Proceedings of ICRC 2001: 3698 © Copernicus Gesellschaft 2001

ICRC 2001

# Effect of Fisk-type heliospheric magnetic fields on the latitudinal transport of cosmic rays

**R. A. Burger and M. Hattingh**

Unit for Space Physics, School of Physics, Potchefstroom University for CHE, 2520 Potchefstroom, South Africa

**Abstract.** A heliospheric magnetic field (HMF) with a meridional component, such as the model of Fisk, leads to a more complicated form of the transport equation (TPE) for cosmic rays than is the case for the Parker model. The number of mixed derivatives increases and as a result the numerical codes used to solve the TPE becomes unstable more easily. In this progress report we circumvent some of these complications by using restrictive transport parameters. Apart from the standard Fisk field, we also consider a second Fisk-type field. Here the motion of the footpoints of the magnetic field on the source surface are assumed to follow circles centred on an axis that is perpendicular to the rotation axis of the Sun. Such footpoint motions may occur, for instance, when the orientation of the solar magnetic dipole changes. We solve the three-dimensional steady-state TPE in a system corotating with the Sun, using spherical coordinates in an ADI numerical scheme. We show that both the standard Fisk field and the second Fisk-type field may reduce the latitudinal cosmic-ray gradient more at low than at high rigidity. Given our choice transport parameters, we see small effects. These should however be indicative of what can be expected when the restrictions on the transport parameters are relaxed.

## 1 Introduction

In the model of the heliospheric magnetic field proposed by Fisk (1996) magnetic field lines at high latitudes can be connected directly to corotating interaction regions in the solar wind at low latitudes at larger heliocentric distances. This effect can account for observations from the Ulysses spacecraft of recurrent energetic particle events at high latitudes.

In this progress report we solve the three-dimensional steady-state TPE in a system corotating with the Sun using spherical coordinates and an ADI numerical scheme. The number of mixed derivatives increases and as a result the numerical code becomes less stable. This restricts the range of parameters that can be used. We regard the results from our code as both preliminary and qualitative.

We introduce a second type of Fisk field and show that both types of fields reduce latitudinal gradients more at low rigidity than at high rigidity. While we neglect drifts in the numerical code, we show an estimate of drift effects in a Fisk field and a comparison to drift effects in other models for the HMF.

## 2 Modulation model

In a coordinate system corotating with the Sun the cosmic-ray transport equation (TPE) of Parker (1965) can be written in terms of the omnidirectional distribution function $f(\mathbf{r}, p)$ (related to the differential intensity by $j_T \propto p^2 f$) as (Kóta and Jokipii, 1983)

$$\nabla \cdot (\mathsf{K}^s \cdot \nabla f) - (\mathbf{v}_d + \mathbf{V}^*) \cdot \nabla f + \frac{1}{3} (\nabla \cdot \mathbf{V}^*) \frac{\partial f}{\partial \ln p} = 0.$$

Here $\mathbf{r}$ is position, $p$ is momentum, $\mathsf{K}^s$ is the symmetric part of the diffusion tensor, $\mathbf{V}^* = \mathbf{V} - \mathbf{\Omega} \times \mathbf{r}$ with $\mathbf{V}$ the solar wind velocity and $\mathbf{\Omega}$ the angular velocity of the Sun, and $\mathbf{v}_d = \frac{pv}{3q} \nabla \times \frac{\mathbf{B}}{B^2}$ the particle drift velocity for a near-isotropic particle distribution due to the curvature and gradient of the HMF, with $v$ and $q$ respectively particle speed and signed charge, and $\mathbf{B}$ the magnetic field.

In our current three-dimensional modulation model (Burger and Hattingh, 1995; Hattingh, 1998) the TPE is solved in spherical coordinates. The diffusion tensor in spherical coordinates $(r, \theta, \phi)$ for the case when the HMF has a meridional component $B_\theta$, is taken from Kobylinski (2001). Some elements are

$$\kappa_{\rm rr} = (\kappa_{||} \cos^2 \Psi + \kappa_\perp \sin^2 \Psi) \cos^2 \zeta + \kappa_\perp \sin^2 \zeta;$$

$$\kappa_{\theta\theta} = (\kappa_{||} \sin^2 \Psi + \kappa_\perp \cos^2 \Psi) \sin^2 \zeta + \kappa_\perp \cos^2 \zeta,$$

with $\tan \Psi = -B_\phi / B_r$ and $\tan \zeta = B_\theta / B_r$. Here $\kappa_{||}$ and $\kappa_\perp$ are the diffusion coefficients parallel and perpendicular to

