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A new approach to the cosmic-ray propagation in the Galaxy in the wide energy range 1 GeV – 1 PeV

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We extend the model of three dimensional cosmic-ray propagation recently developed without the energy change to that with the energy change due to the reacceleration and the ionization energy loss. It is possible to apply the weighted slab technique for the transport equation even including the energy change process, if the diffusion coefficient $D(\boldsymbol{r};R)$ is separable in the energy (R: rigidity) and the space, i.e., $D(\boldsymbol{r};R) = D(\boldsymbol{r})R^{\alpha}$ assumed in the present work, and combining it with the first pole approximation for the path length distribution, we can obtain the analytical solution rather easily. We find the spectral indexes of the secondary component in rigidity are $\gamma + 2\alpha$ in the high energy limit, and $\gamma - 2\alpha$ in the low energy limit, while those of the primary component are $\gamma + \alpha$ and $\gamma - \alpha$, respectively. This means that the rigidity dependence of the secondary-to-primary ratio behaves as $R^{-\alpha}$ in the high energy region, and as R^{α} in the low energy region. We compare our numerical results with the experimental data in both the low and the high energy regions, and find our model reproduces very nicely all components over the wide energy ranges, 1 GeV/nucleon $\sim$ 100 TeV/nucleon, by adopting appropriate scale heights, and the reacceleration efficiency.

1. Introduction

Recently the experimental data on the cosmic-ray (CR) spectrum and composition become available with good statistics and with good accuracy for the various kinds of elements in the very wide energy ranges. Particularly, AMS [1] and BESS [2] groups have reported excellent data on the proton and the helium component, covering the energy ranges 0.2–100 GeV/nucleon. RUNJOB [3] group reports most recently their final results on the nuclear components, proton $\sim$ iron, covering the very high energy region 10–500 TeV for proton, 2–100 TeV/nucleon for helium, and 1–30 TeV/nucleon for heavy elements, and also presents the secondary-to-primary ratio in the energy region around TeV/nucleon, while the data quality is limited due to the large atmospheric correction.

In addition to these stable nuclear components, rare elements such as the antiparticle ($\bar{p}$ and e^{+}) and isotopes (Be^{10}, Al^{23}, and so on) are also available, while the statistics and the energy ranges are still limited for the latter elements, at most 1 GeV/nucleon.

It is also remarkable that with the recent development of the techniques in both space-based and the ground-based γ-ray observation, we can study quantitatively those coming from both point sources and the diffusive ones, covering very wide energy ranges MeV $\sim$ TeV. It might be worth mentioning that the energy regions for the diffusive γ-rays covered by the space-based and the ground-based experiments will overlap each other somewhere around 100 GeV in the near future.

All elements mentioned above, stable rich CR elements, rare CR elements, and γ-ray component, relate each other, and give us important information for both particle physics and astrophysics, in particular those of antiparticle and γ-rays. It is quite essential nowadays in physics to identify either these rare elements come from the secondary products due to the CR interactions in the Galaxy, or from some novel sources such as those due to the annihilation products in the dark matter. Of course, in order to induce a definite conclusion for these challenging problems, we need good experimental data with good statistics and with good accuracy, and also reliable physical parameters related to the CR propagation in the Galaxy, such as the diffusion coefficient, gas density, source distribution and so forth.

Under these motivations, one of the present authors (T. S.) have proposed an approach for the three dimensional CR propagation in the Galaxy ([4]; hereafter referred to Paper I), taking a rather realistic structure of the Galaxy into account, while our model is based on the diffusion model developed extensively by Ginzburg group [5]. Namely we assume no boundary in both the radial spread of the disk and the halo space in our Galaxy, and three critical parameters, the diffusion coefficient D, the gas density n and the CR source Q have spatial distribution with the exponential-type, while the current models have assumed rather uniform-type, typically the leaky-box model. These assumptions make possible to give the analytical solution in a quite simple form despite the complicated process in the CR propagation in the Galaxy.

In Paper I, however, we didn't touch upon the energy change process during the CR propagation, such as the reacceleration and the ionization energy loss, focusing on the high energy region. In the present paper, we study these effects in order to extend our model to the low energy region down to 1 GeV/nucleon and below.

It is not so easy to obtain the analytical solution with the energy change effects, while rather easy in the case of no energy change as presented by Paper I. Fortunately, however, Ptuskin et al. [6] have pointed out that we can apply the weighted slab techniques for the diffusion problem even including the energy change effects, if the diffusion coefficient is separable in the energy (rigidity) and the space.

Moreover, as was presented in Paper I, we found the path length distribution $\Pi(x)$ is well described by the first-pole approximation, while the exact one is quite complicated, namely given by the simple exponential function, $\Pi(x) \propto \exp(-\bar{\sigma}_r x)$, where $\bar{\sigma}_r$ depends on the position of the observer in the Galaxy as well as the scale heights of the diffusion coefficient (see Paper I for the detail). One might think that it is nothing but the form assumed by the leaky-box model, but our model includes the spatial dependence in $\bar{\sigma}_r$ closely related to the Galactic parameters, i.e., depending on the position of observer $\boldsymbol{r}$, while the current model assumes it is independent of the position of the observer.

