Parallel and Perpendicular Transport of Charged Particles in the Solar System

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Abstract: A key problem of cosmic ray astrophysics is the explanation of measured parallel and perpendicular mean free paths in the heliosphere. Previous approaches used quasilinear theory in combination with simple turbulence models to reproduce heliospheric observations. Because of recent progress in transport and turbulence theory we present linear and nonlinear diffusion coefficients within an improved dynamical turbulence model to demonstrate that the observed mean free paths can indeed be reproduced theoretically.

Introduction

Transport of charged cosmic rays in the interplanetary space was discussed by many authors (e.g. Jokipii 1966, Bieber et al. 1994) and remains an interesting and important field of astrophysical research. One theoretical challenge is the understanding of observed mean free paths of the cosmic particles which experience scattering parallel and perpendicular to the magnetic field of the sun $\vec{B}_0$.

In this article we compare different theoretical results for parallel diffusion with the Palmer consensus (Palmer 1982) and pickup ion observations (Gloeckler et al. 1995, Möbius et al. 1998). Theoretical results for perpendicular diffusion are compared with the Palmer consensus (Palmer 1982), Jovian electrons (Chenette et al. 1977), and Ulysses measurements of Galactic protons (Burger et al. 2000).

If a diffusion coefficient is calculated theoretically, the turbulence properties have to be specified by specifying the correlation tensor: $P_{lm}(\vec{k},t) = \left\langle \delta B_l(\vec{k},t)\delta B^*_m(\vec{k},0)\right\rangle$ which is determined by the wave spectrum (wavenumber dependence of P_{lm}), the turbulence geometry (orientation of $\vec{k}$ relative to the background field $\vec{B}_0$), and the time-dependence of $P_{lm}(\vec{k},t)$. To specify the wavespectrum for instance, we can use observations (e.g. Denskat & Neubauer 1982). Such a measured spectrum can be divided into three intervals which can easily be distinguished: for small wavenumber we find a flat spectrum which can be approximated by a constant (energy-range), for intermediate wavenumbers we find a kolmogorov-like behaviour ($\sim k^{-5/3}$, inertial-range), and for large wavenumbers a steep behaviour can be seen ($\sim k^{-3}$, dissipation-range). Also the turbulence geometry can be obtained from measurements. According to Bieber et al. (1994), a composite model which consists of a superposition of a slab model ($\vec{k} \parallel \vec{B}_0$) and a 2D model ($\vec{k} \perp \vec{B}_0$) should be appropriate. Bieber et al. (1994) suggested that 20% slab and 80% 2D should be realistic. More difficult to specify is the time-dependence. By introducing the dynamical correlation function $\Gamma(\vec{k},t)$, the correlation tensor can be written as $P_{lm}(\vec{k},t) = P_{lm}(\vec{k})\Gamma(\vec{k},t)$. Prominent models for $\Gamma(\vec{k},t)$ are the magnetostatic model ($\Gamma(\vec{k},t) = 1$), the plasma wave model ($\Gamma(\vec{k},t) = e^{i\omega t}, \omega =$plasma wave dispersion relation), and dynamical turbulence models (e.g. $\Gamma(\vec{k},t) = e^{-t/\tau}, \tau =$correlation timescale). Furthermore, the turbulence parameters have to be specified. As shown in Table 1, the most parameters can be obtained from observa-

Parameter	Symbol/Value
IR spectral index	$2\nu = 5/3$
DR spectral index	$p = 3$
Alfvén speed	$v_A = 33.5\ km/s$
Mean field	$B_0 = 4.12\ nT$
Turbulence strength	$\delta B / B_0 = 1$
Slab fraction	$\delta B_{slab}^2 = 0.2 \cdot \delta B^2$
2D fraction	$\delta B_{2D}^2 = 0.8 \cdot \delta B^2$
Slab bendover scale	$l_{slab} = 0.030\ AU$
Slab DR wavenumber	$k_{slab} = 3 \cdot 10^6\ (AU)^{-1}$
2D bendover scale	$l_{2D} = 0.1 \cdot l_{slab}$
2D DR wavenumber	$k_{2D} = 3 \cdot 10^6\ (AU)^{-1}$

Table 1: The turbulence parameters used for our calculations. These values should be appropriate for 1 AU heliocentric distance. IR stands for inertial-range and DR for dissipation-range.

tions. In the following we discuss different previous approaches which were proposed to reproduce heliospheric observations of the mean free paths.