*Correspondence to:* R.A. Burger (fskrab@puknet.puk.ac.za)

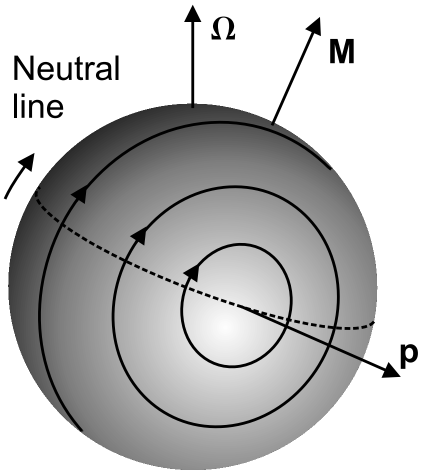


**Fig. 1.** Footpoint trajectories for the type II Fisk field viewed in a frame corotating with the Sun. The **p** axis is in the plane defined by the neutral line and perpendicular to the rotation axis $\mathbf{\Omega}$. $\mathbf{M}$ is the magnetic axis, assumed to be perpendicular to the plane defined by the neutral line.

the mean magnetic field, respectively. We use simple forms for the diffusion coefficients, separable in rigidity and position,

$$\kappa_{||} = v(aP^{0.6} + bP^2)c \qquad (1)$$

$$\kappa_{\perp} = d\kappa_{||} \qquad (2)$$

with $a$, $b$ and $d$ constants with appropriate dimensions and $c = B_{Earth}/B$ or $c = (1 + r/r_{Earth})$. The solar wind speed is assumed constant at 800 km/s and the modulation boundary is set between 30 AU for some runs and at 60 AU for others. Any HMF considered here is assumed to be valid at all latitudes. We neglect drift effects for the present paper.

## 3 Fisk-type magnetic fields

Fisk (1996) argued that the interplay between the differential rotation of the footpoints of heliospheric magnetic field lines in the photosphere of the Sun and the subsequent nonradial expansion of these field lines with the solar wind from rigidly rotating coronal holes can result in extensive excursions of field lines with heliographic latitude. For the case when the footpoint trajectories on the source surface can be approximated by circles offset from the solar rotation axis with an angle $\beta$, an analytical expression for the HMF is obtained (Zurbuchen et al., 1997),

$$\begin{aligned} B_r &= B_0 \left(\frac{r_0}{r}\right)^2 \\ B_\theta &= B_r \frac{r}{V} \omega \sin\beta \sin\left(\phi + \frac{\Omega r}{V}\right) \qquad (3) \\ B_\phi &= B_r \frac{r}{V} \left[\omega \sin\beta \cos\theta \cos\left(\phi + \frac{\Omega r}{V}\right)\right. \end{aligned}$$

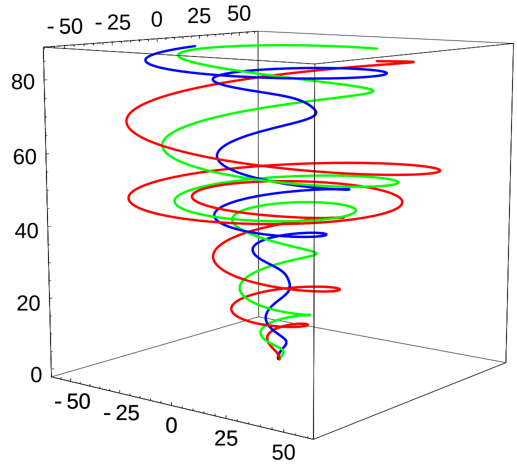


**Fig. 2.** Heliospheric magnetic field lines of the type I Fisk field. The field lines originate at a colatitude of 30° and at different longitudes. Units are in AU.

$$\left. + \sin\theta \left(\omega\cos\beta - \Omega\right)\right]$$

Here $\Omega$ is the equatorial rotation rate of the Sun and $\omega$ is the differential rotation rate. The incorporation of this HMF into numerical modulation models is problematic (Kóta and Jokipii, 1997, 1999) because additional mixed derivatives of the distribution function come into play. The properties of this HMF has been studied quite extensively by e.g. Van Niekerk (2000). Mathematically a value of 90° for $\beta$ is valid in which case Equation 3 becomes

$$\begin{aligned} B_r &= B_0 \left(\frac{r_0}{r}\right)^2 \\ B_\theta &= B_r \frac{r}{V_{sw}} \omega^* \sin\left(\phi + \frac{\Omega r}{V}\right) \qquad (4) \\ B_\phi &= B_r \frac{r}{V} \left[\omega^* \cos\theta \cos\left(\phi + \frac{\Omega r}{V}\right) - \Omega \sin\theta\right]. \end{aligned}$$

