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The inverse problem: determining of energy spectrum and total flux of SEP in the source, time of ejection into solar wind, and propagation parameters in the interplanetary space on the basis of ground and satellite CR measurements

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The observed energy spectrum of SEP and its change with time are determined by the energy spectrum in the source, by the time of SEP ejection into the solar wind and by the parameters of SEP propagation in the interplanetary space in dependence of particle energy. Here we will try to solve the inverse problem: on the basis of cosmic ray (CR) observations by the ground base detectors and detectors in the space to determine the energy spectrum of SEP in the source, the time of SEP ejection into the solar wind and the parameters of SEP propagation in the interplanetary space in dependence of particle energy. In general this inverse problem is very complicated, and we suppose to solve it approximately step by step. In this paper we present the solution of the inverse problem in the frame of the simple model of isotropic diffusion (the first step). We suppose that after start of SEP event, the energy spectrum of SEP at different moments of time is determined with a good accuracy in a broad interval of energies by the method of coupling functions. We show that after this can be determined the time of ejection, diffusion coefficient in the interplanetary space and energy spectrum in source of SEP. This information, obtained on line on the basis of real-time scale data, may be useful also for radiation hazard forecasting.

1. Introduction

It is well known that Solar Energetic Particle (SEP) events in the beginning stage are very anisotropic, especially during great events as in February 1956, July 1959, August 1972, September-October 1989, July 2000, January 2005, and many others. To determine on the basis of experimental data the properties of the SEP source and parameters of propagation, i.e. to solve the inverse problem, is very difficult, and it needs data from many CR stations. By the procedure developed in [1], for each CR station can be automatically determined the moment of SEP event start and then for different moments of time by the method of coupling functions to determine the energy spectrum of SEP out of the atmosphere over the individual CR station. As result we may obtain the planetary distribution of SEP intensity over the atmosphere and then by taking into account the influence of geomagnetic field on particles trajectories – the SEP angle distribution out of the Earth's magnetosphere. By this way by using of the planetary net of CR stations with on-line registration in real time scale can be organized the continue on-line monitoring of great ground observed SEP events [2, 3]. In this paper we practically base on the two well established facts: 1) the time of particle acceleration on the Sun and injection into solar wind is very short in comparison with time of propagation, so it can be considered as delta-function from time; 2) the very anisotropic distribution of SEP with developing of the event in time after few scattering of energetic particles became near isotropic (well known examples of February 1956, September 1989 and many others). Our paper is the first step for solution of inverse problem by using only one on-line detector on the ground for high energy particles and one on-line detector on satellite for small energies. Therefore we will base here on the simplest model of generation (delta function in time and in space) and on the simplest model of propagation (isotropic diffusion). The second step will be

based on anisotropic diffusion, and the third – on kinetic description of SEP propagation in the interplanetary space.

2. The inverse problem for the case when diffusion coefficient $K = K(R)$

In this case the solution of isotropic diffusion from the pointing instantaneous source described by function $Q(R,r,t) = N_o(R)\delta(r)\delta(t)$ at the distance r from the Sun and at the time t after ejection will be

$$N(R,r,t) = N_o(R)\times\left[2\pi^{1/2}(K(R)t)^{3/2}\right]^{-1}\times\exp\left(-\frac{r^2}{4K(R)t}\right), \quad (1)$$

where $N_o(R)$ is the rigidity spectrum of total number of SEP at the source, and $K(R)$ is the diffusion coefficient in the interplanetary space during SEP event. Let us suppose that at distance from the Sun $r = r_1 = 1\,\mathrm{AU}$ and at several moments of time t_i $(i = 1,2,3,...)$ after SEP ejection into solar wind the observed rigidity spectrum out of the Earth's atmosphere $N(R,r_1,t_i) \equiv N_i(R)$ are determined in high energy range on the basis of ground CR measurements by neutron monitors and muon telescopes (by using method of coupling functions, spectrographic and global spectrographic methods, see review in [4]) as well as determined directly in low energy range on the basis of satellite CR measurements. Let us suppose also that the UT time of ejection T_e as well as the diffusion coefficient $K(R)$ and the SEP rigidity spectrum in source $N_o(R)$ are unknown. To solve the inverse problem, i.e. to determine these three unknown parameters, we need information on SEP rigidity spectrum $N_i(R)$ at least at three different moments of time T_1, T_2 and T_3 (in UT). In this case for moments of time after SEP ejection into solar wind we obtain:

