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Great SEP events and space weather: 2. On-line determination of solar energetic particle spectrum, diffusion coefficient in the interplanetary space, time of ejection and source function

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In paper [1] was described how works automatically the program "SEP-Search", determined on the basis of on-line one-minute NM data the onset of great SEP event and formatted in our website the Alert: No or YES. If YES, automatically starts to work on line the program "SEP-Research/Spectrum" based on the coupling functions method. We consider two variants: 1) quiet period (no change in cut-off rigidity), 2) disturbed period (characterized with possible changing of cut-off rigidity). We show that after determining of SEP spectrum out of the atmosphere can be determined the time of SEP ejection into solar wind, diffusion coefficient in the interplanetary space and source function, described by corresponding programs "SEP-Research/Time of Ejection", "SEP-Research/Diffusion", and "SEP-Research/Source". The work of NM on Mt. Hermon is supported by Israel (Tel Aviv University) – Italian (University Rome-3 and IFSI-CNR) Collaboration.

1. Analytical approximation for coupling functions

Based on the latitude survey data [2-4] the polar normalized coupling functions for any secondary CR component of type *m* can be approximated by the so called Dorman function (introduced in [5]):

$$W_{om}(R) = a_m k_m R^{-(k_m+1)} \exp\left(-a_m R^{-k_m}\right), \tag{1}$$

where for neutron super-monitor at the level of air pressure h_o will be $a_m(h_o)$, $k_m(h_o)$, m = tot, 1, 2, 3, … for total neutron intensity and different multiplicities; for muon telescopes with different zenith angles θ will be $a_m(h_o,\theta)$ and $k_m(h_o,\theta)$. Let us note that functions described by Eq. 1 are normalized: $\int_0^\infty W_{om}(R)dR = 1$ at any values of a_m and k_m. The normalized coupling functions for point with cut-off rigidity R_c, will be

$$W_m(R_c,R) = a_m k_m R^{-(k_m+1)}\left(1 - a_m R_c^{-k_m}\right)^{-1} \exp\left(-a_m R^{-k_m}\right), \text{ if } R \ge R_c, \text{ and } W_m(R_c,R) = 0, \text{ if } R < R_c. \tag{2}$$

2. The first approximations of the SEP energy spectrum

In the first approximation the spectrum of primary variation of SEP event can be described by the function

$$\Delta D(R)/D_o(R) = bR^{-\gamma}, \tag{3}$$

where $\Delta D(R) = D(R,t) - D_o(R)$, $D_o(R)$ is the differential spectrum of galactic cosmic rays before the SEP event and $D(R,t)$ is the spectrum at a later time *t*. In Eq. 3 parameters *b* and γ depend on *t*. Eq. (3) can be used for describing a limited interval of energies in the sensitivity range detected by the various components.

3. On-line determining of the SEP spectrum from data of single observatory in the case of magnetically quiet periods

It is inverse problem which can be solved by spectrographic method [6, 7] using coupling functions. The observed variation $\delta I_m(R_c) \equiv \Delta I_m(R_c)/I_{mo}(R_c)$ will be described by function $F_m(R_c,\gamma)$:

$$\delta I_m(R_c) = bF_m(R_c,\gamma) \tag{4}$$

where

$$F_m(R_c,\gamma) = a_m k_m \left(1 - \exp\left(-a_m R_c^{-k_m}\right)\right)^{-1} \int_{R_c}^{\infty} R^{-(k_m+1+\gamma)} \exp\left(-a_m R^{-k_m}\right) dR \tag{5}$$

is a known function. Let us compare data for two components *m* and *n*. According to Eq. 4 we obtain

$$\delta I_m(R_c)/\delta I_n(R_c) = \Psi_{mn}(R_c,\gamma), \text{ where } \Psi_{mn}(R_c,\gamma) = F_m(R_c,\gamma)/F_n(R_c,\gamma) \tag{6}$$

is calculated using Eq. 5. Comparison of experimental results with function $\Psi_{mn}(R_c,\gamma)$ according to Eq. 6 gives the value of γ, and then from Eq. 4 the value of the parameter *b*. The observed SEP increase for different components allows the determination of parameters *b* and γ for the SEP event beyond the Earth's atmosphere and magnetosphere.