Combining the weighted slab techniques given by Ptuskin et al. with the first pole approximation developed by Paper I, we can obtain rather easily the analytical solution with the energy change effect. Due to the limited space, we don't present explicitly the procedures of the derivation of the solution in the present paper, which will appear in a separate paper soon.

2. Basic assumptions

We assume that there is no boundary in both radial spread of the disk and the latitudinal spread of the halo, and that the three critical parameters, the diffusion coefficient D, the gas density n, and the cosmic-ray source density Q, depend on the space position $\boldsymbol{r}$ in the form of $D(\boldsymbol{r}) = D_0 \exp[r/r_D + |z|/z_D]$, $n(\boldsymbol{r}) = n_0 \exp[-(r/r_n + |z|/z_n)]$, and $Q(\boldsymbol{r}) = Q_0 \exp[-(r/r_Q + |z|/z_Q)]$, respectively, where we omit the rigidity-dependent term for $D(\boldsymbol{r})$ and $Q(\boldsymbol{r})$. The set of the scale heights (r_D, r_n, r_Q) correspond to the radial size of the disk, and the scale height of the diffusion coefficient z_D to the thickness of the halo.

The rigidity dependences for D and Q are given by the following equations

$$D(\boldsymbol{r}; R) = vR^{\alpha} D(\boldsymbol{r}), \quad \text{with } \alpha = 1/3 - 1/2, \tag{1a}$$

$$Q(\boldsymbol{r}; R) = Q(\boldsymbol{r}) R^{-\gamma}, \quad \text{with } \gamma = 2.1 - 2.4, \tag{1b}$$

where v is the velocity of the particle in units of the velocity of the light.

It is well known that the average energy gain and its dispersion per interaction with magnetic turbulence in the case of three dimensional scattering model are given by

$$\langle \Delta E \rangle_{\rm rea} \simeq \frac{4}{3} v_M^2 p/v, \quad \langle (\Delta E)^2 \rangle_{\rm rea} \simeq \frac{2}{3} v_M^2 p^2, \tag{2}$$

where v_M is the velocity of the magnetic turbulence, and p the particle momentum per nucleon. Thus the energy gain and the dispersion per unit of time are given by

$$\left\langle \frac{\Delta E}{\Delta t} \right\rangle_{\rm rea} \simeq n(\boldsymbol{r})\zeta_0 \frac{p}{R^\alpha}, \quad \left\langle \frac{(\Delta E)^2}{\Delta t} \right\rangle_{\rm rea} \simeq n(\boldsymbol{r})\zeta_0 \frac{vp^2}{2R^\alpha}; \quad \text{with } \zeta_0 = \frac{4}{9}\frac{v_M^2}{c}\frac{1}{n_{\rm eff} D_{\rm eff}}, \tag{3}$$

where ζ_0 corresponds to the reacceleration efficiency, and we introduced two effective parameters, $n_{\rm eff}$ and $D_{\rm eff}$, denoting the effective gas density and the effective diffusion coefficient. ζ_0 has the dimensions of the cross section, and for a typical set of parameters, $v_M = 20$ km/sec, $n_{\rm eff} = 1\,{\rm cm}^{-3}$, $D_{\rm eff} = 10^{28}\,{\rm cm}^2$/sec, we find $\zeta_0 = 59.3\,{\rm mb}$.

The ionization energy losses per nucleon in the interstellar medium is given by the Bethe's formula,

$$\left\langle \frac{dE}{dt} \right\rangle_{\rm ion} = -n(\boldsymbol{r})vw(p), \quad \text{with } w(p) = \frac{4\pi e^4}{m_e v^2}\frac{Z^2}{A}\left[\ln \frac{2m_e v^2}{\bar{I}(1-v^2)} - v^2\right], \tag{4}$$

and $\bar{I}$ is the mean ionization potential of the interstella medium (mostly composed of hydrogen with approximately 10% helium).

2.1 Transport equation

Let us consider a case that a nucleus with initial kinetic energy E_0 is produced at a position $\boldsymbol{r}_0(r_0, z_0)$ in the cylindrical coordinate system. The transport equation for the CR propagation, $\Phi(t, \boldsymbol{r}, \boldsymbol{r}_0; E, E_0)$, is given by, taking account of the energy change mentioned in the last section,

$$\left[\frac{\partial}{\partial t} - \nabla\cdot D(\boldsymbol{r}; R)\nabla + n(\boldsymbol{r})v\sigma + \frac{\partial}{\partial E}\left\{\left\langle \frac{\Delta E}{\Delta t}\right\rangle_{\rm rea} + \left\langle \frac{dE}{dt}\right\rangle_{\rm ion}\right\} - \frac{1}{2}\frac{\partial^2}{\partial E^2}\left\langle \frac{(\Delta E)^2}{\Delta t}\right\rangle_{\rm rea}\right]\Phi = \delta(E-E_0)\frac{\delta(\boldsymbol{r}-\boldsymbol{r}_0)}{2\pi r_0}, \tag{5}$$

where σ is the inelastic collision cross section with nuclei of the interstellar gas, and $\langle \ldots \rangle_{\rm ion}$ denotes the ionization loss term.