The standard quasilinear approach

An early treatment of particle transport employed the standard quasilinear theory (SQLT, Jokipii 1966) where a simplified turbulence model was combined with the quasilinear approach. This turbulence model assumes magnetostatic slab turbulence and a wave spectrum without dissipation-range. To examine their validity, the SQLT-results can be compared with test particle simulations (e.g. Qin et al. 2002a). Whereas the results for parallel diffusion can be confirmed by these simulations, the results for perpendicular diffusion cannot be confirmed. In the simulations and in recent nonlinear considerations (e.g. Shalchi 2005) a subdiffusive behaviour of perpendicular transport was discovered, which disagrees with the diffusive result of QLT. Therefore, we come to the conclusion that QLT is not appropriate for perpendicular transport.

Palmer (1982) compared the predictions of SQLT for the parallel mean free path with heliospheric observations and noted two major problems:
1) the observed parallel mean free paths are typically much larger than the predicted SQLT results (magnitude problem);
2) the observed parallel mean free paths are generally constant with a rigidity independent mean free path for 0.5 to 5000 MV, but SQLT predicts that the mean free path should increase with increasing rigidity (flatness problem).

The turbulence model of Bieber et al. 94

Because of the disagreement between SQLT and the observed parallel mean free paths, Bieber et al. (1994) proposed an improved turbulence model:
1) They replaced the simple static model by two different dynamical turbulence models. In the damping model of dynamical turbulence the dynamical correlation function is $\Gamma(\vec{k}, t) = \exp(-\alpha v_A \mid k \mid t)$ and in the random sweeping model $\Gamma(\vec{k}, t) = \exp(-(\alpha v_A k t)^2)$. In both models a parameter α was introduced to adjust the strength of dynamical effects.
2) In agreement with observations, they replaced the slab model by a 20% slab / 80% 2D composite model.
3) They assumed that the 2D contribution to parallel scattering can be neglected. A justification for this assumption was not given in the original Bieber at al. (1994) paper, but by using analytical methods this assumption was confirmed by Shalchi & Schlickeiser (2004a).
4) They used a realistic wave spectrum with energy-, inertial- and dissipation-range in agreement with observations.

As demonstrated in several articles (Bieber et al. 1994, Dröge 2000, Shalchi & Schlickeiser 2004a), a combination of QLT with the damping model of dynamical turbulence is able to reproduce the observed parallel mean free path. However, there are several problems assoziated with the Bieber et al. (1994) approach. First, the form $\Gamma(\vec{k}, t) = \exp(-\alpha v_A \mid k \mid t)$ and the parameter α cannot be derived theoretically. Furthermore, plasma wave effects are neglected in the damping and random sweeping model. The most serious problem is that the observed perpendicular mean free paths cannot be reproduced by combining QLT with such dynamical turbulence models (Shalchi & Schlickeiser, 2004b).

The NADT-model

To solve these problems we recently proposed a new turbulence model, which we call the "Non-linear Anisotropic Dynamical Turbulence model" (NADT-model, Shalchi et al. 2006). In this model we still assume composite geometry and the wavespectrum used in Bieber et al. (1994), but we assumed different forms of the slab and the 2D dynamical correlation functions: $P_{lm}(\vec{k},t) = P^{slab}_{lm}(\vec{k})\Gamma^{slab}(k_\parallel,t) + P^{2D}_{lm}(\vec{k})\Gamma^{2D}(k_\perp,t)$.

