STRONG COSMIC-RAY SCATTERING IN LARGE-SCALE ANISOTROPIC RANDOM AND REGULAR MAGNETIC FIELDS

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Kinetic coefficients and parallel (to the mean field) mean free paths of the fast particles in large-scale anisotropic random magnetic field are obtained with use nonlinear collision integral, i.e., by taking into account the strong random scattering. The diffusion of the solar and Galactic cosmic rays in the two-dimensional turbulence is investigated. It is shown that the two-dimensional turbulence can make a principal contribution to the parallel mean free paths of cosmic rays in the heliosphere and interstellar medium.

INTRODUCTION

It follows from the analysis of experimental data performed by Matthaeus et al. (1990) and Bieber et al. (1996) that the distribution of interplanetary magnetic field fluctuations is anisotropic. In the weakly disturbed inner heliosphere, the preferential direction of the magnetic field fluctuations is perpendicular to the regular magnetic field. The wave vectors of the fluctuations are also mainly perpendicular to the regular magnetic field, which gives rise to two-dimensional fluctuations. In the interplanetary medium, the energy of the two-dimensional fluctuations can reach 85% of the energy of the random magnetic field.

The parallel transport mean free paths of high-energy particles in the interplanetary magnetic field, including the anisotropy of random fluctuations, were calculated numerically by Bieber et al. (1994), Teufel and Schlickeiser (2002, 2003), Teufel et al. (2003), Shalchi and Schlickeiser (2004) and analytically by Droge (2003). These authors used a quasi-linear random magnetic field approximation and introduced the cyclotron resonance broadening using a decorrelation in the correlation tensor of the random magnetic field. They showed that particles are scattered weakly by two-dimensional fluctuations. The calculated transport mean free paths of solar cosmic-ray protons exceed their observed values by several tens or hundreds of times. In this paper, we show that for nonlinear broadening of two-dimensional perturbations, the random scattering frequency increases significantly, and the transport mean free path decreases.

TRANSFORMING THE COLLISION INTEGRAL

We will consider the kinetic coe□cients and particle transport mean free paths over a wide energy range from 1 MeV to several GeV in the inner heliosphere and at energies above 10 GeV in the outer heliosphere, including those at the energies at which $R_1 \sim L_{\|}, L_{\perp}$, where R_1 is the gyroradius in the random magnetic field, and $L_{\perp}$ and $L_{\|}$ are the perpendicular and parallel (relative to the regular magnetic field) correlation lengths, respectively. We use the following kinetic equation for the average particle distribution function $F(\vec{r},\vec{p},t)$) with the nonlinear collision integral (Mel'nikov 1996, 2000):

$$\left\{\frac{\partial}{\partial t}+\vec{v}\frac{\partial}{\partial \vec{r}}-\vec{H}_0\vec{D}\right\}F(\vec{r},\vec{p},t)=StF \qquad (1)$$

$$StF=D_{\alpha}\int dx_1 B_{\alpha\beta}(\vec{r},t,\vec{r}_1,t_1)\cdot G_1(x,x_1)\cdot D_{1\beta}F(x_1,x_0) \qquad (2)$$

где $x \equiv \vec{r}, \vec{p}, t$; $\vec{r}$ is the coordinate, $\vec{p}$ is the momentum, $\vec{v}$ is the particle velocity; t is the time; $\vec{H}_0$ is the strength of the regular magnetic field, $\vec{D} = \frac{e}{c}[(\vec{v}-\vec{u})\times\frac{\partial}{\partial \vec{p}}]$, e is the

particle charge, c is the speed of light, u is the velocity of the magnetic field, and $G_1(x,x_1)$ is the one-particle Green function that is the solution of the linear kinetic equation.
We choose the correlation tensor of the random anisotropic magnetic field H_1 for a power-law spectrum in the form (Matthaeus et al. 1990; Toptygin 1985; Chuvilgin and Ptuskin 1993)

