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16-point Discrete Fourier Transform based on the Radix-2 FFT algorithm implemented into Cyclone FPGA as the UHERC trigger for horizontal air showers.

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Extremely rare flux of UHERC requires sophisticated detection techniques. Standard methods oriented on the typical events may not be sensitive enough to capture rare events, crucial to fix a discrepancy in the current data or to confirm/reject some new hypothesis. Currently used triggers in water Cherenkov tanks, selecting events above some amplitude thresholds or investigating a length of traces seems to be not optimized for the horizontal and very inclined showers, interesting as potentially generated by neutrinos. Those showers could be triggered using their signatures: i.e. a curvature of the shower front, transformed on the rise time of traces or muon component giving early peak for "old" showers. Currently available powerful and cost-effective FPGAs provide sufficient resources to implement new triggers not available in the past. The paper describes the implementation of 16-point Discrete Fourier Transform based on the Radix-2 FFT algorithm into Altera Cyclone FPGA. All complex coefficients are calculated online in heavy pipelined routines. The register performance $\sim$ 200 MHz and relatively low resources occupancy $\sim$ 2000 logic elements for 10-bit FADC traces provide a powerful tool to trigger the events on the traces characteristic in the frequency domain.

1. Introduction

Extensive Air Showers are investigated in several experiments utilizing different detection techniques (scintillators, water Cherenkow detectors, fluorescence detectors). Signals in detectors depend on several parameters like the energy and the type of the primary particle, a distance from the core, the angle of registered shower etc. Usually the triggering conditions are chosen to detect wide as possible class of events. However sometimes the standard trigger conditions are not optimized for the specific class of events, which either are not registered at all or the registration efficiency is poor.

In experiments utilizing water Cherenkov detectors signals from photomultipliers are usually digitized in Flash ADCs and next performed by often-sophisticated electronics. Typically signals from PMTs are online analyzed both in amplitude and time domains. Strong signals in all PMT channels, corresponding to energetic showers detected near the core, are registered because of many-fold coincidence single bin trigger with a fixed thresholds calculated for individual PMT during a calibration procedure. Showers detected far from the core give much lower signals usually spread in time. That type of events is detected by the other type of trigger investigating the structure of signal in some period (in sliding time window).

Both types of triggers seem to be not optimized for very inclined or horizontal showers, also generated by neutrinos. The signals from that type of showers are usually too low to be detected by the single bin trigger, from the other hand, the signals could be too short to provide the sufficient occupancy to be detected by Time over Threshold trigger (ToT) [1, 2].

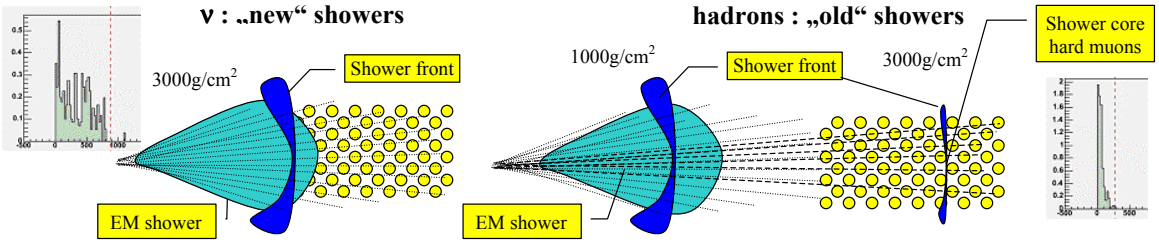

Figure 1. Development of showers generated deeply and early in the atmosphere.

2. Deep showers

The structure of signals detected in water Cherenkov tanks and generated by horizontal showers depends strongly on the point of the EAS initialization. "Old" showers (see Fig. 1) generated by hadrons early in the atmosphere give flat muonic front, showers generated by deeply interacting neutrinos are characterized by a curved front (radius of curvature of a few km), a large electromagnetic component and with particles spread over a few microseconds interval [4]. In both cases muonic front produces a bump, which can be a starting signature of horizontal showers. The bump for the "old" showers is shorter and sharper than for the "new" ones and give a larger contribution in higher Fourier coefficients. For "new" showers with relatively smooth shape of a signal profile, the lower Fourier components should dominate. The online analysis of the Fourier components may trigger specific events.