In earlier treatments of dynamical turbulence, the decorrelation factors $\Gamma^i(\vec{k},t)$ were established using simple approximations to the interactions responsible for temporal decorrelation of excitations near wave vector $\vec{k}$. In random sweeping and damping models, for example, a single parameter is introduced to estimate the rate of decorrelation at scale $1/k$ and this is assumed to be related to the Alfvén speed v_A. To improve these models, we note that in recent years there has been a more complete understanding of the time scales of MHD turbulence (e.g. Zhou et al. 2004), and the relation these may have to interactions between excitations that may be associated with either low frequency or wavelike components of the turbulence spectrum (Matthaeus et al. 1990, Tu & Marsch, 1993, Oughton et al. 2006). In the context of the two component model, these ideas may be used to determine reasonable approximations to the functions $\Gamma^{slab}(\vec{k},t)$ and $\Gamma^{2D}(\vec{k},t)$ (for details see section 2.2 of Shalchi et al. 2006):

$$\begin{aligned}\Gamma^{slab}(k_\parallel,t) &= e^{-t/\tau_{slab}}\cdot e^{i\omega t},\\ \Gamma^{2D}(k_\perp,t) &= e^{-t/\tau_{2D}}\end{aligned} \quad (1)$$

with the dispersion relation of shear Alfvén waves $\omega = v_A k_\parallel$, the slab correlation time-scale

$$\tau^{-1}_{slab} = \sqrt{2}\frac{v_A}{l_{2D}}\frac{\delta B_{2D}}{B_0} \quad (2)$$

and the 2D correlation time-scale

$$\tau^{-1}_{2D} = \sqrt{2}\frac{v_A}{l_{2D}}\frac{\delta B_{2D}}{B_0} \times \begin{cases} 1 & \text{for} \quad k_\perp l_{2D} \leq 1 \\ (k_\perp l_{2D})^{2/3} & \text{for} \quad k_\perp l_{2D} \geq 1 \end{cases} \quad (3)$$

The NADT-model is defined through Eqs. (1) - (3).

Another problem is the invalidity of QLT for per-

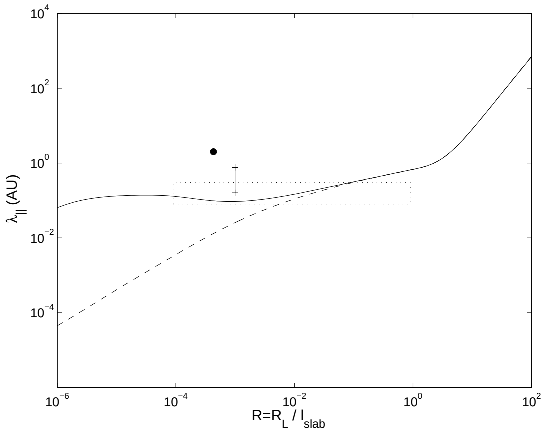

Figure 1: The parallel mean free path $\lambda_\parallel$ versus $R = R_L/l_{slab}$ (R_L =Larmor-radius, l_{slab} =slab bendover scale) obtained within the NADT-model. Shown are QLT results for electrons (solid line) and protons (dashed line) in comparison with the Palmer consensus (Palmer 1982, box), Ulysses observations (Gloeckler et al. 1995, dot) and AMPTE spacecraft observations (Möbius et al. 1998, vertical line).

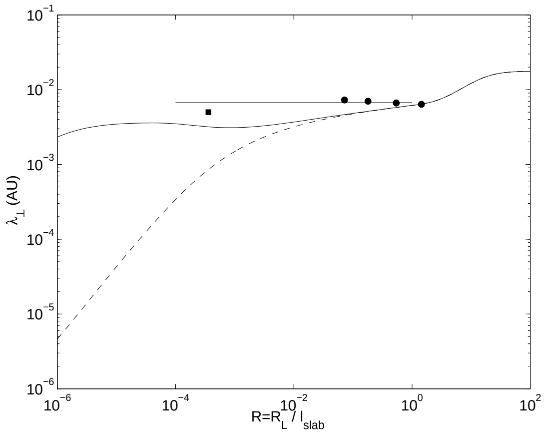

Figure 2: The perpendicular mean free path $\lambda_\perp$ versus $R = R_L/l_{slab}$ obtained within the NADT-model. Shown are the NLGC-results for electrons (solid line) and protons (dashed line) in comparison with the Palmer consensus (1982, horizontal line), Jovian electrons (Chenette et al. 1977, square) and Ulysses measurements of Galactic protons (Burger et al. 2000, dots).

pendicular transport. By using test-particle simulations, it can be demonstrated that diffusion is recovered if the slab model is replaced by a composite model (see Giacalone & Jokipii 1999; Qin et al. 2002b). Within QLT, however, we find superdiffusive transport for composite geometry (see Shalchi & Schlickeiser 2004b). So far only two theories are able to achieve agreement with these simulations: the NLGC-theory of Matthaeus et al. 2003 and the weakly nonlinear theory (WNLT) of Shalchi et al. 2004. Although the WNLT has some advantages (e.g. one theory for parallel and perpendicular diffusion) we employ the NLGC-approach because this theory is more tractable. For our calculations we used the parameters illustrated in Table 1.

