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Propagation of Cosmic Ray Electrons Including the Source Region II : Diffusion Model

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Galactic supernova remnants(SNRs) are currently considered to be the source of galactic electrons. In SNRs it is supposed that electrons escape from the acceleration site, diffuse in the magnetic field and lose energy through the synchrotron radiation. Such propagation and the energy losses certainly affect the observed electron spectrum. We treat the electron propagation in the diffusion model and estimate the electron spatial distribution in SNRs and its effects to the escape rate into the interstellar space. The result shows that the location of the acceleration site influences the escape rate from SNRs, if the energy dependence of the diffusion coefficient is small. In the case of plerions the escape rate has an abrupt decrease at the high energy side.

1. Introduction

The supernova remnants(SNRs) are most likely sources of cosmic ray electrons because the non-thermal radio and X-ray emission from SNRs indicate the existence of high energy electrons in SNRs. In this paper the source region is approximately treated as the diffusion space; electrons ejected from the shock front of blast waves or pulsar winds, diffuse in a SNR and escape from it at the specific rate. The electron propagation and its escape rate from SNRs may be distinguished by shell-type SNRs or plerions. We assume the simple spatial distributions of the acceleration site and calculate the electron distribution in the source region and the escape rate from it. The distribution is calculated at various energies which reflect the observed radiation profiles with different frequencies. Thus the consistency with the observed data will decide the validity of the model.

2. Formulation

The electron spectrum in the source region is given by $f(\mathbf{r}, E)$ with the space variable $\mathbf{r}$. If the acceleration site has the source distribution $q(\mathbf{r}, E)$ and ejected electrons diffuse with the diffusion coefficient $D(\mathbf{r}, E)$, the propagation equation in SNRs is represented by

$$-\mathrm{div}(D\nabla f(\mathbf{r}, E)) + \frac{\partial}{\partial E}(\frac{dE}{dt} \cdot f(\mathbf{r}, E)) = q(\mathbf{r}, E) \ . \tag{1}$$

In this paper the spherical equation with the radial variable $x = r/r_s$ (r_s : SNR radius, $x = (0, 1)$) is solved. The electron spectrum $f(x, E)$ and its source spectrum $q(x, E)$ are Fourier transformed as

$$\{ \ f(x, E), \ \ q(x, E) \ \} = \sum_{n=0}^{\infty} \{ \ a_n(E), \ \ q_n(E) \ \} \cos[(n + 1/2)\pi x$$

with the boundary condition $f(1, E) = 0$ and $q(1, E) = 0$. The Fourier expressions are substituted into Eq. (1) and consequently the coefficient a_n satisfies the leaky box equation,

$$-\frac{a_n}{\tau_n} + \frac{d}{dE}(bE^2 \cdot a_n) + q_n = 0 \ , \tag{2}$$

in which τ_n is defined as

$$D(E)[(n+1/2)\pi]^2 \equiv 1/\tau_n \ ,$$

and the diffusion coefficient $D(E) = D_0 E^{\delta_d}$ is assumed to be spacially uniform.

Next, we calculate the electron flux escaping from the source region. Integrating the Eq. (1) over the whole source region with the volume V (surface S), we get

$$\int_V -\mathrm{div}(D\nabla f(\mathbf{r},E))dV + \frac{\partial}{\partial E}(\frac{dE}{dt}\cdot N(E)) = Q(E) \ . \tag{3}$$

The first term represents the electron spectrum $S(E)$ escaping from the source region into the interstellar space.

$$S(E) = \int_V -\mathrm{div}(D\nabla f(\mathbf{r},E))dV = \int_S -D\nabla f(\mathbf{r},E)dS \ .$$

In the spherical case, $S(E)$ is obtained from the solution a_n of Eq. (2),

$$S(E) = -D\frac{df}{dx}|_{x=1} = D\sum_{n=0}^{\infty}(-1)^n(n+1/2)\pi\cdot a_n(E) \ .$$

The electron spectrum $N(E)$ and the source spectrum $Q(E)$ in Eq. (3) are given by

$$\{\ N(E),\ \ Q(E)\ \} = \int_0^1 \{\ f(x,E),\ \ q(x,E)\ \}dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{(n+1/2)\pi}\{\ a_n(E),\ \ q_n(E)\ \} \ .$$

The escape rate from the source region is calculated from

$$\frac{1}{\tau_s(E)} = \frac{S(E)}{N(E)} = D(E)\cdot\frac{\sum_{n=0}^{\infty}(-1)^n a_n(E)\cdot(n+1/2)\pi}{\sum_{n=0}^{\infty}(-1)^n a_n(E)/[(n+1/2)\pi]} \ . \tag{4}$$

In the next section. the above spectra are calculated in some simple cases, in which $q(x,E) = Q_0 E^{-\gamma}\cdot Q(x)$ is assumed.