4. On-line determining of the SEP spectrum from data of single observatory in the case of magnetically disturbed periods

For magnetically disturbed periods the observed CR variation instead of Eq. 4 will be described by

$$\delta I_k(R_c) = -\Delta R_c W_k(R_c,R_c) + bF_k(R_c,\gamma), \tag{7}$$

where ΔR_c is the change of cut-off rigidity due to change of the Earth's magnetic field, and $W_k(R_c,R_c)$ is determined according to Eq. 2 at $R = R_c$:

$$W_m(R_c,R_c) = a_m k_m R_c^{-(k_m+1)} \left(1 - a_m R_c^{-k_m}\right)^{-1} \exp\left(-a_m R_c^{-k_m}\right). \tag{8}$$

Now for the first approximation of the SEP energy spectrum we have unknown variables γ, *b*, ΔR_c, and for their determination we need data from at least 3 different components *k=l, m, n* in Eq. 7. In accordance with the spectrographic method [6] let us introduce the function

$$\Psi_{lmn}(R_c,\gamma) = \frac{W_l(R_c,R_c)F_m(R_c,\gamma) - W_m(R_c,R_c)F_l(R_c,\gamma)}{W_m(R_c,R_c)F_n(R_c,\gamma) - W_n(R_c,R_c)F_m(R_c,\gamma)}. \tag{9}$$

Then from equation

$$\Psi_{lmn}(R_c,\gamma) = \frac{W_l(R_c,R_c)\delta I_m(R_c) - W_m(R_c,R_c)\delta I_l(R_c)}{W_m(R_c,R_c)\delta I_n(R_c) - W_n(R_c,R_c)\delta I_m(R_c)}, \tag{10}$$

the value of γ can be determined. Using the found value of γ, for each time *t*, we determine

$$\Delta R_c = \frac{F_l(R_c,\gamma)\delta I_m(R_c) - F_m(R_c,\gamma)\delta I_l(R_c)}{F_m(R_c,\gamma)\delta I_n(R_c) - F_n(R_c,\gamma)\delta I_m(R_c)}, \quad b = \frac{W_l(R_c,R_c)\delta I_m(R_c) - W_m(R_c,R_c)\delta I_l(R_c)}{W_l(R_c,R_c)F_m(R_c,\gamma) - W_m(R_c,R_c)F_l(R_c,\gamma)}. \tag{11}$$

So, in magnetically disturbed periods the observed SEP increases for different components also allows the determination of parameters γ and *b,* for the SEP spectrum beyond the Earth's atmosphere as well as ΔR_c, giving information on the magnetospheric ring currents.

5. On-line determining of the SEP spectrum from data of two observatories in the case of magnetically disturbed periods

In this case instead of Eq. 7 we will have for stations 1 and 2 following four equations:

$$\delta I_k(R_{c1}) = -\Delta R_{c1} W_k(R_{c1}, R_{c1}) + bF_k(R_{c1}, \gamma),\ \delta I_l(R_{c1}) = -\Delta R_{c1} W_l(R_{c1}, R_{c1}) + bF_l(R_{c1}, \gamma); \quad (12)$$

$$\delta I_m(R_{c2}) = -\Delta R_{c2} W_m(R_{c2}, R_{c2}) + bF_m(R_{c2}, \gamma),\ \delta I_n(R_{c2}) = -\Delta R_{c2} W_n(R_{c2}, R_{c2}) + bF_n(R_{c2}, \gamma), \quad (13)$$

In this case we will have 4 unknown variables: γ, *b*, ΔR_{c1}, ΔR_{c2}. It is possible to exclude *b*, ΔR_{c1}, ΔR_{c2} from the system of Eq. 12-13 and finally obtain a non-linear equation for determining γ:

$$\frac{W_k \delta I_l(R_{c1}) - W_l \delta I_k(R_{c1})}{W_m \delta I_n(R_{c2}) - W_n \delta I_m(R_{c2})} = \Psi_{klmn}(R_{c1}, R_{c2}, \gamma)\ , \quad (14)$$

where

$$\Psi_{klmn}(R_{c1}, R_{c2}, \gamma) = \frac{W_k F_l(R_{c1}, \gamma) - W_l F_k(R_{c1}, \gamma)}{W_m F_n(R_{c2}, \gamma) - W_n F_m(R_{c2}, \gamma)} \quad (15)$$