According to Ptsukin et al., in the case of separable diffusion coefficient in space and the energy (rigidity) as in the case of Eq. $(1a)$, it is possible to apply the weighted slab technique by introducing a new path length parameter y, and we can write down Φ for the steady state ($\partial\Phi/\partial t = 0$)

$$\Phi(\boldsymbol{r}, \boldsymbol{r}_0; E, E_0) = \int_0^\infty \Pi(y; \boldsymbol{r}, \boldsymbol{r}_0) f(y; E, E_0)\, dy, \tag{5}$$

where Π and f satisfy

$$\left[n(\boldsymbol{r})c\frac{\partial}{\partial y} - \nabla\cdot D(\boldsymbol{r})\nabla\right]\Pi = \delta(y)\frac{\delta(\boldsymbol{r}-\boldsymbol{r}_0)}{2\pi r_0}, \tag{6a}$$

$$\left[v\left(R^\alpha \frac{\partial}{\partial y} + \sigma\right) + \frac{\partial}{\partial E}\left(\zeta_0 \frac{p}{R^\alpha} - vw\right) - \frac{1}{2}\frac{\partial^2}{\partial E^2}\left(\zeta_0 \frac{vp^2}{2R^\alpha}\right)\right] f = \delta(y)\frac{\delta(E-E_0)}{v_0 R_0^\alpha}. \tag{6b}$$

3. Numerical results and discussion

While it seems to be cumbersome to perform the above integration, we can obtain rather easily the CR density in the analytical form at the Galactic plane ($z = 0$) with use of the results presented in Paper I. However, as we have no space here to present the procedure of the derivation and the explicit form of the solution, we show only two examples of numerical results on the proton and the helium components. Other examples and the comparison with the experimental data will be presented orally in the conference.

In Fig. 1, we demonstrate the energy spectra of proton and the helium component in the energy range 0.1 GeV/n – 1 PeV/n, for two sets of ζ_0, 0, and 50 mb, with the modulation parameter of $\phi =$ 0.6, 1.2GV, where experimental data are also plotted. One finds that the reacceleration effect boost the intensity considerable around the energy region ~ 10 GeV/n, and ζ_0 is as large as 50 mb. Detail discussions will be presented at the conference, including the heavier components as well as secondary-to-primary ratio.

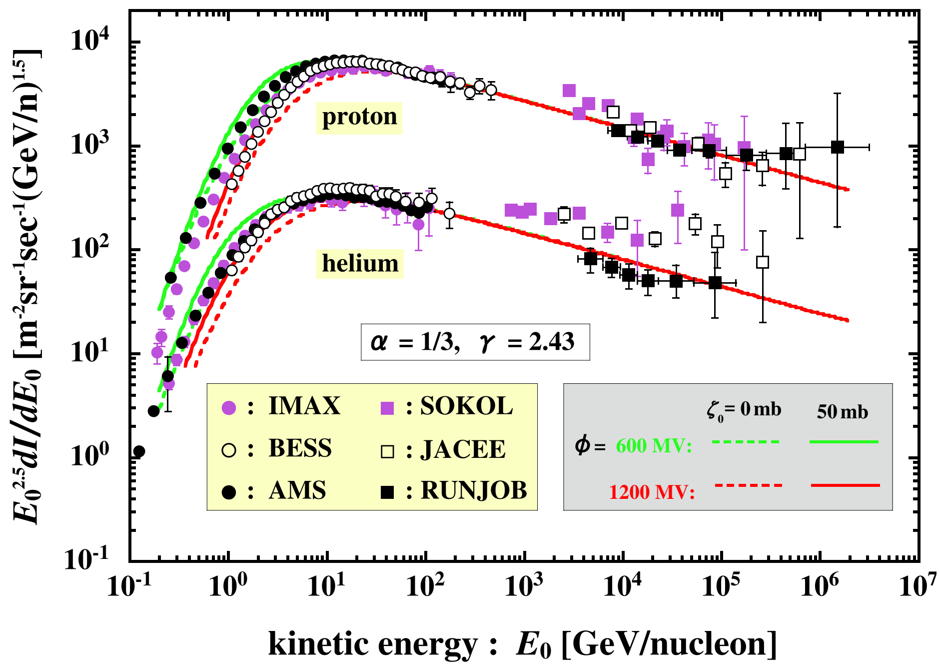

Figure 1. Examples of the numerical calculation on the energy spectra of the proton and helium components. See Derbina et al. [3] and references therein for the data.

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