The existing software procedures for FPGAs, available as commercial IP routines, can calculate Fourier coefficients effectively utilizing a FFT algorithm. However the software implementation is too slow to be able to trigger events in the real time. Online triggering requires the hardware implementation calculating multipoint DFT with a sufficient speed. The modern powerful FPGAs can do this job, however the resource requirement increases dramatically with the number of points. The analysis time interval should be a reasonable compromise between a time resolution and the resources occupancy in the FPGA.

3. Timing

16-point structure clocked with 40 MHz gives the DFT in 400 ns window. This interval seems to be sufficient for the preliminary analysis of the muonic bump, especially for the "old" showers. Nevertheless for "new" showers can be treated as pre-trigger, turning on the channel investigating next the signal over longer microsecond time, corresponding to the electromagnetic component of the shower. The enlarged length of shift registers (by the factor 16) can provide an analysis in 6.4 μs sliding window. If the pre-trigger appears, the same hardware FFT structure is multiplexed to the boxcar integration circuit, integrating data over 16 time bins (see Fig. 5 in [2]).

4. Algorithms

Let us consider a DFT $\bar{X}$ of dimension N [3] $\bar{X}_k = \sum_{m=0}^{N-1} x_m W^{mk}$, where W = $e^{-j2\pi/N}$ and k = 0,...N-1

If N is the product of two factors, with $N = N_1 N_2$, the indices m and k we can redefined as follows: $m = N_1 m_2 + m_1$, where m_2 = 0,...,N_2-1 and m_1 = 0,...,N_1-1.

$$\bar{X}_k = \sum_{m_1=0}^{N_1-1} W^{N_2 m_1 k_1} W^{m_1 k_2} \sum_{m_2=0}^{N_2-1} x_{N_1 m_2 + m_1} W^{N_1 m_2 k_2}$$

For the Radix-2 algorithm: N = 2^t N_1 = 2, $N_2 = 2^{t-1} = \frac{N}{2}$ and $W^{\frac{N}{2}} = -1$ we get

$$\bar{X}_k = \sum_{m=0}^{N/4-1} (x_{2m} + (-1)^k x_{2(m+N/4)}) W^{2mk} + W^k (\sum_{m=0}^{N/4-1} (x_{2m+1} + (-1)^k x_{2(m+N/4)+1}) W^{2mk}$$

If we introduce new variables (l=0,...,N/2-1) A_l=x_l+$x_{l+N/2}$, $A_{l+N/2}$=x_l-$x_{l+N/2}$,

$$\bar{X}_{k\binom{even}{odd}} = \sum_{m=0}^{N/4-1} (A_{\binom{2m}{2m+N/2}} + W^k A_{\binom{2m+1}{2m+1+N/2}} W^{2mk}$$

x_m represent signals in time domain. They can be easily available from outputs of shift registers clocked synchronously with FADC. In the 1st pipeline stage $\bar{X}_k$ can be expressed by A_m, which are simple linear combination of x_m and can be calculated by typical adders/subtracters in a single clock cycle. The implementation of the multi-points algorithm requires multiple pipeline stages and also multipliers, corresponding to the W^k coefficients of the fractional "angle" $e^{-j2k\pi/N}$. The Radix-2 algorithm used in the next stage reduces again the abundance of W^k coefficients due to the next twiddle factors' related to the next stage of the pipeline. Symmetries of trigonometrical functions reduce additionally independent multipliers.

Let us consider the FFT algorithm for N = 16. Because of the real values given by FADC, the final number of coefficients is reduced due to the following symmetry:

$\binom{Re}{Im} \{\bar{X}_k\} = \pm \binom{Re}{Im} \{\bar{X}_{N-k}\}$

The repetition of the FFT Radix-2 algorithm for 16 points requires totally 5 pipeline stages and only 3 real coefficients: $\alpha = \cos(\pi/8)$, $\beta = \sin(\pi/8)$ and $\gamma = \cos(\pi/4)$. The structure of data flow is shown on Fig. 2.

5. FPGA implementation

The structure of the routines implemented into FPGA is presented on the Fig. 2. Two first pipeline stages utilize adders/subtracters only and can be implemented as the single bin routines, however multipliers need at least two clock cycles to assure a sufficient registered performance. Some signals (i.e. $B_8 = A_8$), which do not require any mathematical calculation, have to be delayed in appropriate registers in order to assure the correct synchronization.