3. Calculations

Spatial distribution in SNRs : In two type of SNRs, a plerion and a shell-type SNR, the spatial distribution of electrons is calculated. A plerion has the acceleration site at the pulsar wind shock. If we adopt the typical SNR of Crab nebula, the shock radius of $r_s = 8$" and the nebular size $r_{nebula} = 200$"[2] give the source function described as

$$Q_p(x) \ = \ 1,\ \ 0 \le |x| < 0.04, \ \ = \ 0,\ \ 0.01 \le |x| \le 1 \ .$$

The result of $f(E,x)$ is shown in Fig. 1(a). The parameters used in the calculation are the average mangetic field strength of $(1.7\pm0.3)\times10^{-4}$G[1] and the diffusion coefficient at 1GeV of $4\times10^{25}\mathrm{cm}^2/\mathrm{s}$ with the energy index of $\delta_d = 0.3, 0.6$. As shown in Fig 1(a), the δ_d significantly affects the distribution. The calculated curves with $\delta_d = 0.3$ are consistent with the data larger than equipartition point ($r/r_{nebula} > 0.3$). Inner part(<0.3) may have different propagation from a diffusion process or the different magnetic field strength, so that the optical and X-ray curves do not fit the data. The value of 0.6 does not reproduce the data because 0.6 means the weak confinement and electrons escape too rapidly.

The distribution $f(E,x)$ of a shell-type SNR is also shown in Fig 1(b). We assume that the distribution is described as

$$Q_s(x) \;=\; 0,\;\; 0 \le |x| < 0.9, \;\;\; = \; 1,\;\; 0.9 \le |x| \le 1\;,$$

The explanation of the low radio intensity in the inner region as shown in the SN1006[6] needs the weak magnetic field or a small value of the diffusion coefficient. For an example shown in Fig. 1(b), the magnetic field of 10μG and $D(1\mathrm{GeV}) \;=\; 5 \times 10^{22}\mathrm{cm}^2/\mathrm{s}$ are adopted. As shown in Fig. 1(b), the energy index of diffusion coefficient does not affect the distribution and the X-ray profile is almost same as source distribution because of the small value of $D(E)$.

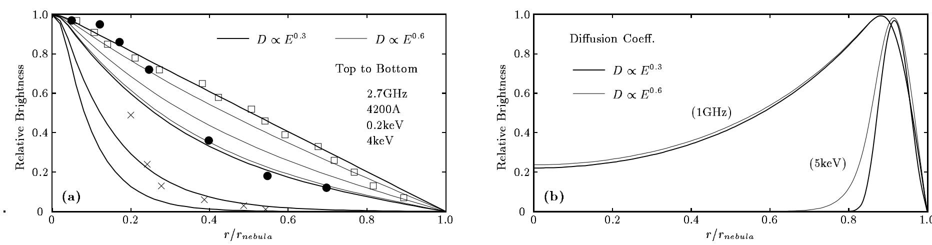

Figure 1. (a) The spatial distribution $f(x,E)$ is shown with the observed radio, optical and X-ray profile data of Crab Nebula given in DeJager et al.(1992)[2]. The source spectral index γ has the values of 1.6(radio), 2.7(optical) and 3.3(X-ray) that were estimated from the radiation data[7]. (b) The distribution $f(x,E)$ of a shell-type SNR is shown. The the radio profile at 1GHz and the X-ray of 5keV are indicated when the energy dependence of $D(E)$ is 0.3 and 0.6.

Escape rate from SNRs : In the three distributions of Q_p , Q_s and a linearly decreased-type $Q_l \;=\; |1-x|$, the escape probability $1/\tau_1$ are calculated from Eq. (4) and shown in Fig. 2(a). The parameters adopted in the calculation are the source spectral index $\gamma \;=\; 2.2$ and the diffusion coefficient $D \;=\; 2 \times 10^{25} E^{\delta_d}\mathrm{cm}^2/\mathrm{s}$ with $\delta_d = 0.3,\; 0.6$.

