Secondary electron spectrum in the upper atmosphere

Y. KOMORI[1], T. KOBAYASHI[2], K. YOSHIDA[3] AND J. NISHIMURA[4]
[1]*Kanagawa University of Human Services, Yokosuka, Japan*
[2]*Department of Physics and Mathematics, Aoyama gakuin University, Sagamihara, Japan*
[3]*Department of Electronic Information Systems, Shibaura Institute of Technology, Saitama, Japan*
[4]*Institute of Space and Astronautical Science, JAXA, Sagamihara, Japan*
komori-y@kuhs.ac.jp

Abstract: The secondary electron spectrum in the upper atmosphere($< 10\ \mathrm{g/cm^2}$) is investigated on the basis of the atmospheric gamma ray spectrum above 30 GeV obtained from our emulsion chamber experiments at balloon altitudes. We have to subtract these electrons produced by nuclear interactions from observed electrons to get the primary electron spectrum in the Galaxy. Thus it is required to precise estimates of secondary electron abundance and the study of variation of electron and gamma ray flux in the upper atmosphere. Both electron and gamma ray intensities are calculated using the one-dimensional cascade shower theory with the extra gamma ray spectrum produced by nuclear interactions. The result shows that the number of secondary electrons at depth $t\ \mathrm{g/cm^2}$ increases almost proportionally with t^2 and becomes nearly t % of that of gamma rays. The ratio of secondary to primary electrons rapidly increases in the TeV region, so that the high altitude is essential for TeV electron experiments.

Introduction

In balloon-borne electron experiments, primary electrons coming from interstellar space and secondary electrons produced in the residual atmosphere are observed together. Thus we have to subtract the secondary electrons from the observed electrons for estimating the primary electron spectrum. Secondary electrons above 30 GeV mostly originate in pair production process of atmospheric gamma rays produced in the hadronic interactions in the atmosphere.

The atmospheric gamma-ray spectrum J_γ has been simultaneously observed in our primary electron experiments as follows[4].

$$J_\gamma(E)|_{4g/cm^2}\ (\mathrm{m^2 \cdot s \cdot sr \cdot GeV})^{-1} = (1.12 \pm 0.13) \times 10^{-4} (\frac{E}{100\ \mathrm{GeV}})^{-2.73 \pm 0.06} \quad (1)$$

This spectrum is normalized at 4.0 g/cm^2 and only includes gamma rays in nuclear interactions since the gamma rays from primary electrons have already been subtracted.

Calculations

When we treat the electron and gamma ray energy above 30 GeV, we are able to use the one-dimensional cascade shower theory called Approximation A [2], which has a complete screening cross section of radiation and pair creation process and neglects the ionization loss. The cross section in this paper agrees with that used in Geant4 simulation code[1] within a few percents. We adopt the radiation length in air, 36.7 $\mathrm{g/cm^2}$.

The number of electrons $\pi(E,t)dE$ and gamma-rays $\gamma(E,t)dE$ satisfies the following simultaneous equations. The notations are seen in the Handbuch der Physik[2].

$$\begin{aligned} \frac{\partial \pi(E,t)}{\partial t} &= -A'\pi(E,t) + B'\gamma(E,t) \\ \frac{\partial \gamma(E,t)}{\partial t} &= C'\pi(E,t) - \sigma_0\gamma(E,t) \\ &\quad +\gamma_{ex}(E)\ , \end{aligned}$$

in which we add the extra gamma ray spectrum $\gamma_{ex}(E)dE \propto E^{-\beta-1}$ produced by nuclear interactions. It is assumed to be a constant in the up-

ICRC 2007 Proceedings - Pre-Conference Edition

ICRC 2007 Proceedings - Pre-Conference Edition

per atmosphere(< 10 g/cm^2) and estimated from eq. (1). The intial values are assumed as

$$\begin{aligned}\pi(E,0) &= 1.6\times 10^{-4}(\frac{E}{100\ \mathrm{GeV}})^{-3.3} \equiv \pi_0\\ \gamma(E,0) &= 0\end{aligned}$$

where primary electron spectrum is given by [3] and $\alpha + 1 = 3.3$.

The solution of electron intensity at the depth t becomes

$$\pi(E,t) = \pi(E,0)\zeta(t) + \gamma_{ex}(E)\xi(t) \qquad (2)$$

$$\begin{aligned}\zeta(t) &= \frac{[(\lambda_1+\sigma_0)e^{\lambda_1 t} - (\lambda_2+\sigma_0)e^{\lambda_2 t}]}{\lambda_1-\lambda_2}\\ \xi(t) &= \frac{B(\beta)}{\lambda_1-\lambda_2}[\frac{e^{\lambda_1 t}-1}{\lambda_1} - \frac{e^{\lambda_2 t}-1}{\lambda_2}]\end{aligned}$$

where $\zeta(t)$ represents the energy loss rate of electrons by bremsstrahrung in the atmosphere and $\xi(t)$ represents the pair production rate from gamma rays.