$$t_1 = T_1 - T_e = x,\;\; t_2 = T_2 - T_e = T_2 - T_1 + x,\;\; t_3 = T_3 - T_e = T_3 - T_1 + x\,, \quad (2)$$

where T_2 - T_1 and T_3 - T_1 are known values and $x = T_1 - T_e$ is unknown value to be determined. From three equations for t_1, t_2 and t_3 of the type of Eq. (1), by taking into account Eq. (2) and dividing one equation on other for excluding unknown parameter $N_o(R)$, we obtain two equations for determining unknown two parameters x and $K(R)$:

$$\frac{T_2 - T_1}{x(T_2 - T_1 + x)} = -\frac{4K(R)}{r_1^2}\times\ln\left\{\frac{N_1(R)}{N_2(R)}\left(x/(T_2 - T_1 + x)\right)^{3/2}\right\}, \quad (3)$$

$$\frac{T_3 - T_1}{x(T_3 - T_1 + x)} = -\frac{4K(R)}{r_1^2}\times\ln\left\{\frac{N_1(R)}{N_3(R)}\left(x/(T_3 - T_1 + x)\right)^{3/2}\right\}. \quad (4)$$

To exclude unknown parameter $K(R)$ let us divide Eq. (3) by Eq. (4); in this case we obtain equation for determining unknown $x = T_1 - T_e$:

$$x = \left[(T_2 - T_1)\Psi - (T_3 - T_1)\right]/(1 - \Psi), \quad (5)$$

where

$$\Psi = \left[(T_3 - T_1)/(T_2 - T_1)\right]\times\left\{\ln\left[N_1(R)\left(x/(T_2 - T_1 + x)\right)^{3/2}/N_2(R)\right]/\ln\left[N_1(R)\left(x/(T_3 - T_1 + x)\right)^{3/2}/N_3(R)\right]\right\}. \quad (6)$$

Eq. (5) can be solved by the iteration method: as a first approximation, we can use $x_1 = T_1 - T_e \approx 500$ sec which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit. Then, by Eq. (6) we determine $\Psi(x_1)$ and by Eq. (5) we determine the second approximation x_2. To put x_2 in Eq. (5) we compute $\Psi(x_2)$, and then by Eq. (6) we determine the third approximation x_3, and so on. After solving Eq. (5) and determining the time of ejection, we can compute very easily diffusion coefficient from Eq. (3) or Eq. (4):

$$K(R)=-\frac{r_1^2(T_2-T_1)/4x(T_2-T_1+x)}{\ln\left\{\frac{N_1(R)}{N_2(R)}(x/(T_2-T_1+x))^{3/2}\right\}}=-\frac{r_1^2(T_3-T_1)/4x(T_3-T_1+x)}{\ln\left\{\frac{N_1(R)}{N_3(R)}(x/(T_3-T_1+x))^{3/2}\right\}}. \quad (7)$$

After determining the time of ejection and diffusion coefficient, it is easy to determine the source SEP spectrum:

$$N_o(R)=2\pi^{1/2}N_1(R)\times(K(R)x)^{3/2}\exp\left(r_1^2/(4K(R)x)\right)=2\pi^{1/2}N_2(R)\times(K(R)(T_2-T_1+x))^{3/2}\times$$
$$\times\exp\left(r_1^2/(4K(R)(T_2-T_1+x))\right)=2\pi^{1/2}N_3(R)\times(K(R)(T_3-T_1+x))^{3/2}\exp\left(r_1^2/(4K(R)(T_3-T_1+x))\right). \quad (8)$$