There are two physical arguments to support the existence of such an HMF. Kóta and Jokipii (1997) suggested and illustrated (Kóta and Jokipii, 1999) that a global reorganization of the HMF, such as a tilted dipole model with the tilt axis varying in time, can cause a regular meridional component in the HMF. If this variation is smooth and all other footpoint motions are neglected the resulting HMF could be represented by Equation 4 with $\omega^*$ proportional to the time rate of change of the tilt axis. The footpoint trajectories for this scenario are shown in Figure 1.

Fisk (2001) suggests that the diffusion of magnetic field lines on the Sun may cause the current sheet to become vertical and then overturn, thus providing a simple description of the HMF during solar maximum conditions. Equation 4 may also provide a simplified description for this scenario, with $\omega^*$ some appropriated time rate of change. While Thomas et al. (1986) also proposed that an overturning current sheet may be used to model the HMF near solar maximum, their model appears to be quite different from Equation 4.

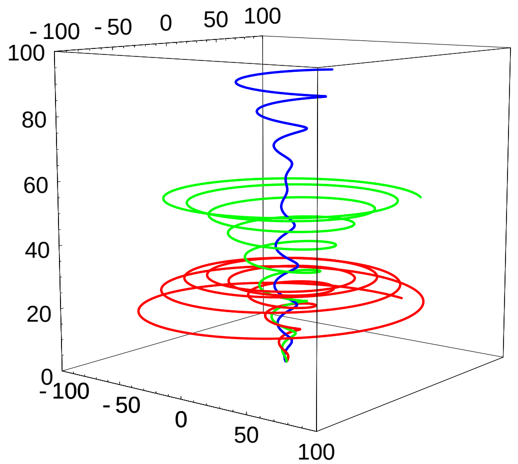


**Fig. 3.** Heliospheric magnetic field lines of the type II Fisk field. The field lines originate at a colatitude of $30°$ and at different longitudes. Units are in AU.

In what follows we will refer to Equation 3 as the type I Fisk field and to Equation 4, whatever it's cause, as the type II Fisk field. A representation of the type I Fisk field is shown in Figure 2 with $\beta = 10°$ and $\omega = \Omega/4$, and of the type II Fisk field in Figure **??** with $\omega^* = \Omega/56$.

The field lines all originate at a colatitude of $30°$ but at different longitudes. In both cases the solar wind speed is 800 km/s and these patterns are therefore set up over a period of approximately 7 months. During this period the footpoints on the source surface of the type I field has completed almost two rotations while those of the type II field has moved through approximately $45°$. The type II field is much smoother than the type I field and consequently somewhat easier to handle in a numerical modulation model.

An interesting alternative to models with smooth motions of magnetic footpoints are given in Giacalone (1999) who considers random motions.

## 4 Sample solutions

The fact that the type I Fisk field reduces latitudinal gradients has been illustrated by Kóta and Jokipii (1997). In this paper we consider how this reduction occurs as a function of rigidity. As we have pointed out above our results should be seen as qualitative. The procedure is to calculate latitudinal gradients for the type I field with $\beta = 0$ and the type II field with $\omega^* = 0$. These gradients are then divided by the gradients with $\beta = 10°$ and $\omega^* = \Omega/56$ respectively.

The results are shown in Figure 4 for the type I field and in Figure 5 for the type II field. For the former the modulation boundary is at 30 AU and $d = (1 + r/r_{Earth})$, and for the latter the modulation boundary is at 60 AU and $d = B_{Earth}/B$. Although there are quantitative differences between the two figures, they show the same qualitative behaviour. The latitudinal gradients are reduced more at smaller rigidity than at larger rigidity.