In the case of the small value of $\delta_d \;=\; 0.3$, we notice the slope of the curves strays from the value of δ_d. The escape rate of plerions(Q_p) drastically decreases at the high energy side because high energy electrons lose the most part of energy by the synchrotron radiation during the propagation in a plerion and cannot escape from it. On the other hand the escape rate of the shell-type SNRs(Q_s) gradually increases than 0.3 as energy increases. The linear decreased type q_l maintains the slope of 0.3, so that it has no effect of the spatial distribution.

In the case of the large value of $\delta_d \;=\; 0.6$, that means easily escape of high energy electrons, the escape rate has little changes from 0.6 for each curve. It means that the spatial distribution hardly affects the escape rate and the observed spectrum may have no energy cut-off. Those results indicate that the spatial distribution of the acceleration site gives the large influence to the escape rate only with a small energy dependence of the diffusion coefficient.

Interstellar spectrum : The interstellar spectrum of electrons is calculated, which originate in plerions and shell-type SNRs respectively. In the calculation the interstellar space is treated as a leaky box model that assumes the uniform distribution of space and time; the resulting curves only present the effectiveness of the propagation of electrons in SNRs to the observed spectrum. Fig. 2(b) shows the results in which the propagation parameters used in the calculation are the same as in Fig. 1.

As shown in the figure, Both curves do not exceed the data above several hundereds GeV, even if the source index is 2.2 . In the case that shell-type SNRs are major contributors, that is realistic because 80% of SNRs in the Galaxy are shell-type[3], they cover the whole observed spectrum, even if the energy index δ_d is small.

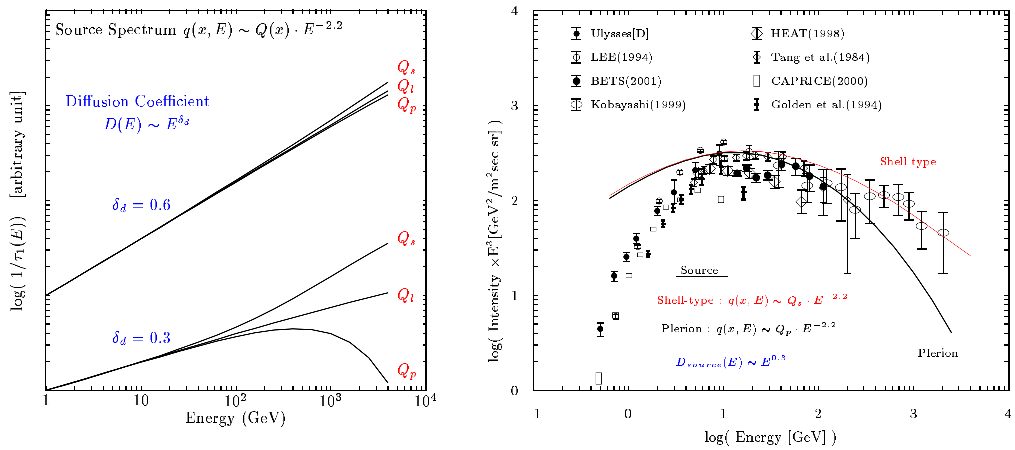

Figure 2. (a) Escape rate $1/\tau_s$ from SNRs: The source spectrum $Q(x)E^{-2.2}$ has three types of a shell-type SNR(Q_s), a linear decrease(Q_l) and a plerion(Q_p), which expression is indicated in the text. The diffusion coefficient is $2\times10^{25}E^{\delta_d}\mathrm{cm}^2/\mathrm{s}$ with $\delta_d = 0.3,\ 0.6$. (b) Interstellar spectrum from two types of SNRs: shell-type SNRs and plerions. The energy dependence of the diffusion coeffeicient in SNRs of $\delta_d = 0.3$ and the source index of 2.2 are assumed.

4. Conclusions

The electron propagation including the source region as the diffusion space has been formulated and discussed. The electron spatial distribution which reflects the radiation profile in a SNR and the escape rate have been calculated in some cases having the simple source distribution of the acceleration site.

The escape rate from a SNR is influenced by the spatial distribution of the acceleration site, when the energy dependence of the diffusion coefficient is small. High-energy electrons lose major energy in a plerion, so that the escape rate has a abrupt decrease above TeV,

The galactic electron spectrum is influenced whether major contributors are plerions or shell-type SNRs. If the shell-type SNRs are major contributors, it is possible to cover the observed spectrum up to a few TeV[5].

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