is a special function that can be calculated for any pair of stations with cut-off rigidities R_{c1} and R_{c2}, using known functions $F_k(R_{c1}, \gamma)$, $F_l(R_{c1}, \gamma)$, $F_m(R_{c2}, \gamma)$, $F_n(R_{c2}, \gamma)$ (calculated from Eq. 5), and known values $W_k \equiv W_k(R_{c1}, R_{c1})$, $W_l \equiv W_l(R_{c1}, R_{c1})$, $W_m \equiv W_m(R_{c2}, R_{c2})$, and $W_n \equiv W_n(R_{c2}, R_{c2})$ (calculated from Eq. 8). After determining γ we can solve for the three other unknown variables:

$$\Delta R_{c1} = \frac{F_k(R_{c1}, \gamma)\delta I_l(R_{c1}) - F_l(R_{c1}, \gamma)\delta I_k(R_{c1})}{W_k F_l(R_{c1}, \gamma) - W_l F_k(R_{c1}, \gamma)},\ \Delta R_{c2} = \frac{F_m(R_{c2}, \gamma)\delta I_n(R_{c2}) - F_n(R_{c2}, \gamma)\delta I_m(R_{c2})}{W_m F_n(R_{c2}, \gamma) - W_n F_m(R_{c2}, \gamma)} \quad (16)$$

$$b = \frac{W_k \delta I_l(R_{c1}) - W_l \delta I_k(R_{c1})}{W_k F_l(R_{c1}, \gamma) - W_l F_k(R_{c1}, \gamma)} = \frac{W_m \delta I_n(R_{c2}) - W_n \delta I_m(R_{c2})}{W_m F_n(R_{c2}, \gamma) - W_n F_m(R_{c2}, \gamma)}. \quad (17)$$

6. On line determining time of SEP ejection, diffusion coefficient and source function

Let us assume that by described above methods the SEP rigidity spectrum $N_i(R)$ is on-line determined at least at three different moments of time T_1, T_2 and T_3 (in UT). In this case for moments of time after SEP ejection into solar wind we obtain:

$$t_1 = T_1 - T_e = x,\ \ t_2 = T_2 - T_e = T_2 - T_1 + x,\ \ t_3 = T_3 - T_e = T_3 - T_1 + x\ , \quad (18)$$

where T_2 - T_1 and T_3 - T_1 are known values and $x = T_1 - T_e$ is unknown value to be determined. From three equations for t_i (*i* = 1, 2, 3) of SEP propagation

$$N_i(R, r, t) = N_o(R) \times \left[2\pi^{1/2}\left(K(R)t_i\right)^{3/2}\right]^{-1} \times \exp\left(-\frac{r^2}{4K(R)t_i}\right), \quad (19)$$

by taking into account Eq. (18) and dividing one equation on other for excluding unknown parameter $N_o(R)$, we obtain two equations for determining unknown *x* and $K(R)$:

$$\frac{T_2-T_1}{x(T_2-T_1+x)}=-\frac{4K(R)}{r_1^2}\ln\left\{\frac{N_1(R)}{N_2(R)}\left(\frac{x}{T_2-T_1+x}\right)^{3/2}\right\},\frac{T_2-T_1}{x(T_2-T_1+x)}=-\frac{4K(R)}{r_1^2}\ln\left\{\frac{N_1(R)}{N_2(R)}\left(\frac{x}{T_2-T_1+x}\right)^{3/2}\right\}$$

From these two equations we exclude unknown parameter $K(R)$ and obtain equation for determining x:

$$x=\left[(T_2-T_1)\Psi-(T_3-T_1)\right]/(1-\Psi), \qquad (20)$$

where

$$\Psi=\left[(T_3-T_1)/(T_2-T_1)\right]\times\left\{\ln\left[N_1(R)(x/(T_2-T_1+x))^{3/2}/N_2(R)\right]/\ln\left[N_1(R)(x/(T_3-T_1+x))^{3/2}/N_3(R)\right]\right\}. \qquad (21)$$

Eq. (20) can be solved by the iteration method: as a first approximation, we can use $x_1=T_1-T_e\approx 500$ sec which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit. Then, by Eq. (21) we determine $\Psi(x_1)$ and by Eq. (20) we determine the second approximation x_2 and so on. After determining the time of ejection, we can compute very easily diffusion coefficient and source function:

$$K(R)=-\frac{r_1^2(T_2-T_1)/4x(T_2-T_1+x)}{\ln\left\{\frac{N_1(R)}{N_2(R)}(x/(T_2-T_1+x))^{3/2}\right\}}=-\frac{r_1^2(T_3-T_1)/4x(T_3-T_1+x)}{\ln\left\{\frac{N_1(R)}{N_3(R)}(x/(T_3-T_1+x))^{3/2}\right\}}. \qquad (22)$$

$$N_o(R)=2\pi^{1/2}N_1(R)\times(K(R)x)^{3/2}\exp\left(r_1^2/(4K(R)x)\right)=2\pi^{1/2}N_2(R)\times(K(R)(T_2-T_1+x))^{3/2}\times$$
$$\times\exp\left(r_1^2/(4K(R)(T_2-T_1+x))\right)=2\pi^{1/2}N_3(R)\times(K(R)(T_3-T_1+x))^{3/2}\exp\left(r_1^2/(4K(R)(T_3-T_1+x))\right). \qquad (23)$$