According to Figs. 1 and 2, a combination of the NADT-model, QLT and NLGC-theory can explain the observed parallel and perpendicular mean free paths in the heliosphere. The discrepancy between the different observations can easily be understood: the Gloeckler et al. (1995) result for instance, was at a heliocentric distance of 2.34 AU, whereas the other observations were at 1 AU. Although the deviation between the damping model and NADT-model results is small for parallel diffusion, there are some differences between these two models (for a detailed discussion see Shalchi et al. 2006). Within the NADT-model for instance, we can no longer neglect the 2D contribution to scattering. Thus, the unknown ratio l_{2D}/l_{slab} has a strong influence upon the mean free paths.

Conclusion and future work

As demonstrated, the NADT-model in combination with QLT for parallel diffusion and NLGC-theory for perpendicular diffusion can reproduce the heliospheric observations (see Figs. 1 and 2).

In recent articles (e.g. Shalchi et al. 2004), however, it was demonstrated that nonlinear effects are in general also important if the parallel mean free path is calculated. Although the nonlinear effects are strong for the turbulence parameters considered in the simulations, QLT could be recovered for parallel diffusion for other parameter regimes (e.g. $l_{2D} \approx l_{slab}$) and if we assume dynamical turbulence. However, one has to prove this statement by applying more appropriate transport theories for parallel diffusion such as WNLT.

References

[1] Bieber, J. W., Matthaeus, W. H., Smith, C. W., et al. 1994, ApJ 420, 294

[2] Burger, R. A., Potgieter, M. S. & Heber, B., 2000, J. Geophys. Res., 105, 27447

[3] Chenette, D. L., Conlon, T. F., Pyle, K. R. & Simpson, J. A., 1977, ApJ., 215, L95

[4] Denskat, K. U. & Neubauer, F. M., 1982, JGR, 87, 2215

[5] Dröge, W., 2000, ApJ, 537, 1073

[6] Giacalone, J. & Jokipii, J. R., 1999, ApJ, 520, 204

[7] Gloeckler, G., Schwadron, N. A., Fisk, L. A. & Geiss, J., 1995, Geophys. Res. Lett., 22, 2665

[8] Jokipii, J. R., 1966, ApJ, 146, 480

[9] Matthaeus, W. H., Goldstein, M. & Roberts, D. A., 1990, J. Geophys. Res., 95, 20673

[10] Matthaeus, W. H., Qin, G., Bieber, J. W., et al. 2003, ApJ, 590, L53

[11] Möbius, E., Rucinski, D., Lee, M. A. & Isenberg, P. A., 1998, J. Geophys. Res., 10, 257

[12] Oughton, S., Dmitruk, P. & Matthaeus, W. H., 2006, Physics of Plasmas 13, 042306

[13] Palmer, I. D., 1982, Rev. Geophys. Space Phys., 20, 2, 335

[14] Qin, G., Matthaeus, W. H. & Bieber, J. W., 2002a, Geophys. Res. Lett., 29

[15] Qin, G., Matthaeus, W. H. & Bieber, J. W., 2002b, ApJ, 578, L117

[16] Shalchi, A., Bieber, J. W., Matthaeus, W. H. & Qin, G., 2004, ApJ, 616, 617

[17] Shalchi, A. & Schlickeiser, R., 2004a, ApJ, 604, 861

[18] Shalchi, A. & Schlickeiser, R., 2004b, A&A, 420, 821

[19] Shalchi, A., 2005, J. Geophys. Res., 110, A09103

[20] Shalchi, A., Bieber, J. W., Matthaeus, W. H. & Schlickeiser, R., 2006, ApJ, 642, 230

[21] Tu, C.-Y. & Marsch, E., 1993, J. Geophys. Res., 98, 1257

[22] Zhou, Y., Matthaeus, W. H. & Dmitruk, P., 2004, Rev. Mod. Phys., 76, 1015