$$B_{\alpha\beta}(\vec{k}) = P(\vec{k})\frac{[\vec{k}\times\vec{h}_0]_\alpha[\vec{k}\times\vec{h}_0]_\beta}{k_\perp^2}, \quad P(\vec{k}) = A_\nu (q_\perp q_\parallel)^{-1-\frac{\nu}{2}}\left(\frac{k_\parallel^2}{q_\parallel^2}+\frac{k_\perp^2}{q_\perp^2}\right)\left[1+\frac{k_\parallel^2}{q_\parallel^2}+\frac{k_\perp^2}{q_\perp^2}\right]^{-2-\frac{\nu}{2}}, \quad (3)$$

where $\vec{k}$ - is the wave vector, ν is the spectral index , $\vec{h}_0 = \vec{H}_0 \cdot H_0^{-1}$, $\vec{k}_\parallel = (\vec{k}\vec{h}_0)\vec{h}_0$,
$\vec{k}_\perp = \vec{k} - \vec{k}_\parallel$, $\vec{q}_\parallel = (\vec{q}\vec{h}_0)\vec{h}_0$, $\vec{q}_\perp = \vec{q} - \vec{q}_\parallel$, $q_\perp = 2\pi L_\perp^{-1}, q_\parallel = 2\pi L_\parallel^{-1}$,

$$A_\nu = 2\cdot\Gamma(2+\tfrac{\nu}{2})\cdot q_\perp^{\frac{\nu}{2}-1} q_\parallel^{\frac{\nu}{2}} \langle H_1^2\rangle [3\cdot\pi^{\frac{3}{2}}\Gamma(\tfrac{\nu-1}{2})]^{-1},$$

$\Gamma(n)$ – is the Gamma function. Passing to the drift approximation, we obtain :

$$\left\{\frac{\partial}{\partial t} + v\mu\frac{\partial}{\partial z} - \frac{1}{2}v\, div\vec{h}_0 \sin\vartheta\frac{\partial}{\partial\vartheta}\right\}\Phi = \langle StF\rangle_\varphi,$$

(4)
where the coordinate z is along the vector координата $\vec{h}_0$, $\Phi = \langle F\rangle_\varphi$, ϑ is the angle between $\vec{p}$ и $\vec{h}_0$, $\mu = cos\vartheta$, φ is the azimuthally angle between $\vec{p}$ и $\vec{h}_0$. The nonlinear average
collision integral is

$$\langle StF\rangle_\varphi = \frac{\partial}{\partial\mu}(1-\mu^2)b(\mu)\frac{\partial}{\partial\mu}\Phi(\vec{r},p,\mu,t), \quad (5)$$

where the kinetic coefficient is

$$b(\mu) = \frac{e^2}{2m^2c^2}\int_0^\infty d\tau\int d\vec{k}\cdot P(\vec{k})\cdot\cos(\varphi_k-\varphi)\cdot\cos(\varphi_k-\varphi-\Omega\tau)\cdot\Gamma_0(\omega)\cdot\exp\{i\vec{k}\Delta\vec{r}(\tau)\}, \quad (6)$$

m is the particle mass, φ_k is the azimuthal angle of the vector $\vec{k}$, Ω is the gyrofrequency in the regular magnetic field, ω is the gyrofrequency in the random magnetic field, $\Delta\vec{r}(\tau)$ is the
change in the radius vector of the particle in the regular magnetic field, $\Gamma_0(\omega)$ is a factor that is related to the additional Green function of the particle in the nonlinear collision integral and that yields the damping of the resonant wave–particle interaction.

$$\Gamma_0(\omega) = \exp\left\{-\frac{\omega_\perp^2}{16}v_\parallel^2 k_\perp^2\tau^4 - \frac{\omega_\perp^2}{4\Omega^2}v_\perp^2 k_\parallel^2\tau^2\right\}. \quad (7)$$

