THE BASIC CORRELATION LENGTHS OF THE INTERPLANETARY MAGNETIC FIELD FLUCTUATIONS

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ABSTRACT

The theoretical momentum dependence of a mean free path, obtained by solving the nonlinear kinetic equation taking into account nonlinear damping of a resonant interaction of particles with isotropic random magnetic field, and the experimental data on the momentum dependence of a mean free paths of cosmic rays in the interplanetary magnetic field (IMF) in a wide energy interval are analyzed and compared. The conditions of applicability of strong large-scale scattering are considered. As a result, basic correlation scales of a random interplanetary magnetic field, which are consistent with its multifractal topology, are found.

1.INTRODUCTION

The IMF spatial structure is directly experimentally determine by using data from one or several satellites and parameters of the magnetosphere [Veselovsky, 1996; Zelenyi and Milovanov, 1993, 1997; Ivanov, 1996; Ivanov and Kharshiladze, 1998; Burlaga and Klein, 1986; Burlaga, 1991a; Burlaga et al., 2003]. However, this does not allow one to accurately determine the spatial structure of a random magnetic field. Therefore, theoretically and experimentally consistent models of RIMF are used. Zelenyi and Milovanov [1997], Burlaga and Klein [1986], and Burlaga [1991a, 1991b] determined the characteristics of the turbulent IMF component, whose correlation properties are consistent with the multifractal field topology, and the fluctuation spectrum by using experimental data and statistical properties of a random magnetic field. These researchers obtained the IMF representation as a flow of fractal clusters on magnetic field tubes frozen in the solar wind.

Cosmic-ray particles propagating in IMF experience averaged interaction with random magnetic structures and with the regular magnetic field. Therefore, the energy and spatial dependences of kinetic coefficients are determined by the averaged structure of the random and regular magnetic fields. The kinetic coefficients taking into account specific features of the structure of the turbulent IMF component can be theoretically obtained by using the kinetic equation with a nonlinear collision integral. This allows us to take into account strong random scattering, which arises at a Larmor radius in a random magnetic field R_1 of the order of or less than correlation length L_c [Toptygin, 1985; Melnikov, 1996].

2. TRANSPORT PATH WITH REGARD TO STRONG RANDOM SCATTERING

Let us assume that random scattering is basically resonant and large-scale and broadening of resonant interaction takes place on small-scale irregularities of the magnetic field. The nonlinear effects related to strong random scattering can be described by using the nonlinear collision integral quadratic with respect to the Green function [Melnikov, 2000]. In this integral the broadening of resonant scattering is described by small-scale isotropic random scattering. The averaged nonlinear kinetic coefficient $b(\mu)$ for $R_0 \ll L_c$, where R_0 is the Larmor radius in a regular magnetic field, nonlinear with respect to the Green function, is written as [Mel’nikov, 2000]:

$$b(\mu)=\frac{e^{2}}{\;^{2}\;^{2}}\int_{0}^{\infty} d\tau \iiint d^{3}k\, B(k)\left[\cos\Omega\tau+\frac{k_{\perp}^{2}}{k^{2}}\sin\varphi\,\sin(\Omega\tau-\varphi)\right] W_{D}(k_{||})\, W_{T}(k_{||})\times \quad (1)$$

$$\times\exp\left\{-\frac{1}{16}\,\omega_{1}^{2}\,\tau^{4}\, v^{2}\, k_{\perp}{}^{2}\right\}\cos\{k_{||}v_{||}\tau+k_{\perp}R_{\perp}[\sin(\Omega\tau-\varphi)+\sin\varphi]\},\ k_{||}=(\vec{k}\,\vec{h}_{0}),\ \vec{k}_{\perp}=\vec{k}-\vec{k}_{||},$$

where $\vec{v}$ is the particle velocity; $\vec{H}_0 = H_0 \cdot \vec{h}_0$ is the strength of the regular magnetic field. ϑ is the angle between the $\vec{p}$ and $\vec{h}_0$, $v_{||} = v\cos\vartheta$, $v_\perp = v\sin\vartheta$, and φ is the azimuthal angle of the $\vec{p}$ in the plane perpendicular to $\vec{h}_0$, $\mu=\cos\vartheta$, $R_\perp = v_\perp/\Omega$, $\Omega = eH_0/mc$, $\omega_1 = e\sqrt{<H_1^2>}/mc$, m is the relativistic mass of a particle, c is the velocity of light, e is the particle charge, and φ is the angle between $\vec{k}_\perp$ and $\vec{v}_\perp$. Here B(k) is

$$B(k)=A_\nu k^2(k_0^2+k^2)^{-2-\nu/2}\ ,\ A_\nu=\Gamma(2+\nu/2)\,k_0^{\nu-1} <H_1^2> \left[3\pi^{3/2}\Gamma((\nu-1)/2)\right]^{-1}\quad ,$$