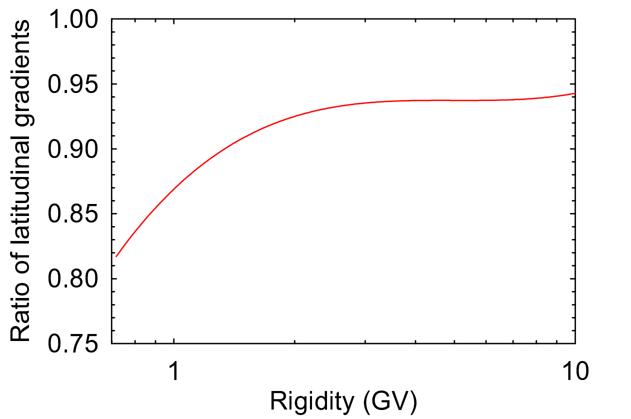


**Fig. 4.** The ratio of latitudinal gradients for the type I Fisk field with $\beta = 10°$ to those with $\beta = 0$, as function of rigidity.

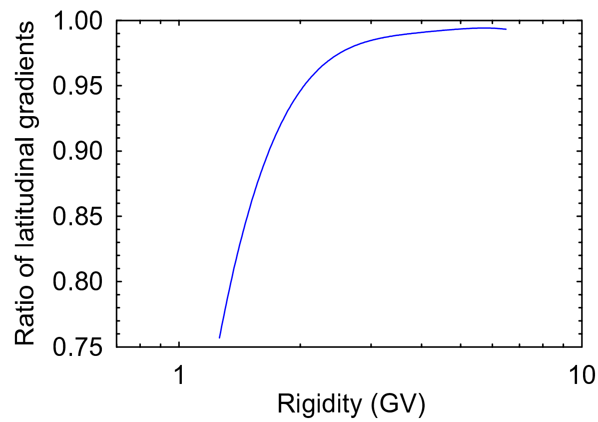


**Fig. 5.** The ratio of latitudinal gradients for the type II field with $\omega^* = \Omega/56$ to those with $\omega^* = 0$, as function of rigidity.

## 5 Discussion and conclusions

To arrive at the results of the previous section we have had to make a number of assumptions. Obviously a time-dependent approach is needed (Kóta and Jokipii, 1999) which aggravates the problem of incorporating the Fisk field into a numerical modulation model.

Although we neglect drifts in the current paper, Burger et al. (2001) give an estimate of how effective drifts are in the type I Fisk field. The spatial average of the drift term $\frac{pv}{3q}\nabla \times \frac{\mathbf{B}}{B^2} \cdot \nabla f$ is calculated assuming that $\nabla f$ is constant. This is done in a cone centred on the heliographic pole, with half-angle $60°$ to take into account the fact that the Fisk field is valid at high latitudes only. The value of this average, referred to as the drift factor, is set equal to 1 for the Parker field, as shown in Figure 6. For comparison, the same calculation is shown for the modifications proposed by Jokipii

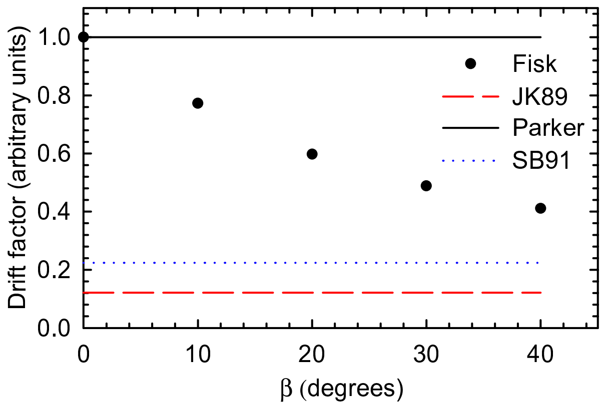


**Fig. 6.** An estimate of drift effects in the type I Fisk field, the field of Jokipii and Kóta (1989) (JK89) and that of Smith and Bieber (1991) (SB91), compared to drift effects in the standard Parker field. The quantity $\beta$ is from Eq. 3. Smaller numbers indicate smaller drifts. Adapted from Burger et al. (2001).

and Kóta (1989) and Smith and Bieber (1991), referred to as JK89 and SB91, respectively. Both the SB91 and the JK89 modifications reduce the drift factor considerably. As the angle $\beta$ in Equation 3 is increased, drift effects are reduced, but the drift factor at $\beta = 40°$ is still higher than for the other two fields. This simple comparison suggests that the type I Fisk field may not reduce drift effects any more than either of the two modifications of the Parker field. However, in the present paper we show that the Fisk-type fields yields smaller latitudinal gradients at all energies considered which will further reduce drift effects.

The type I Fisk field has been invoked to explain a variety of cosmic-ray and other observations. The type II Fisk field seems to be easier to use in our numerical modulation model and may have more than one physical explanation. If it due to a global reorganization of the HMF (Kóta and Jokipii, 1999) one would expect to see some correlation between the meridional component of the HMF and changes in the tilt angle of the heliospheric current sheet. One would also expect to see a correlation between cosmic-ray intensity and changes in the tilt angle, as modelled by Kóta and Jokipii (1999).

