Effect of muon-nuclear inelastic scattering on high-energy atmospheric muon spectrum at large depth underwater

S. I. SINEGOVSKY[1], A. MISAKI[2], K. S. LOKHTIN[1] AND N. TAKAHASHI[3].
[1]*Irkutsk State University, Irkutsk, Russia*
[2]*Advanced Research Institute for Science and Technology, Waseda University, Tokyo, Japan*
[3] *Hirosaki University, Hirosaki, Japan*
misakiakeo@air.ocn.ne.jp

Abstract: The energy spectra of hadron cascade showers produced by the cosmic ray muons travelling through water as well as the muon energy spectra underwater at the depth up to 4 km are calculated with two models of muon inelastic scattering on nuclei, the recent hybrid model (two-component, $2C$) and the well-known generalized vector-meson-dominance model for the comparison. The $2C$ model involves photonuclear interactions at the low and moderate virtualities as well as the hard scattering including the weak neutral current processes. For the muon scattering off nuclei substantial nuclear effects, shadowing, nuclear binding and Fermi motion of nucleons are taken into account. It is shown that deep underwater muon energy spectrum calculated with the $2C$ model are noticeably distorted at energies above 100 TeV as compared to that obtained with the GVMD model.

Introduction

The muon inelastic scattering on nuclei contributes noticeably to the total energy loss of cosmic rays muons. The influence of this interaction on the shape of ultra-high energy muon spectra at the great depth of a rock/water is still unknown in detail. Of interest is also to estimate the number of cascade showers produced by very high-energy muons in inelastic interactions with nuclei and to study the influence of this process on the energy spectra of cosmic-ray muons in water at the depths of the underwater/ice neutrino telescopes – NT200+, AMANDA, ANTARES, NESTOR, IceCube and NEMO.

In this work, we compute the energy spectra of hadron cascade showers produced by cosmic-ray (atmospheric) muons in water by inelastic scattering on nuclei, as well as the integral energy spectra of atmospheric muons in water at depths up to 4 km. Calculations are performed with two models: the hybrid model of inelastic scattering of leptons on nuclei [1, 2] and, for a comparison, the known generalized vector-meson-dominance (GVMD) model of photonuclear muon interactions by Bezrukov and Bugaev [3].

Muon-nucleus inelastic scattering

The hybrid two-component ($2C$) model [1, 2] for inelastic scattering of high-energy charged leptons on nuclei involves photonuclear interactions at low and moderate Q^2 as well as the deep inelastic scattering processes at high Q^2 values. For the virtuality $0 < Q^2 \leq 5$ GeV^2 the Regge based parametrization [4] for the electromagnetic structure function F_2^γ is applied and the muon-nucleon inelastic scattering cross section at $Q^2 \leq 5$ GeV^2 is computed with the formula

$$\frac{d^2\sigma}{dQ^2dy} = \frac{4\pi\alpha^2}{yQ^4}F_2^\gamma(x,Q^2)\left[1-y-\frac{Q^2}{4E^2}+\right.$$
$$\left.\frac{y^2}{2(1+R)}\left(1-\frac{2m_\mu^2}{Q^2}\right)\left(1+\frac{Q^2}{E^2y^2}\right)\right], \quad (1)$$

where the ratio $R=\sigma_L/\sigma_T$ is taken into account according to Ref. [5], $y=\nu/E$ is the fraction of muon energy transfered to the hadron system, Q^2 and $x=Q^2/(2MEy)$ are the Bjorken variables.

Table 1: Comparison of calculations of the muon energy loss due to inelastic scattering in standard rock.

E, GeV	$b_n(E)$, 10^{-6} cm$^2\cdot$ g^{-1} [1, 2] ($2C$)	[14]	[13]	[15]	[16]
10^5	0.62	0.60	0.68	0.70	0.70
10^6	0.82	0.80	0.88	1.08	1.00
10^8	1.53	1.60	–	2.25	2.50
10^9	2.16	2.18	–	3.10	4.00

In the range $Q^2 > 5$ GeV^2 the cross section of μN-scattering can be written in the form

$$\frac{d^2\sigma}{dQ^2dy} = \frac{4\pi\alpha^2}{yQ^4}\left[F_2^{NC}\left(1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2} - \frac{y^2m_\mu^2}{Q^2}\right) \pm \left(\frac{y^2}{2} - y\right)xF_3^{NC}\right], \quad (2)$$

where we put $R = Q^2/\nu^2$, that is equivalent to the Callan-Gross relation, $F_2 = 2xF_1$. Signs "$\pm$" stand for $\mu^\pm$. In Eq. (2) used notations are:

$$F_2^{NC} = F_2^\gamma - g_V^\mu \eta_{\gamma Z} F_2^{\gamma Z} + ({g_V^\mu}^2 + {g_A^\mu}^2)\eta_{\gamma Z}^2 F_2^Z,$$
$$F_3^{NC} = -g_A^\mu \eta_{\gamma Z} F_3^{\gamma Z} + 2g_V^\mu g_A^\mu \eta_{\gamma Z}^2 F_3^Z; \quad (3)$$
$$\eta_{\gamma Z} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\frac{Q^2}{M_Z^2 + Q^2}, \quad (4)$$
$$g_V^\mu = -\frac{1}{2} + 2\sin^2\theta_W,\ g_A^\mu = -\frac{1}{2}. \quad (5)$$

