1 - cos^2\theta} cos\phi$ where $\phi(0,2\pi)$ is the azimuth angle with respect to the original momentum direction.A relativistic transformation is performed to the local plasma frames each time the particle scatters across the shock following it according to particle *jump conditions* and it is made to leave the system from the moment that it 'escapes' far downstream at the spatial boundary at 100λ or if it reaches a well defined maximum energy E_{max}. The particle splitting technique is employed to improve statistics at high energies. The particles ($\sim 10^6$) of weight equal to one, are injected far upstream at a constant energy of high gamma, which supposes that a pre-acceleration of the particles has already taken place. They left to move towards the shock where along the way they collide with the pressumed scattering centers and consequently as they keep scattering between the upstream and downstream regions they gain each time an amount of energy. At the shock, for the non-parallel cases, transmission or reflection is determined by transforming the momentum into the de Hoffmann-Teller frame and assuming conservation of the first adiabatic invariant. Particles are followed in either the upstream or downstream frames with Lorentz transformation at the interface, but at each computational step, the position is checked in shock frame coordinates.

3 Parallel and Oblique Subluminal Shock Results

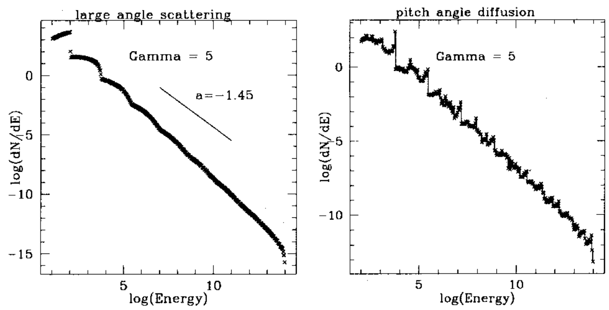
Although true parallel shocks may be rare in practise, there may be sufficient turbulence at a shock interface to destroy reflection and the following results may be more generally applicable. A compression ratio of 4 is used to allow immediate comparison with non-relativistic unmodified shock results, but later we will claim the qualitative trend of the results is insensitive to the exact value. Figure 1 shows the log of the mean particle energy γ against crossing number where we have three crossings per up-down-upstream cycle and for large angle and pitch angle scattering respectively. The upstream flow is at $\Gamma = 200$. In both cases, the energy gain on the first complete cycle (up-down-upstream) is $\sim \Gamma^2$ but subsequent crossings show a reducing gain ($\sim \Gamma$). These gains on subsequent cycles appear larger than the factor two predicted by Gallant and Achterberg (1999), but qualitatively, the trend is similar. The ratio of energies crossing downupstream in the respective fluid frames is $\Gamma(1 + \beta_r \mu'_{\rightarrow u})$ where β_r is the relative velocity of the two streams as a fraction of c and $\mu'_{\rightarrow u}$ is cosine pitch angle for down to upstream crossing in the downstream frame which only needs to exceed $1/4 \rightarrow 1/3$ kinematically. Hence the gain can still be $\sim \Gamma$ whereas following the arguments of Gallant and Achterberg (1999) the gain ratio up to down is $\Gamma(1 - \beta_r \mu_{\rightarrow d})$ and $\mu_{\rightarrow d}$ the upstream frame cosine pitch angle on up to down transmission is nearly unity and small angle scattering must be used and the energy gain is limited. Results for $log\gamma > 11$ for protons and a lower number for electrons of course are unrealistic in that trapping breakdown and synchrotron radiation are not in the model. The effects of the enhanced energy gains on the *total* acceleration time are seen in figure 2 (top for large angle, bottom for pitch angle diffusion) where we show the ratio of the 'experimental' or numerically computed time constant to the non-relativistic, analytic theory prediction mentioned above. Note the very dramatic speed-up for pitch angle scatter depends on the π phase reversal by random walk definition of λ. The large angle results are similar to those obtained previously at lower Γ by Lieu et al, (1994) for various shock inclinations. We show contrasting spectral results in figures 3 (left and right) for $\Gamma = 5$ for large and pitch angle scattering and figures 4 (left and right) for $\Gamma = 990$ for large and pitch angle scattering. The smooth spectra of the relativistic flow with $\Gamma = 5$ become plateau like at $\Gamma = 990$ where the effects of individual acceleration cycles are clearly evident but more pronounced in the large angle case. The spectra are taken at the shock in the shock frame. For oblique subluminal shocks we show in the figure 5 (two top, two bottom) the mean energy gain per crossing for large angle scattering with ψ_1 equal to 15° and 35° and for r=4 and r=3 respectively, all for $\Gamma = 200$. The relative independence of r but the similarity in the trend from Γ^2 to Γ in energy dependence to the parallel case are seen. Finally distinctive spectra shapes are seen in figure 6 for $\Gamma = 500$ and $\Gamma = 990$ for pitch angle and large angle scattering at $\psi_1 = 15°$.



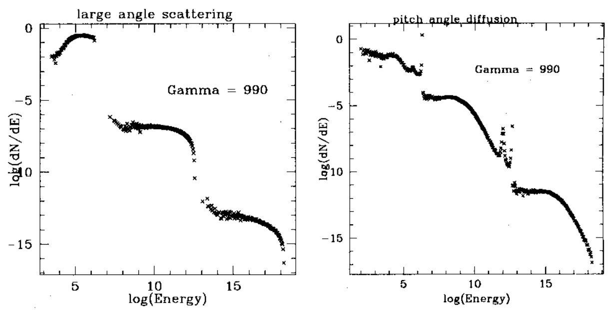
. **Fig. 1.** The upstream flow is at $\Gamma = 200$. In both cases, the energy gain on the first complete cycle (up-down-upstream) is $\sim \Gamma^2$ but subsequent crossings show a reducing gain ($\sim \Gamma$).