The atmospheric gamma-ray spectrum at depth t is given by

$$\gamma(E,t) = \pi(E,0)\eta_1(t) + \gamma_{ex}(E)\eta_2(t) \qquad (3)$$

$$\begin{aligned}\eta_1(t) &= \frac{C(\alpha)}{\lambda_1-\lambda_2}[e^{\lambda_1 t} - e^{\lambda_2 t}]\\ \eta_2(t) &= \frac{1}{\lambda_1-\lambda_2}[\frac{(\lambda_1+A(\beta))e^{\lambda_1 t} - A(\beta)}{\lambda_1}\\ &\quad -\frac{(\lambda_2+A(\beta))e^{\lambda_2 t} - A(\beta)}{\lambda_2}]\end{aligned}$$

where η_1 represents the gamma ray production rate from primary electrons and η_2 represents the attenuation rate of gamma rays by pair creation. The coefficients λ_1, λ_2 are given by

$$\begin{aligned}\lambda_1(s) &= \frac{1}{2}[-(A+\sigma_0)\\ &\quad +\{(A+\sigma_0)^2 - 4(A\sigma_0 - BC)\}^{\frac{1}{2}}]\\ \lambda_2(s) &= \frac{1}{2}[-(A+\sigma_0)\\ &\quad -\{(A+\sigma_0)^2 - 4(A\sigma_0 - BC)\}^{\frac{1}{2}}]\\ A(s) &= 1.36\frac{d}{ds}\ln\Gamma(s+2)\\ &\quad -\frac{1}{(s+1)(s+2)} - 0.075\end{aligned}$$

$$\begin{aligned}B(s) &= 2(\frac{1}{s+1} - \frac{1.36}{(s+2)(s+3)})\\ C(s) &= \frac{1}{s+2} + \frac{1.36}{s(s+1)}\\ \sigma_0 &= 0.773\end{aligned}$$

where s has a value of spectral index $\alpha = 2.3$ or $\beta = 1.73$.

The zenith angle distribution

The intensities of eq.(2) and eq.(3) are integrated over the zenith angle θ. At the depth t , the electron differential spectrum $j_{ob}(E,t)$ and the atmospheric gamma-ray spectrum $\gamma_{ob}(E,t)$ have a unit of $(\mathrm{m}^2\cdot\mathrm{sec}\cdot\mathrm{GeV})^{-1}$ and are given by

$$\begin{aligned}j_{ob}(E,t)/2\pi &= \int_0^\theta \pi(E,\frac{t}{\cos\theta})\cos\theta\sin\theta d\theta\\ &= \pi_0\cdot\zeta(\alpha,\theta,t) + \gamma_{ex}\cdot\xi(\beta,\theta,t)\end{aligned} \qquad (4)$$

$$\begin{aligned}\gamma_{ob}(E,t)/2\pi &= \int_0^\theta \gamma(E,\frac{t}{\cos\theta})\cos\theta\sin\theta d\theta\\ &= \pi_0\cdot\eta_1(\alpha,\theta,t) + \gamma_{ex}\cdot\eta_2(\beta,\theta,t)\end{aligned} \qquad (5)$$

The coefficients are calculated in the series as

$$\begin{aligned}\zeta(s,\theta,t) &= O_1(\alpha,\theta,t) + \sigma_0 O_2(\alpha,\theta,t)\\ \xi(s,\theta,t) &= B(\beta)O_3(\beta,\theta,t)\\ \eta_1(s,\theta,t) &= C(\alpha)O_2(\alpha,\theta,t)\\ \eta_2(s,\theta,t) &= O_2(\beta,\theta,t) + A(\beta)O_3(\beta,\theta,t)\end{aligned}$$

where function series are represented by

$$\begin{aligned}O_1(s,\theta,t) &= \sum_{n=0}^{\infty}\Lambda_n(s)T_n(\theta,t)\\ O_2(s,\theta,t) &= \sum_{n=0}^{\infty}\Lambda_n(s)T_{n+1}(\theta,t)\\ O_3(s,\theta,t) &= \sum_{n=0}^{\infty}\Lambda_n(s)T_{n+2}(\theta,t)\end{aligned}$$

$$T_0 = \frac{1-\cos^2\theta}{2},\quad T_1 = t(1-\cos\theta),$$

$$T_2 = \frac{t^2}{2!}\log(\frac{1}{\cos\theta}),\quad T_3 = \frac{t^3}{3!}(\frac{1}{\cos\theta} - 1),$$

$$\cdots\quad T_n = \frac{t^n}{n!}(\frac{1}{\cos^{n-2}\theta} - 1)\quad\cdots$$

$$\Lambda_0 = 1, \quad \Lambda_1 = \lambda_1 + \lambda_2,$$
$$\Lambda_2 = (\lambda_1 + \lambda_2)\Lambda_2 - \lambda_1\lambda_2\Lambda_1, \cdots$$
$$\Lambda_n = (\lambda_1 + \lambda_2)\Lambda_{n-1} - \lambda_1\lambda_2\Lambda_{n-2}, \quad \cdots$$