7. Appendix: calculations of coupling and special functions for the problem of SEP spectrum determining on the basis of ground observations

7.1. On the basis of latitude surveys [2-4] and theoretical calculations [8, 9], the dependence of a_m and k_m in Eq. (1) from the average station pressure h (in bar) and solar activity level characterized by the logarithm of CR intensity (we used here monthly averaged of Climax NM $\ln(I_{Cl})$) can be approximated by the functions:

$$a_{tot}=\left(-2.9150h^2-2.2368h-8.6538\right)\ln(I_{Cl})+\left(24.5842h^2+19.4600h+81.2299\right), \qquad (A1)$$

$$k_{tot}=\left(0.1798h^2-0.8487h+0.7496\right)\ln(I_{Cl})+\left(-1.4402h^2+6.4034h-3.6975\right), \qquad (A2)$$

$$a_m=\left[\left(-2.92h^2-2.24h-8.638\right)\ln(I_{Cl})+\left(24.584h^2+19.46h+81.23\right)\right]\left(0.987m^2+0.225m+6.913\right)/9.781 \qquad (A3)$$

$$k_m=\left[\left(0.1798h^2-0.8487h+0.7496\right)\ln(I_{Cl})+\left(-1.4402h^2+6.4034h-3.6975\right)\right]\left(0.0808m+1.8186\right)/1.940 \qquad (A4)$$

Instead of Climax, other stations can be used in Eq. A1-A4 with the appropriate coefficients. For example, the coefficients for ESO 6NM-64 on Mt. Hermon are

$$\ln(I_{Cl})=2.1612\times\ln(I_{ESO})-9.6646 \qquad (A5)$$

with correlation coefficient 0.899 based on comparison of $\ln(I_{Cl})$ and $\ln(I_{ESO})$ for about two years.

7.2. The integrals $F_m(R_c,\gamma)$ of Eq. 5 are calculated numerically for values of γ from −1 to +7, for different average station air pressure h, cut-off rigidities R_c and different levels of solar activity characterized by the value $\ln(I_{Cl})$ according to

$$F_m(R_c,\gamma)=a_m k_m\left(1-\exp\left(-a_m R_c^{-k_m}\right)\right)^{-1}R_c^{-k_m-\gamma}\sum_{n=0}^{\infty}(-1)^n a_m^n R_c^{-nk_m}\left(n!((n+1)k_m+\gamma)\right)^{-1}. \qquad (A6)$$

Results of calculations of Eq. A6 at different values of γ from -1 to +7 shows that integrals $F_m(R_c,\gamma)$ can be approximated, with correlation coefficients between 0.993 and 0.996, by the function

$$F_m(R_c,\gamma,h,\ln(I_{Cl})) = A_m(R_c,h,\ln(I_{Cl}))\exp(-\gamma B_m(R_c,h,\ln(I_{Cl}))). \quad \text{(A7)}$$

7.3. Special functions determined by Eq. 6, 9 and 16 now by using Eq. A7 becomes

$$\Psi_{mn}(R_c,\gamma) = F_m(R_c,\gamma)/F_n(R_c,\gamma) = (A_m/A_n)\exp(-\gamma(B_m - B_n)), \quad \text{(A8)}$$

$$\Psi_{lmn}(R_c,\gamma) = \frac{W_l A_m \exp(-B_m\gamma) - W_m A_l \exp(-B_l\gamma)}{W_m A_n \exp(-B_n\gamma) - W_n A_m \exp(-B_m\gamma)}, \quad \text{(A9)}$$

$$\Psi_{klmn}(R_{c1},R_{c2},\gamma) = \frac{W_k A_l \exp(-B_l\gamma) - W_l A_k \exp(-B_k\gamma)}{W_m A_n \exp(-B_n\gamma) - W_n A_m \exp(-B_m\gamma)}. \quad \text{(A10)}$$

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