The structure functions (SFs) F_2^Z, F_3^Z represent the weak neutral current (NC) contribution, $F_2^{\gamma Z}$, $F_3^{\gamma Z}$ are taking into account the electromagnetic and weak current interference. The nucleon SFs, F_2^γ, $F_2^{\gamma Z}$, F_2^Z, $F_3^{\gamma Z}$, F_3^Z, are defined in the quark-parton picture through the parton distributions (see [6, 7]). For the range of $Q^2 > 6$ GeV^2 the electroweak nucleon SFs are computed with the CTEQ6 [8] and MRST [9] sets of the parton distributions. Linear fits for the nucleon SFs are used in the range $5 < Q^2 < 6$ GeV^2.

Nuclear modifications of the nucleon SFs due to coherent and incoherent effects [10, 11] are taken into account according to Ref. [12] (see also [13]) by the factor $r_A(x, Q^2) = F_2^A/F_2^N$ that is the ratio of the structure function per nucleon of a nucleus with mass number A and the isospin averaged nucleon structure function F_2^N. Thus,

$$\frac{d\sigma^{\mu A}(E,y)}{dy} = A\int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2\, r_A \frac{d^2\sigma}{dQ^2dy}. \quad (6)$$

The contribution of muon-nuclear interactions to the continous energy loss is given by integral

$$b_n(E) \equiv -\frac{1}{E}\frac{dE}{dh} = N_0\int_{y_{\min}}^{y_{\max}} dy\, y\frac{d\sigma^{\mu A}}{dy}, \quad (7)$$

where $N_0 = N_A/A$ is the number of nuclei per gramme of matter.

Table 1 presents the muon energy loss due to muon-nucleus interactions in rock, computed with various models: the $2C$ model values of b_n are given in the second column, the rest are results predicted in Refs. [13, 14, 15, 16]. One may see that the muon energy loss $b_n(E)$ at $E > 10^6$ GeV obtained in Refs. [1, 2, 14] differ apparently from the predictions [15, 16].

The energy dependence of the inelastic scattering energy loss for muons traveling through water may be parametrized in the energy range $10^2 - 10^9$ GeV as

$$b_n(E) = c_0 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4, \quad (8)$$

where $\eta = \lg(E/1\,\mathrm{GeV})$ and coeffecients c_i (in units of 10^{-6} cm^{2}g^{-1}) are

$$c_0 = 1.06416, \quad c_1 = -0.64629,\ c_2 = 0.20394,$$
$$c_3 = -0.02465,\ c_4 = 0.00130. \quad (9)$$

Figure 1 shows the energy loss $b_n(E)$ for the inelastic scattering of muons in water calculated with the $2C$ model(solid curve) and the GVMD one (dashed).

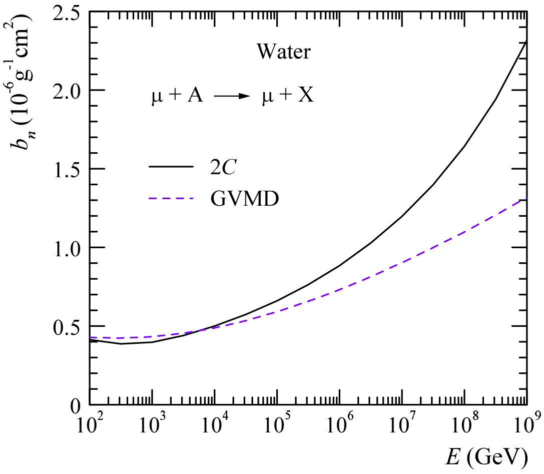

Figure 1: The muon energy loss for muon-nucleus interactions in water.

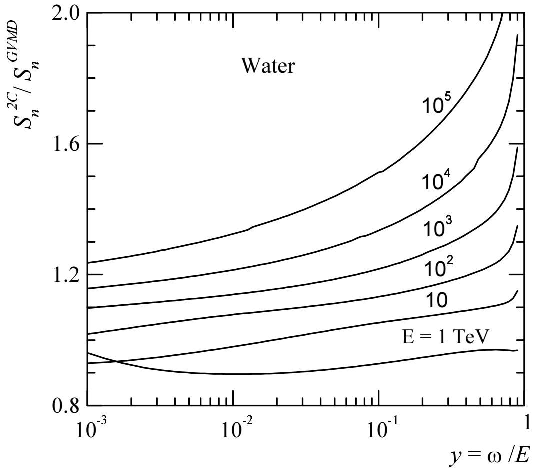

Figure 2: The ratio of the energy spectra of hadron showers computed with the $2C$ model to those obtained with use of the GVMD one.