KINETIC COEFFICIENTS AND TRANSPORT MEAN FREE PATHS. COMPARISON WITH EXPERIMENTAL DATA

Let us first consider the limiting case of the absence of resonance broadening, ω =0 and $\Gamma_0(\omega)$ =1. In the kinetic coefficient (6), we expand the corresponding functions in terms of Bessel functions. We transform the series of Bessel functions and add the series using the addition formula for the Bessel functions

$$J_n(\rho)\cdot\exp\left\{in\frac{\pi-\beta}{2}\right\} = \sum_{k=-\infty}^{+\infty} J_{n+k}(z)\cdot J_k(z)\cdot\exp\{ik\beta\},$$

where $\rho = 2z\cdot\sin\left\{\frac{\beta}{2}\right\}$, $J_n(\rho)$ is the Bessel function of order n. The integrations in (6) yield a kinetic coefficient in the form

$$b_0(\mu) = c_0(\nu)\omega_\perp\left(\frac{\omega_\perp}{\Omega}\right)(q_{\|}\mu R)^{\nu-1}, \quad c_0(\nu) = \frac{\sqrt{\pi}\cdot\Gamma(\frac{\nu}{2}+2)}{3\nu\cdot\Gamma(\frac{\nu-1}{2})}, \quad R = v\cdot\Omega^{-1}. \tag{8}$$

Let us now turn to the di□usion approximation using the formulae

$$\Lambda_0 = \frac{3v}{4}\int_0^1 d\mu\,\frac{1-\mu^2}{b\ (\mu)}.$$

In this case

$$\Lambda_0 = \left(\Omega^2\omega^{-2}\right)3R\cdot(q_{\|}R)^{1-\nu}\left[4(2-\nu)(2-\frac{\nu}{2})\cdot c_0(\nu)\right]^{-1}.$$

In the case of strong random scattering at $v_{\|}^2 q_\perp^2 >> v_\perp^2 q_{\|}^2$ following factor makes a major contribution to the resonance damping:

$$\Gamma_1(\omega,\tau) = \exp\left\{-\frac{\omega_\perp^2}{16}v_{\|}^2 k_\perp^2\tau^4\right\}.$$

The integrations in (6) yield a kinetic coe□cient in the form

$$b_1(\mu) = c_1(\nu)\cdot\omega_\perp\left(\frac{\omega_\perp}{\Omega}\right)^{\nu}(\mu\, q_\perp R)^{\nu-1}, \quad c_1(\nu) = \left(\frac{5}{4}\right)^{\nu-\frac{1}{2}}\frac{\sqrt{\pi}\,\Gamma(\frac{1}{4})(\frac{\nu}{2}+1)\Gamma(\nu-\frac{1}{2})}{3\cdot 2^{\frac{\nu}{2}+1}\cdot\Gamma(\frac{\nu-1}{2})}. \tag{9}$$

For $v_{\|}^2 q_\perp^2 << v_\perp^2 q_{\|}^2$, the following integrand factor makes a major contribution to the damping function :

$$\Gamma_2(\omega,\tau) = \exp\left\{-\frac{\omega_\perp^2}{4\Omega^2}v_\perp^2 k_{\|}^2\tau^2\right\}.$$

Substituting it into (6), we obtain after transformations and integrations

$$b_2(\mu) = c_2(\nu)\cdot\omega_\perp\left(\frac{\omega_\perp}{\Omega}\right)\left(q_{\|}R\right)^{\nu-1}\left(\frac{\sqrt{\pi}}{\Gamma(\frac{\nu}{2})}\mu^{\nu-1} + \left(\frac{\omega_\perp}{\Omega}\right)^{\nu-1}\sqrt{1-\mu^2}\right), \tag{10}$$

where

$$c_2(\nu) = \frac{\Gamma(\frac{\nu}{2})\Gamma(\frac{\nu}{2}+2)}{3\nu\cdot\Gamma(\frac{\nu-1}{2})}.$$

It is convenient to combine (8), (10), (12) into a general interpolation formula for $b(\mu)$ that is valid at any pitch angle, we obtain

$$b(\mu) = \omega\left(\frac{\omega}{\Omega}\right)(q_{\|}R)^{\nu-1}\left[c_0\mu^{\nu-1} + c_1\left(\frac{\omega q_\perp}{\Omega\, q_{\|}}\right)^{\nu-1}\mu^{\nu-1} + c_2\left(\frac{\omega}{\Omega}\right)^{\nu-1}\sqrt{1-\mu^2}\right]. \tag{11}$$