$\Gamma(n)$ is the gamma function, and ν is the spectral index of a random field, $\nu\sim1.5$-2 of the scales 10^7- 10^{11}m [Toptygin, 1985]. For protons with energy E >1 MeV, broadening produced by damping of the correlation of a random magnetic field is less than damping related to nonlinear random broadening. Therefore, we will subsequently neglect thermal and dispersion broadening assuming that $W_D(k_{||}) = W_T(k_{||})=1$. After transformations and integrations, passing to a diffusion approximation, we obtain [Melnikov, 2000]:

$$\Lambda_{||}=\frac{6R_1^2L_c^{\nu-1}}{\sqrt{\pi}(1+\nu/2)(\nu-1)R_0^\nu}\int_0^1 d\mu(1-\mu^2)\left[1+\frac{2\sqrt{\pi}\Gamma(\frac{\nu}{2})R_1L_c^{\nu-1}}{\Gamma(\frac{\nu+1}{2})R_0^\nu}\exp\{-\frac{4R_1\mu}{L_c}\}\right]^{-1}, \qquad (2)$$

where $R_1= cp/e\sqrt{<H_1^2>}$. From this formula it follows that, at weak random scattering the transport path is

$$\Lambda_{||} = 9R_1^2L_c^{\nu-1}\left[\sqrt{\pi}(\nu-1)(1+\nu/2)R_0^\nu\right]^{-1}\ . \qquad (3)$$

For a sufficiently low particle energy in the case of strong random scattering

$$\Lambda_{||} = 3\Gamma((\nu-1)/2)R_1\left[\pi\Gamma(\nu/2)(1+\nu/2)\right]^{-1}\ . \qquad (4)$$

The energy interval where random resonant scattering is weak becomes narrower if damping is taken into account. To prove this, we introduce $b_1=H_1/H_0$ ratio. This ratio is close to 1/4-1/3 in the heliosphere [Toptygin, 1985]. Assume that the maximum energy of particles, for which random resonant scattering is weak, is E_1; i.e., the conditions $R_1(E=E_1) > L_c$ and $R_0(E=E_1) \le L_c$ are satisfied for these particles. If the particle energy decreases to E_2, random scattering becomes strong, $R_1(E=E_2) = L_c \ge R_0(E=E_1)$. From this it follows that

$$R1(E=E2) \ge R0(E=E1)\quad \text{и}\quad p_1\cdot p_2^{-1} \le b_1^{-1}\ . \qquad (5)$$

We assume that transverse diffusion is related to the perpendicular components of a random large-scale magnetic field, [Toptygin, 1985; Völk and Alpers, 1975]. The perpendicular transport paths with regard to strong random scattering, [Melnikov, 2000], are

$$\Lambda_\perp=\frac{\sqrt{\pi}\Gamma(\nu/2)<H_1^2>L_c}{2\Gamma((\nu-1)/2)H_0^2},\ \text{for } R_1 \gg L_c;\ \Lambda_\perp=\frac{8\pi<H_1^2>R_1^{1/2}L_c^{1/2}}{5H_0^2},\ \text{for } R_1\approx L_c;$$

$$\Lambda_\perp=\frac{4\Gamma((\nu-1)/2)<H_1^2>R_1}{3\pi\Gamma(\nu/2)(1+\nu/2)H_0^2},\ \text{for } R_1 \ll L_c. \qquad (6)$$

From this formulas it follows that, for $\nu= 2$ and $b_1= 1/3$, the transverse path is $\Lambda_\perp\approx 0{,}05\ L_c$ at $R_1 >> L_c$, $\Lambda_\perp\approx0.56\sqrt{R_1L_C}$ at $R_1 \sim L_c$, and $\Lambda_\perp\approx 0.04\ R_1$ at $R_1<< L_c$. Thus, the $\Lambda_\perp$ resonantly increases at $R_1 \sim L_c$. The same increase in random changes in μ and an increase in transverse diffusion take place in a nonuniform magnetic field [Chen and Palmadesso, 1986; Buchner and Zelenyi, 1989].

3.MAXIMUM SCALE OF A RANDOM INTERPLANETARY MAGNETIC FIELD

Experimental data on the momentum dependence of $\Lambda_{||}$ of cosmic rays with energies of 5-300 GeV indicate that $\Lambda_{||} \propto p$ [de Koning and Mathews, 1996; Miroshnichenko et al., 1998; Gerasimova et al., 1999]. We now assume that the particle distribution function is almost isotropic and scattered particles have a pitch angle of the order of $\pi/4$. From formula (4) it follows that the longitudinal path has the form $\Lambda_{||} = 0.9\ R_1$ at a strong broadening of resonant scattering, i.e., at $R_1 \leq L_c$ and $R_0 < L_c$., Using (4), we can theoretically describe the momentum dependence of the experimental transport path assuming that particles with energies of 5-100 GeV are strongly scattered by random irregularities with scale $L_4 = 10^{12}$ m, whose extents are about the Larmor radius of particles with maximum energies (100 GeV). The scale L_4 is about the distance covered by the solar wind during one solar rotation, [Burlaga et al., 2003].