3. The inverse problem for the case when diffusion coefficient $K = K(R,r)$

Let us suppose, according to [5], that the diffusion coefficient

$$K(R,r)=K_1(R)\times(r/r_1)^{\beta}. \quad (9)$$

In this case

$$N(R,r,t)=\frac{N_o(R)\times r_1^{3\beta/(2-\beta)}(K_1(R)t)^{-3/(2-\beta)}}{(2-\beta)^{(4+\beta)/(2-\beta)}\Gamma(3/(2-\beta))}\times\exp\left(-\frac{r_1^{\beta}r^{2-\beta}}{(2-\beta)^2K_1(R)t}\right), \quad (10)$$

where t is the time after SEP ejection into solar wind. So now we have four unknown parameters: time of SEP ejection into solar wind T_e, β, $K_1(R)$, and $N_o(R)$. Let us assume that according to ground and satellite measurements at the distance $r=r_1=1$ AU from the Sun we know $N_1(R),N_2(R),N_3(R),N_4(R)$ at UT times T_1,T_2,T_3,T_4. In this case

$$t_1=T_1-T_e=x,\ \ t_2=T_2-T_e=T_2-T_1+x,\ \ t_3=T_3-T_e=T_3-T_1+x,\ \ t_4=T_4-T_e=T_4-T_1+x, \quad (11)$$

For each $N_i(R,r=r_1,T_i)$ we obtain from Eq. (10) and Eq. (11)

$$N_i(R,r=r_1,T_i)=\frac{N_o(R)\times r_1^{3\beta/(2-\beta)}(K_1(R)(T_i-T_1+x))^{-3/(2-\beta)}}{(2-\beta)^{(4+\beta)/(2-\beta)}\Gamma(3/(2-\beta))}\times\exp\left(-\frac{r_1^2(2-\beta)^{-2}}{K_1(R)(T_i-T_1+x)}\right), \quad (12)$$

where i = 1, 2, 3, and 4. To determine x let us step by step exclude unknown parameters $N_o(R)$, $K_1(R)$, and then β. In the first we exclude $N_o(R)$ by forming from four Eq. (12) three equations for ratios

$$\frac{N_1(R,r=r_1,T_1)}{N_i(R,r=r_1,T_i)}=\left(\frac{x}{T_i-T_1+x}\right)^{-3/(2-\beta)}\times\exp\left(-\frac{r_1^2}{(2-\beta)^2K_1(R)}\left(\frac{1}{x}-\frac{1}{T_i-T_1+x}\right)\right), \quad (13)$$

where i = 2, 3, and 4. To exclude $K_1(R)$ let us take logarithm from both parts of Eq. (13) and then divide one equation on another; as result we obtain following two equations:

$$\frac{\ln(N_1/N_2)+(3/(2-\beta))\ln(x/(T_2-T_1+x))}{\ln(N_1/N_3)+(3/(2-\beta))\ln(x/(T_3-T_1+x))}=\frac{(1/x)-(1/(T_2-T_1+x))}{(1/x)-(1/(T_3-T_1+x))}, \quad (14)$$

$$\frac{\ln(N_1/N_2)+(3/(2-\beta))\ln(x/(T_2-T_1+x))}{\ln(N_1/N_4)+(3/(2-\beta))\ln(x/(T_4-T_1+x))}=\frac{(1/x)-(1/(T_2-T_1+x))}{(1/x)-(1/(T_4-T_1+x))}. \quad (15)$$

