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The Theory of Synchrotron Emission of Supernova Remnants

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Time-dependent nonlinear kinetic theory for cosmic ray (CR) acceleration in supernova remnants (SNRs) is applied in order to study the properties of the synchrotron emission from SNRs, in particular, the surface brightness-diameter relation. Detailed numerical calculations are performed for the expected range of the relevant physical parameters, namely the ambient density and the supernova explosion energy. The magnetic field in SNRs is assumed to be significantly amplified by the efficiently accelerating nuclear CR component. The theoretically predicted brightness-diameter relation in the radio range fits the observational data in a very satisfactory way.

1. Introduction

Supernova remnants (SNRs) are the main sources of energy for the Interstellar Medium (ISM). And they control the physical state of the ISM which presumably includes the nonthermal component of Interstellar matter, often called the Galactic Cosmic Rays (CRs). The synchrotron emission of relativistic electrons plays an important role in the general study of SNR properties and in CR production inside SNRs in particular. All known SNRs are sources of radio-synchrotron emission. Several Galactic SNRs were recently detected as sources of nonthermal X-ray emission which is presumably also of synchrotron origin.

The determination of the distances to the Galactic SNRs is an important task which is often based on radio observations. When there is no direct distance determination, estimates can be made by using the radio surface brightness-to-diameter relationship ($\Sigma_R - D$). It is however not clear whether any functional correlation between $\Sigma_R(t)$ and $D(t)$ exists for individual objects during their evolution in time t, and if so, for which physical reason [1].

Nonlinear kinetic theory of diffusive CR acceleration in SNRs [2, 3] is used for a systematic study of the synchrotron emission expected during the different evolutionary epochs of SNRs, possibly leading in particular to a $\Sigma_R - D$ relation.

2. Results and discussion

A supernova explosion ejects an expanding shell of matter with total energy E_{sn} and mass M_{ej} into the surrounding ISM. The interaction of the ejecta with the ISM creates a strong shock which accelerates particles.

Our nonlinear model [2, 3] is based on a fully time-dependent solution of the CR transport equation together with the gas dynamic equations in spherical symmetry.

Due to the streaming instability CRs efficiently excite large-amplitude magnetic fluctuations upstream of the SN shock. Since these fluctuations scatter CRs extremely strongly, the CR diffusion coefficient is assumed to be as small as the Bohm limit $\kappa_B(p)$. The nonlinear description of the magnetic field evolution in a numerical model [4] concluded that a considerable amplification to what we call an effective magnetic field should occur. Formally then, the magnetic field in the upstream preshock medium becomes time dependent: $B = \sqrt{8\pi\delta \cdot P_c}$. We use the moderate parameter value $\delta = 10^{-3}$.