The algorithm has been implemented for 10-bit FADC bus into the Cyclone EP1C12Q240I7 Altera chip, with 40 MHz clock (first level surface detector trigger in the Pierre Auger Observatory [2]). It requires 12 multipliers, 28 adders, 29 subtracters and 12 delaying routines (simple registers) for synchronization - totally ∼2000 logic elements (occupancy of resources increases from 54% to 78%). Fixed point multiplication provides an accuracy on the level 0.05% (multiplication of two 12-bit factors gives as result 24-bit value however, the result from multiplier is shrinked to 15 bits only (in order to save resources), which gives additional 3 bits of better accuracy (in comparison with the pure shrink of the input 12-bit signed data by the scaling factor).

Very high registered performance (≥200 MHz) allows on utilization of the same logical structure either for three FADC channels multiplexing with 120 MHz (additional 200 LE more) or for enlarged sliding window with an additional boxcar integration circuits. More useful than pure Re and Im coefficients seems to be "power spectrum density": $\xi = \{\text{Re } \bar{X}_k\}^2 + \{\text{Im } \bar{X}_k\}^2$ which can be calculated by the "square devices" (see Fig.6 in [2]). However, "square devices" and multipliers require a lot of logic elements and their implementation would be much more effective in chips with hardcore DSP blocks like i.e. Altera® Stratix™.

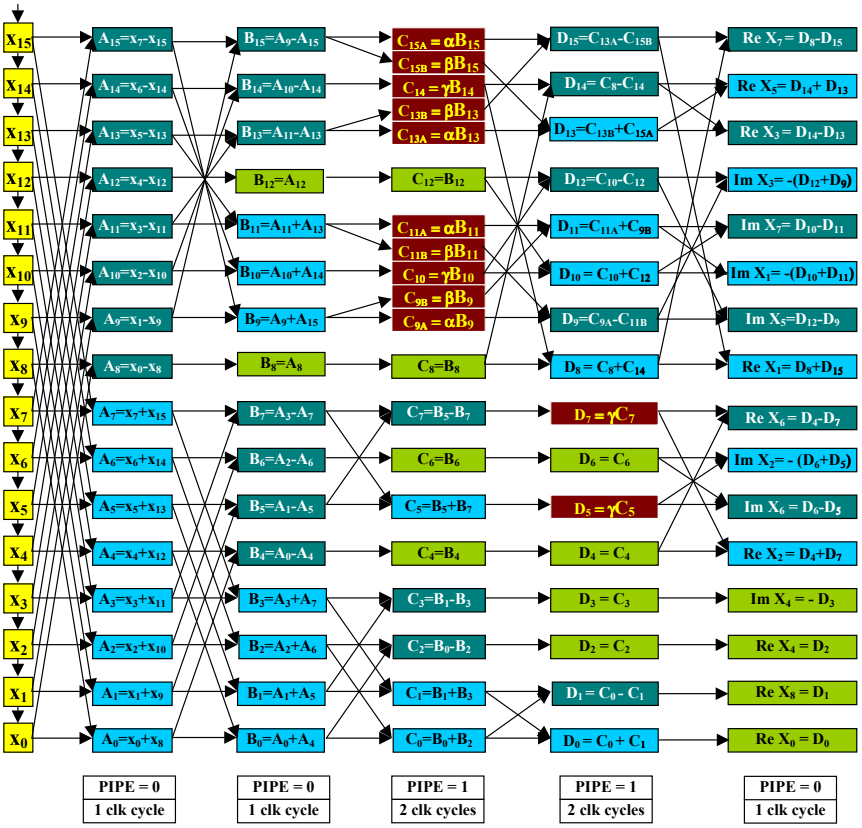

Figure 2. Global pipeline internal structure of FFT

6. Conclusions

The 16-point implementation of the FFT into FPGA seems to be powerful tool for the spectral analysis of the FADC traces. The results presented above focus rather on the hardware FPGA implementation, precise conditions for triggering are still subjected to simulate and optimize of relations between different Fourier coefficients or power spectral densities. For test purposes the algorithm has been successfully merged into the code of the first level trigger based on the Cyclone chip in the Pierre Auger Observatory and is planned to be tested soon in the real environmental condition.

References

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