Muon induced hadron showers in water

The number of hadron showers with energies above ω per cm^2 per second per steradian, generated in water column $\Delta h = h_2 - h_1$ through muon-nucleus interactions at muon energy above E, may be defined as

$$S_n(\omega, E, \Delta h, \theta) = N_0 \int_{h_1}^{h_2} dh \int_{E}^{\infty} d\varepsilon\, D_\mu(\varepsilon, h, \theta) \times \int_{\omega/\varepsilon}^{y_{\max}} dy\, \frac{d\sigma^{\mu A}(\varepsilon, y)}{dy}. \qquad (10)$$

Here $D_\mu(E, h, \theta)$ is the muon flux (cm^{-2}s^{-1}sr^{-1}GeV^{-1}) at depth h, θ is zenith angle, $y_{\max} = 1 - m_\mu/E$. The ratio of the spectra of showers computed with the $2C$ model to those obtained with the GVMD one is shown in figure 2. At the muon energy $E = 10$ TeV numbers of the muon-induced hadron showers, calculated with two models, the $2C$ and GVMD, differ slightly (about 10%). However the discrepancy between the models grows with increasing energy: for $E = 100$ TeV, it is as great as $\sim 30\,\%$, and for $E = 10^5$ TeV, the result obtained for the $2C$ model in the range of the large energy transfer exceeds by a factor of ~ 2.5 the corresponding result for the model of photonuclear interaction [3]. For energies $E > 10^3$ TeV, the number of showers calculated with the $2C$ model exceeds that obtained with use of the GVMD model by 20-50 % even in the region of small energy loss ($y \sim 0.1$). For the catastrophic energy loss ($y > 0.5$), the number of showers, obtained with using of the $2C$ model, exceeds that of the GVMD prediction by factor about 2.

Atmospheric muon spectra underwater

The solution of the muon transport equation allows [17] to obtain the differential energy spectra $D_\mu(E, h, \theta)$ at large depth in homogeneous media and therefore to compute the flux of cosmic ray muons with energy above specified one:

$$N_\mu(E, h, \theta) = \int_{E}^{\infty} d\varepsilon\, D_\mu(\varepsilon, h, \theta). \qquad (11)$$

Figure 3 shows the ratio $N_\mu^{2C}(E, h)/N_\mu^{GVMD}(E, h)$ of the near vertical muon flux underwater computed at depth 1–4 km with two models of muon-nuclear scattering, the 2C model [2] and GVMD one [3]. The effect is rather noticeable at the $E > 10$ TeV, especially for the depth 3–4 km. For $h = 4$ km this ratio decreases to 0.75 at $E = 10^3$ TeV. Thus a sizeable

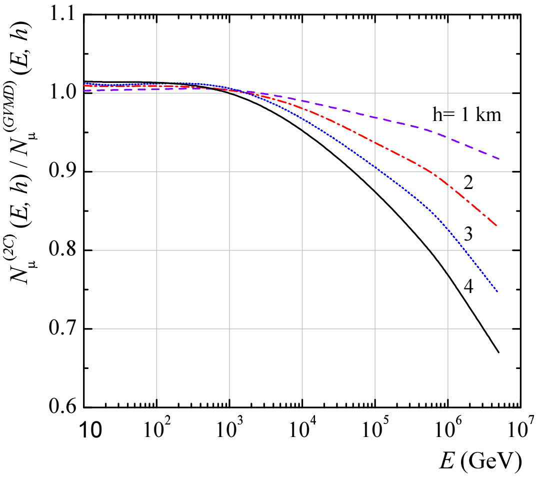

Figure 3: The ratio of muon fluxes underwater calculated with the $2C$ model to those obtained with the GMVD ones.

increase of the muon inelastic scattering cross section may results in an appreciable decrease of the deep underwater muon flux as compared to that obtained with the GVMD model [18, 19]. It should be noted that this result refers only to the atmospheric conventional (π, K) muons. As concerns muons produced in charmed particle decays (prompt muons), which become presumably dominant at $E > 10^5$ GeV (see e. g. [18, 19]), the role of the muon-nucleus inelastic scattering needs further study.

Conclusions

Evidently the increase of the cross section of inelastic muon scattering in matter, while leading to diminished cosmic-ray muon flux deep underwater, results in growing efficiency of muon registration. This last factor is positive for neutrino astronomy since neutrino-induced muons may yield the signal from astrophysical high-energy muon neutrinos.

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