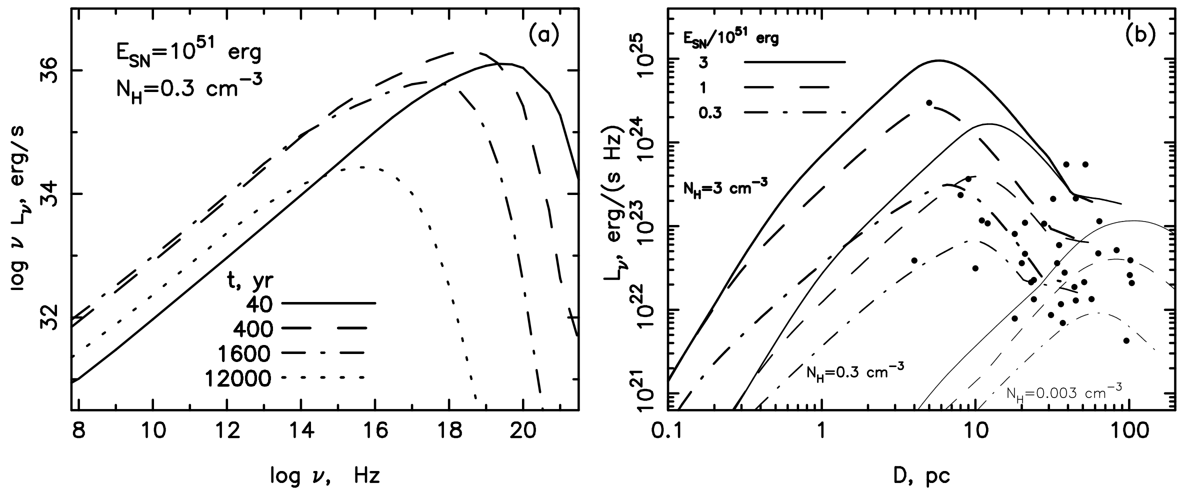

Figure 1. Synchrotron luminosity as a function of frequency at four different times (a) and synchrotron luminosity at $\nu = 1$ GHz as a function of SNR diameter for different explosion energies E_{sn} and ISM number densities N_H together with observational data, comprising 37 Galactic SNRs of known distances [1] (b).

The number of suprathermal protons injected into the acceleration process is described by a dimensionless injection parameter η which is a fixed fraction of the number of ISM particles entering the shock front. we adopt here a value $\eta = 10^{-4}$. It is assumed that electrons are injected into the acceleration process also at the shock front, with the same initial momentum and with an injection rate which is the proton injection rate times some factor K_{ep}. In the sequel we shall use a value $K_{ep} = 10^{-2}$.

The solution of the transport equations for the energetic protons and electrons, and of the gas dynamic equations at each instant of time yields the CR spectra and the spatial distributions of CRs and thermal plasma. This allows to calculate the expected nonthermal emission produced by CRs in SNRs.

We restrict our consideration to the case of a uniform ISM and type Ia SNe which means $M_{ej} = 1.4 M_\odot$. Since the correlation between the SN sites and the ambient ISM density structure is not known we consider three different phases of the ISM with hydrogen number densities $N_H = 3$, 0.3 and 0.003 cm^{-3}, which determine the ISM mass density as $\rho_0 = 1.4 m_p N_H$, where m_p is the proton mass.

To illustrate the time variation of the synchrotron spectrum during the evolution of SNR, corresponding to the SN explosion energy $E_{sn} = 10^{51}$ erg, we present in Fig.1a the calculated synchrotron luminosity $L_\nu(\nu)$ in an ISM with number density $N_H = 0.3$ cm^{-3}, for different ages. The shape of the spectrum $L_\nu(\nu)$ is directly related to the overall electron spectrum $N_e(p)$. For relatively low frequencies $\nu < 10^{14}$ Hz it has a power law form $L_\nu \approx A_\nu B^{\alpha+1} \nu^{-\alpha}$, where the power law index $\alpha = (\gamma - 1)/2$ is related to the power law index γ which determines the electron spectrum $N_e \approx A_e p^{-\gamma}$ near the electron momentum $p \propto \sqrt{\nu}$ which gives the maximum contribution to the synchrotron emission at frequency ν. Due to the shock modification the electron spectrum has a concave shape characterized by an index γ which slowly decreases with increasing energy. Therefore the synchrotron spectrum has also a concave shape with $\alpha > 0.5$ at the lowest frequencies $\nu \sim 1$ GHz, decreasing to $\alpha = 0.5$ at $\nu \gg 1$ GHz.

The current luminosity L_ν is determined by the magnetic field value at the current epoch, which decreases with time due to the shock deceleration, and by the total number of electrons produced during all the previous stages $A_\nu \propto A_e$. During the initial period, $t < 10^3$ yr, the factor $A_\nu \propto A_e$ grows more rapidly than the factor

$B^{\alpha+1}$ decreases and this leads to an increase of L_ν in this period. In the later epoch $t > 2 \times 10^3$ yr, the number of accelerated electrons increases only slowly with time. Therefore the decrease of the magnetic field strength leads to a decrease of L_ν.