Using the interpolation formula for the integral in (16), we obtain the final formula for the parallel transport mean free path, including strong random scattering

$$\Lambda_{\|}(R) = \Lambda_0\sqrt{\pi}(2-\nu)\left(2-\frac{\nu}{2}\right)\left(\frac{\Omega}{\omega}\right)^{\nu-1}\Gamma^{-1}\left(\frac{\nu}{2}\right)\left[1{,}63+\frac{\nu-1}{3}\left(f_1(\omega)\right)^{0{,}7}+(2{,}13-\nu)f_1(\omega)\right]^{-1}, \tag{12}$$

$$f_1(\omega_\perp) = c_0(\nu)c_2^{-1}(\nu)\left(\Omega\omega^{-1}\right)^{\nu-1} + c_1(\nu)c_2^{-1}(\nu)\left(q_\perp q_{\|}^{-1}\right)^{\nu-1}.$$

In this case, $\Lambda_{\|}\propto p^{2-\nu}$. The contribution of strong random scattering is significant at any strengths of the random magnetic field. The momentum dependence of $\Lambda_{\|}$ in this case is similar to that numerically calculated by Teufel , Schlickeiser (2002, 2003) and Shalchi, Schlickeiser (2004).

For protons with energy of 200 MeV scattered in a weakly disturbed interplanetary medium, substituting $R = 4 \cdot 10^8$ m, $L_{\|} = L_{\perp} = 2 \cdot 10^{10}$ m, $\nu = 1.67$, and $\Omega^2/\omega^2 = 8$ yields $\Lambda_{\|} \approx 0.25$ AU. This value of $\Lambda_{\|}$ is close to the mean experimental values from Palmer (1982). The values of $\Lambda_{\|}$ are close to those from Shalchi, Schlickeiser (2004), in which, however, slab turbulence produces the main scattering. Thus, the weak momentum dependence of $\Lambda_{\|}$ for solar cosmic rays over a wide energy range from several MeV to several GeV can be explained in terms of strong (moderate) random scattering by two-dimensional turbulence in the solar wind.

For Galactic cosmic rays with energies above 4 GeV scattered in the interstellar medium, when the random magnetic field has a Kolmogorov spectrum with $\nu \approx 1.7$, we obtain the following order-of-magnitude estimate from formula (12):

$$\Lambda_{\|} = 1{,}6 \cdot q_{\|}^{-1} (Rq_{\|})^{2-\nu} \left(\Omega\omega^{-1}\right)^{\nu+1}.$$

Assuming that $\omega \approx 1.8\Omega$, $L_{\|} \approx 100$ pc, and $H_1 \approx 0.3$ nT, we obtain for relativistic protons

$$\Lambda_{\|} \approx 1{,}8 \cdot 10^{18} \left(\frac{E}{1\ \text{ÃýÂ}}\right)^{2-\nu} \text{sм} , \qquad (13)$$

where E is the particle kinetic energy in GeV. Calculated value of $\Lambda_{\|}$ is close to the experimental mean free path, Ptuskin 2001,1993.

In the case of the very strong turbulence in corotating interaction region of the outer heliosphere

$$\Lambda_{\|} = \frac{3\nu \cdot 2^{\frac{3\nu}{2}-2} \cdot \Gamma(\frac{\nu-1}{2})}{\pi \cdot \Gamma(\frac{\nu}{2}+2)} (\sqrt{2}\,\frac{\Omega}{\omega})^{\nu+1} (\frac{q_{\|}}{q_{\perp}})^{\frac{\nu}{2}-1} R \quad . \qquad (14)$$

Calculations using (14) for $\omega \approx 0.7\Omega$ yield $\Lambda_{\|} \approx 1$ AU for Galactic cosmic rays with an energy of 10 GeV. The numerical value and rigidity dependence of $\Lambda_{\|} \propto R$ matchthe experimental data for Galactic cosmic rays in the outer heliosphere obtained by Gerasimov et al. (1999).

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