4. RANDOM MAGNETIC FIELD SCALE CORRESPONDING TO THE ENERGY AT A BEND OF THE TRANSPORT PATH MOMENTUM DEPENDENCE

The experimental data averaged for many observations indicate that $\Lambda_{||} \propto p^{0.3}$ of cosmic rays with energies of 0.001-3 GeV and $\Lambda_{||} \propto p$ at energies of 2-5 GeV, [Palmer, 1982; Bieber et al., 1994; de Koning and Mathews, 1996; Miroshnichenko et al., 1998,Gerasimova et al., 1999]. Thus, the energy dependence has a bend . The change of the momentum dependence at energies of 2-4 GeV is related to the third scale, L_3. The Larmor radius in a random field, which corresponds to the bend energy, is equal to $L_3 = 10^{10}$ m.

The cross sections of the sets (ropes of the solar wind magnetic filaments) caused by the supergranulation structure of the solar photosphere are about L_3 [Zelenyi and Milovanov, 1993, Zelenyi and Milovanov, 1997]. The supergranulation structure can reflect larger scale granulation structure of deep subphotospheric flows. The scale $L_3 = 10^{10}$ m is close to the cross size of plasma flows near the Earth, which are formed from disappearing solar filaments and from composite flows caused by other solar sources: filamentary streamer, flare filamenttary, and coronal-hole filamentary [Ivanov, 1998; Ivanov, Kharshiladze, 1998].The change of the energy dependence of longitudinal and transverse transport paths are not observed in numerical simulations of particle diffusion in a magnetic field for a power-law spectrum of a random field, which has a uniform isotropic structure and a single correlation length [Giacalone, Jokipii, 1999]. Thus, the bend in the momentum dependence of the longitudinal transport path can be related to the filamentary structure of a random interplanetary magnetic field whose correlation properties are consistent with the idea of the multifractal topology of a field [Burlaga, 1991a, 1991b; Zelenyi and Milovanov, 1993, 1997].

5. MINIMUM SCALES of A RANDOM MAGNETIC FIELD

Experimental data indicate that the longitudinal transport path of particles with energies 0.001-2 GeV, i.e, in a wide energy range, slightly depends on a momentum, $\Lambda_{||} \propto p^n$, n=0.3. The numerical values of the transport path vary from 0.08 to 0.3 AU. These values of the longitudinal path were determined as a result of a consensus during averaging of experimental data [Palmer, 1982, Bieber et al., 1994]. Such a behavior of the longitudinal transport path can be explained by a weak resonant scattering of particles with random field. In this case $n=2-\nu$, $\nu=1.7-2$ for random field scales smaller than or close to L_3 [Toptygin, 1985;Droge, 2003]. In the case of weak random scattering, $R_1 >> L_4$. Therefore, $L_2 << L_3$ for particles with energies $E < 2$ GeV. For particles with a boundary energy of 2 GeV (when $R_1 = 10^{10}$ m, $R_0 = 2\cdot 10^9$, and $b_1 = 1/5$), we choose a correlation scale of $L_c \approx R_0 = 2\cdot 10^9$ m. If the energy of these particles decreases to 200 MeV, their Larmor radius in a random field is $R_1 \approx L_c$, and particles will experience strong random scattering. That is, the path $\Lambda_{||}$ of particles with an energy of 200 MeV should sharply decrease. This explains a decrease in $\Lambda_{||}$ for particles with an energy of 200 MeV observed experimentally, which has not yet been convincingly explained

[Palmer, 1982]. Thus, a scale of $L_2 = 10^9$ m will be about the Larmor radius of a particle with an energy of 200 MeV in a random magnetic field. We should note that a scale of L_2 is close to the cross size of the fine structure of near-Earth plasma flows, which are formed from disappearing solar filaments and from flows caused by other solar sources: filamentary streamer, flare filamentary, and coronal-hole filamentary [Ivanov, 1996, 1998; Ivanov and Kharshiladze, 1998]. The following energy interval of a weak dependence of $\Lambda_{||}$ on the momentum, 1-100 MeV, requires the introduction of a smaller correlation scale L_1. The scale $L_1$is about the radius R_0 of particles with E $\approx$100 MeV, i.e., $L_1 \approx 3 \cdot 10^8$ m. On this interval, E and R_0 change by factors of 10^2 and 10, $b_1 = 1/10$. A correlation scale of $L_1 \approx 3 \cdot 10^8$ m is close to the minimum extent of magnetic clouds that are formed from the granulation structure of the solar photosphere at the expansion of the solar wind [Zelenyi and Milovanov, 1993].

6. CONCLUSIONS

This paper compare the theoretical momentum dependence of the transport paths obtained from the nonlinear theory. As a result, we established the system of scales of a random interplanetary magnetic field taking into account a resonant increase in the transverse transport path. The structure of a random magnetic field is basically consistent with the concept of a multifractal topology of a random magnetic field in the heliosphere.

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