The synchrotron emission at the highest frequencies $\nu > 10^{14}$ Hz is produced by the electrons with $p > p_l$, which suffer dramatic synchrotron cooling. Therefore, in addition to the above physical factors, the behavior of the synchrotron spectrum L_ν in this frequency range is influenced by the values of $p_l(t)$ and $p^e_{max}(t)$. Taking into account the expression for the electron cutoff momentum, one can easily derive that the cutoff frequency $\nu_{max} \propto V_s^2$. Since the cutoff frequency is mainly determined by the shock speed, its value ν_{max} goes down quickly during the SNR evolution due to the shock deceleration, as seen from Fig.1a.

Note that at frequency $\nu \sim 10^{18}$ Hz, corresponding to the typical hard X-ray energy $\epsilon_\nu = 4$ keV, only relatively young SNRs of age $t \lesssim 10^3$ yr produce very intense nonthermal X-ray emission which belongs to the power law part of the synchrotron spectrum $L_\nu(\nu)$. Over the subsequent epochs $t \gtrsim 10^3$ yr the shock speed becomes so small that the above X-ray frequency range falls into to the exponential tail of the spectrum $L_\nu(\nu)$. This leads to a fast decrease of the nonthermal X-ray emission during this period of SNR evolution.

To illustrate the evolution of the synchrotron flux we present in Fig.1b the luminosity L_ν at $\nu = 1$ GHz as a function of SNR diameter for the case of SNRs with explosion energy $E_{sn}/(10^{51}$ erg) = 0.3, 1 and 3 in an ISM of density $N_H/(1$ cm$^{-3})$ = 0.003, 0.3 and 3. The variation of the synchrotron flux is due to variation of R_s, V_s and B. During a short initial period, which lasts from $t \approx 1$ yr for $N_H = 3$ cm^{-3} to $t \approx 10$ yr for $N_H = 0.003$ cm^{-3}, the SN shock speed is constant. Since the SNR shock is not significantly modified $A_e \propto N_H V_s R_s^3$, $B \propto \sqrt{N_H}$ and $L_\nu \propto N_H^{7/4} D^3$, taking into account that in the case of unmodified shock $\gamma = 2$ and $\alpha = 0.5$. This explains the rising part of the calculated curves $L_\nu(D)$ which corresponds to small diameter values D in Fig.1b.

During the subsequent part of the free expansion phase the SN shock decelerates. Since the shock as in the previous epoch is not yet strongly modified the CR pressure goes like $P_c \propto N_H V_s$, leading to a magnetic field variation $B_d \propto \sqrt{N_H V_s}$. The expected luminosity dependence on the SNR diameter is then $L_\nu \propto (E_{sn}^2/M_{ej})^{7/16} N_H^{21/16} D^{27/16}$.

In the subsequent Sedov phase a substantial fraction of the total energy E_{sn} goes into the CR component whose overall number remains nearly constant so that $A_e \propto E_{sn}$. Therefore, for a rough estimate, E_c and $A_e \propto E_c$ can be considered as independent of N_H. In this stage the SN shock is significantly modified. This is the consequence of the fact that the CR pressure is an important fraction of the shock ram pressure, with $P_c \propto \rho_0 V_s^2$. Then, taking into account the expansion law we have $L_\nu \propto E_{sn}^{7/4} D^{-9/4}$ independently of N_H and M_{ej}, if for simplicity we again assume the electron spectrum $N_e \propto p^{-\gamma}$ with power law index $\gamma = 2$, that implies $\alpha = 0.5$. The calculated electron spectra corresponding to the Sedov phase are steeper due to the shock modification. In the late Sedov phase, when the effective magnetic field drops to $B_0 = B_{ISM}$ and remains constant thereafter, the expected SNR luminosity $L_\nu \propto E_{sn}$ is roughly independent of D.

According to the calculated dependence of L_ν on D almost all observed SNRs are at the very end of their free expansion phase or in the Sedov phase. This is a consequence of the low expected luminosity of SNRs at the free expansion phase. In addition, it can also be due to a selection effect: SNRs with small size and low luminosity are more difficult to recognize.