The possibility that these fields (and probably any HMF with a meridional component) reduce near-Earth latitudinal gradient more at low than at high rigidity has important consequences for our study of the properties of the diffusion tensor (Burger et al., 2000; Parhi et al., 2001). In two-dimensional numerical modulation models with Parker-type HMFs latitudinal transport is strongly influenced by the element $\kappa_{\theta\theta}$ of the diffusion tensor. Conclusion drawn from such models may have to be re-evaluated once the effect of a meridional component of the HMF can be studied in more detail.

*Acknowledgements.* RAB thanks Len Fisk, Randy Jokipii, József Kóta and Marius Potgieter for many useful discussion. This work was supported in part by NASA grant NAG5-8134(SECTP).

## References

Burger, R. A. and Hattingh, M., Steady-state drift-dominated modulation models for galactic cosmic rays, Astrophys. Space Sci., 230, 375-382, 1995.

Burger, R. A., Potgieter, M. S. and Heber, B., 2000, Rigidity dependence of cosmic-ray proton latitudinal gradients measured by the Ulysses spacecraft: Implications for the diffusion tensor, J. Geophys. Res., 105(A12), 27,447-27,456.

Burger, R. A., Van Niekerk, Y. and Potgieter, M. S., An estimate of drift effects in various models of the heliospheric magnetic Field, Space Sci. Rev., In press, 2001.

Fisk, L. A., Motion of the footpoints of heliospheric magnetic field lines at the Sun: Implications for recurrent energetic particle events at high heliographic latitudes, J. Geophys. Res., 101, 15547-15553, 1996.

Fisk, L. A., Origin of the slow solar wind at solar maximum: Theory, Space Sci. Rev., In press, 2001.

Giacalone, J., Particle transport and acceleration at corotating interaction regions, Adv. Space Res., 23(3), 581-590, 1999.

Hattingh, M., The modulation of galactic cosmic rays in a three-dimensional heliosphere, Ph.D. thesis, Potchefstroomse Universiteit vir Christelike Hoër Onderwys, South Africa, pp 165, 1998.

Jokipii, J. R. and Kóta, J., The polar heliospheric magnetic field, Geophys. Res. Lett., 16, 1-4, 1989.

Kobylinski, Z., Comparison of the Fisk magnetic field with the standard Parker IMF: Consequences for diffusion coefficients, Adv. Space Res., In press, 2001.

Kóta, J. and Jokipii, J. R., Effect of drift on the transport of cosmic rays. VI. A three-dimensional model including diffusion, Astrophys. J., 265, 573-581, 1983.

Kóta, J. and Jokipii, J. R., 3-D simulation of heliospheric transport: a comparison of models, Proc. 25th Int. Cosmic Ray Conf., 2, 25-28, 1997.

Kóta, J. and Jokipii, J. R., Cosmic-ray modulation and the structure of the heliospheric magnetic field, Proc. 26th Int. Cosmic Ray Conf., 7, 9-12, 1999.

Parhi, S., Burger, R. A., Bieber, J. W. and Matthaeus, W. H., Challenges for an *ab initio* theory of cosmic ray modulation, These proceedings, 2001.

Parker, E. N., Dynamics of the interplanetary gas and magnetic fields, Astrophys. J., 128, 664-676, 1958.

Parker, E. N., The passage of energetic charged particles through interplanetary space, Planet. Space Sci., 13, 9-49, 1965.

Smith, C. W. and Bieber, J. W., Solar cycle variation of the interplanetary magnetic field spiral, Astrophys. J., 370, 435-441, 1991.

Thomas, B. T., Goldstein, B. E. and Smith, E. J. The effect of the heliospheric current sheet on cosmic ray intensities at solar maximum: two alternative hypotheses, J. Geophys. Res. 91, 2889-2895, 1986.

van Niekerk, Y., An investigation into the causes and consequences of North-South asymmetries in the heliosphere, M.Sc. thesis, Potchefstroomse Universiteit vir Christelike Hoër Onderwys, South Africa, pp 95, 2000.

Zurbuchen, T. H., Schwadron, N. A. and Fisk, L. A., Direct observational evidence for a heliospheric magnetic field with large excursions in latitude, J. Geophys. Res., 102, 24172-24181, 1997.