After excluding from Eq. 14 and Eq. 15 unknown parameter β, we obtain equation for determining x:

$$x^2(a_1a_2-a_3a_4)+xd(a_1b_2+b_1a_2-a_3b_4-b_3a_4)+d^2(b_1b_2-b_3b_4)=0, \quad (16)$$

where

$$d=(T_2-T_1)(T_3-T_1)(T_4-T_1),\ a_1=(T_2-T_1)(T_4-T_1)\ln(N_1/N_3)-(T_3-T_1)(T_4-T_1)\ln(N_1/N_2),\ a_2=(T_3-T_1)\times$$
$$\times(T_4-T_1)\ln(x/(T_2-T_1+x))-(T_2-T_1)(T_3-T_1)\ln(x/(T_4-T_1+x)),\ a_3=(T_2-T_1)(T_3-T_1)\ln(N_1/N_4)-$$
$$-(T_3-T_1)(T_4-T_1)\ln(N_1/N_2),\ a_4=(T_3-T_1)(T_4-T_1)\ln(x/(T_2-T_1+x))-(T_2-T_1)(T_4-T_1)\ln(x/(T_3-T_1+x)),$$
$$b_1=\ln(N_1/N_3)-\ln(N_1/N_2),\ b_2=\ln(x/(T_2-T_1+x))-\ln(x/(T_4-T_1+x)),\ b_3=\ln(N_1/N_4)-\ln(N_1/N_2),$$
$$b_2=\ln(x/(T_2-T_1+x))-\ln(x/(T_3-T_1+x)). \qquad (17)$$

As it can be seen from Eq. (17), coefficients a_2, a_4, b_2, b_4 very weekly (as logarithm) depend from x. Therefore Eq. (17) we solve by iteration method, as above we solved Eq. (5): as a first approximation, we use $x_1 = T_1 - T_e \approx 500$ sec (which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit). Then, by Eq. (17) we determine $a_2(x_1), a_4(x_1), b_2(x_1), b_4(x_1)$ and by Eq. (16) we determine the second approximation x_2, and so on. After determining x, i.e. according Eq. 11 determining t_1, t_2, t_3, t_4, the final solutions for β, $K_1(R)$, and $N_o(R)$ can be found. Unknown parameter β in Eq. (9) we determine from Eq. (14) and Eq. (15):

$$\beta = 2-3\left[(\ln(t_2/t_1))-\frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\ln(t_3/t_1)\right]\times\left[(\ln(N_1/N_2))-\frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\ln(N_1/N_3)\right]^{-1}. \qquad (18)$$

Then we determine unknown parameter $K_1(R)$ in Eq. (9) from Eq. (13):

$$K_1(R)=\frac{r_1^2\left(t_1^{-1}-t_2^{-1}\right)}{3(2-\beta)\ln(t_2/t_1)-(2-\beta)^2\ln(N_1/N_2)}=\frac{r_1^2\left(t_1^{-1}-t_3^{-1}\right)}{3(2-\beta)\ln(t_3/t_1)-(2-\beta)^2\ln(N_1/N_3)}. \qquad (19)$$

After determining parameters β and $K_1(R)$ we can determine the last parameter $N_o(R)$ from Eq. (12):

$$N_o(R)=N_k(2-\beta)^{(4+\beta)/(2-\beta)}\Gamma(3/(2-\beta))r_1^{-3\beta/(2-\beta)}(K_1(R)t_k)^{3/(2-\beta)}\times\exp\left(\frac{r_1^2}{(2-\beta)^2K_1(R)t_k}\right), \qquad (20)$$

where index k = 1, 2 or 3.

4. Discussion

Above we show that for some simple model of SEP propagation is possible to solve inverse problem and on the basis of ground and satellite measurements at the beginning of the event. It is important that in each case the obtained results may be checked by data in the next moments of time by comparison of predicted SEP time variation in different energy ranges with observed data. Let us note that described solutions of inverse problem may be partly useful for solving more complicated inverse problems in case of SEP propagation described by anisotropic diffusion and by kinetic equation [6, 7]. Obtained results we used in the method of great radiation hazard forecasting based on on-line CR one-minute ground and satellite data [8].

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