In Fig.2 we present the surface brightness $\Sigma_\nu = L_\nu/(\pi^2 D^2)$ at frequency $\nu = 1$ GHz as a function of SNR diameter $D = 2R_s$. Calculations were performed for the above three different ISM densities and for the three values of the SN explosion energy $E_{sn}/(10^{51})$ erg $= 0.3$, 1 and 3, respectively. Galactic shell SNR with known distances [1] and to SNRs in the Magellanic Clouds, M31 and M 33 [5] are also shown in Fig.2. It was assumed that this energy interval covers the majority of the observed SNRs.

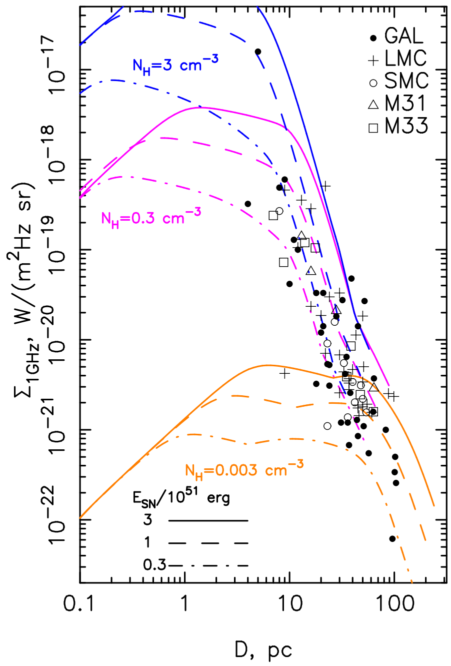

Figure 2. Surface brightness-diameter diagram at frequency $\nu = 1$ GHz. Different styles of lines correspond to the same cases as in Fig.1b. Experimental data for the Galaxy [1] and for the Magellanic Clouds, M 31 and M 33 [5] are shown.

It was assumed that this energy interval covers the majority of the observed SNRs. The experimental values corresponding to 37 The relation $\Sigma_\nu(D)$ is a direct consequence of the $L_\nu(D)$ dependence. In the Sedov phase we have $\Sigma_R \propto E_{sn}^{7/4} D^{-17/4}$, independent of N_H and M_{ej}. Note that this dependence $\Sigma_R \propto D^{-17/4}$ is valid only for the electron spectrum $N_e \propto p^{-2}$ which is created by the unmodified shock. The actual shock during the Sedov phase is significantly modified. The modification, characterized by the deviation of the shock and subshock compression ratios σ and σ_s from the classical value 4, is larger for denser ISM. Therefore the calculated dependence $\Sigma_R(D)$ presented in Fig.2 is close to $\Sigma_R \propto D^{-17/4}$ in the case of a diluted ISM with number density $N_H = 0.003$ cm^{-3} for which the shock is only slightly modified and it becomes steeper with the increase of the ISM density.

The fact that the observational data points lie in a relatively compact region on the $\Sigma_R - D$ diagram can be explained from a theoretical point of view if we suggest that nearly all of the identified SNRs with known distance are in the Sedov phase, or at least at the end of the free expansion phase. The lack of a significant number of SNRs detected in the early free expansion phase can be due to the small SNR size and correspondingly small synchrotron flux expected in this stage, as discussed before.

Even though the expected relation $\Sigma_R \propto E_{sn}^{7/4} D^{-17/4}$ does not depend on the ISM gas density, it is clear from Fig.2 that for small diameters $D < 10$ pc only SNRs located in a dense ISM with $N_H > 1$ cm^{-3} are contained in the sample, whereas for the largest diameters $D > 30$ pc the dominant SNRs are those which are in a tenuous ISM with $N_H < 0.01$ cm^{-3}. If in addition we assume that SNRs in the early Sedov phase are predominantly detected, then one would expect that the relation $D \propto N_H^{-1/3}$ is roughly valid, which leads to $\Sigma_R \propto N_H^{17/12}$ that very well agrees with the dependence $\Sigma_R \propto N_H^{1.37}$, determined from observations [5].

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