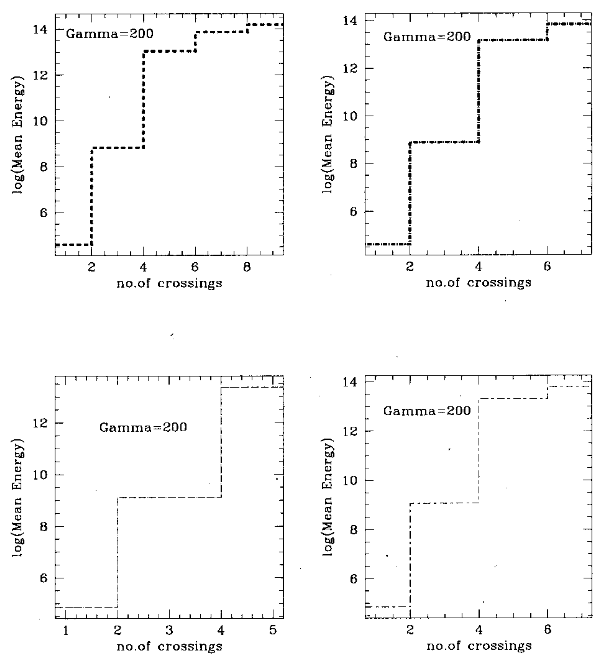
. **Fig. 2.** The *total* 'experimental' time spent upstream and downnstream of the shock to the non-relativistic 'theoretical' time.



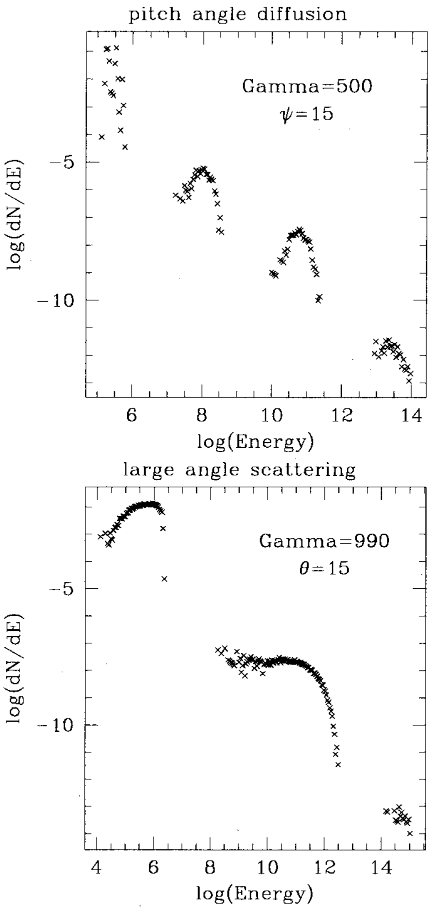
. **Fig. 3.** Parallel shocks. Spectra for $\Gamma = 5$ for large and pitch angle scattering.



. **Fig. 4.** Parallel shocks. Spectra for $\Gamma = 990$ for large and pitch angle scattering respectively.



. **Fig. 5.** Here for oblique subluminal shocks we show the mean energy gain per crossing for large angle scattering with ψ_1 equal to 15° and 35° and for r=4 and r=3 respectively (two top, two bottom), all for $\Gamma = 200$. The relative independence of r but the similarity in the trend from Γ^2 to Γ in energy dependence to the parallel case are seen.



. **Fig. 6.** Oblique subluminal shocks. We observe distinctive spectra shapes. Top figure for $\Gamma = 500$ and pitch angle diffusion, bottom figure $\Gamma = 990$ for large angle scattering. Both for $\psi_1 = 15°$.

4 Conclusions

Numerical simulation of shock acceleration in the parallel and subluminal oblique shock cases for both large angle and pitch angle scattering exhibits a shock gamma squared (Γ^2) energy enhancement for the first up-down-up cycle followed by a $\sim \Gamma$ enhancement but decreasing at very high energies. Speed-up of acceleration by factors ~ 10 or more over the non-relativistic diffusive shock estimate appear to occur. At mildly relativistic plasma flow speeds, smooth spectra result, but at $\Gamma \geq 100$, particularly for large-angle scattering, structure indicating each acceleration cycle becomes apparent. GRB acceleration to proton energies of $\sim 10^{20}$ eV for $\Gamma \sim 1000$ in a very few cycles seems possible.

References

Baring, M.G., 26th ICRC, 1999.
Bednarz, J., and Ostrowski, M., MNRAS, 283, 447, 1996.
Gallant, Y. A., and Achterberg, A., MNRAS, 305L, 6G, 1999.
Kirk, J. G., and Schneider, P., Ap. J., 315, 425, 1987.
Lieu,R., Quenby,J., Drolias,B. and Naidu,K., Ap. J., 421, 211, 1994.
Lucek, S. G., and Bell, A. R., MNRAS, 268, 581L, 1994.
Meli, A and Quenby, J. J., This Conference, 2001.
Meli, A and Quenby, J. J., ECRS 2000, astro-ph/0101245.
Meli, A and Quenby, J. J., ECRS 2000, astro-ph/0101246.
Quenby, J. J., Nature, 342, 654, 1989.
Vietri, M., Ap. J. Lett., 448 L105, 1998.