In the upper atmosphere(< 10 g/cm^2), the following expressions give a good approximation.

$$\begin{aligned} \zeta &\sim \frac{1-\cos^2\theta}{2} = const \\ \xi &\sim B(\beta)\cdot\frac{t^2}{2}\log\frac{1}{\cos\theta} \propto t^2 \\ \eta_1 &\sim C(\alpha)\cdot t\,(1-\cos\theta) \propto t \\ \eta_2 &\sim t\,(1-\cos\theta) \equiv \check{\eta}_2 \propto t \end{aligned}$$

The production rate of gamma rays is calculated from

$$2\pi\eta_2 \cdot \gamma_{ex}(E) = \Omega \cdot J_\gamma(E)|_{4g/cm^2}$$

where $\Omega = 2\pi(1-\cos\theta)$, and we should notice γ_{ex} is the production rate from nuclear interaction because eq.(1) does not include gamma rays produced from primary electrons.

$$\begin{aligned} \gamma_{ex}(E) &= (1.09 \pm 0.13) \times 10^{-3}\left(\frac{E}{100}\right)^{-2.73\pm0.06} \\ &\quad (\mathrm{m}^2\cdot\mathrm{sr}\cdot\mathrm{sec}\cdot\mathrm{GeV}\cdot\mathrm{c.u.})^{-1} \end{aligned}$$

Results

The secondary electron spectrum at the depth t is given by

$$j_{sec}(E,t) = 2\pi\xi\cdot\gamma_{ex}(E)\ (\mathrm{m}^2\cdot\mathrm{s}\cdot\mathrm{GeV})^{-1}. \quad (6)$$

The vertical secondary spectrum($= j_{sec}/\Omega$) is shown in Figure 1.

Atmospheric gamma ray spectrum is also given by

$$\gamma_{ob}(E,t) = 2\pi\eta_2\cdot\gamma_{ex}(E)\ (\mathrm{m}^2\cdot\mathrm{s}\cdot\mathrm{GeV})^{-1}. \quad (7)$$

The ratio of secondary electrons of eq. (6) to atmospheric gamma rays of eq. (7), ξ/η_2, is approximately proportional to the depth t. Figure 2 shows the constant of proportionality, $\xi/\eta_2\cdot(1/t)$ which value is around 0.01 at the zenith angle of 60°. Thus the secondary electron number at the depth t g/cm^2 is approximately t% of gamma rays

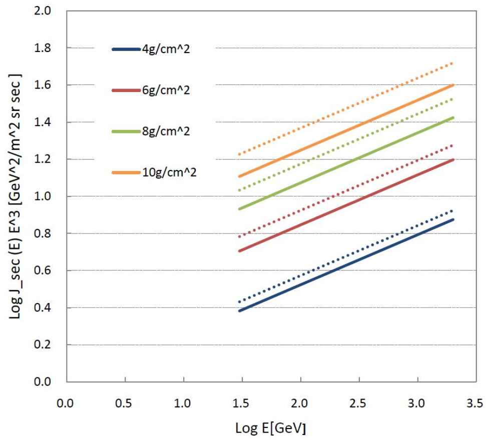

Figure 1: Secondary spectrum at different depths 4, 6, 8, 10 g/cm^2. Dotted lines are approximate expression of $j_\perp \sim B(1.73)\cdot t^2/2\cdot\gamma_{ex}(E)$ $= 2.33\times10^{-7}(t\ \mathrm{g/cm^2})^2 \times (E/100\ \mathrm{GeV})^{-2.73}$.

at the same depth t. Figure 3 shows the secondary/primary electron ratio at the upper atmosphere. For electron measurements above 1TeV with large zenith angle of 60°, if the balloon altitude is lower than 8 g/cm^2, secondary electrons exceeds primary electrons, so that we require a higher altigude than 8 g/cm^2.

Discussions

The electron spectrum at the top of the atmosphere is deduced from eq. (4) as

$$\pi(E,0) = \frac{j_{ob}(E,t)}{2\pi\zeta} - \gamma_{ex}(E)\frac{\xi}{\zeta}$$

The second term in the right side represents the contribution of secondary electrons,

$$\begin{aligned} \hat{j}_{\mathrm{sec}}(E,t) &\equiv \gamma_{ex}(E)\frac{\xi(\beta,\theta,t)}{\zeta(\alpha,\theta,t)} \\ &\quad (\mathrm{m}^2\cdot\mathrm{sr}\cdot\mathrm{sec}\cdot\mathrm{GeV})^{-1} \end{aligned}$$

In the balloon experiments of electrons, the floating altitude varies with time. We have to take into account the distribution of altitude for estimating the value of ξ/ζ. It becomes larger than that from the average altitude in proportion to the dispersion of altitude.

ICRC 2007 Proceedings - Pre-Conference Edition

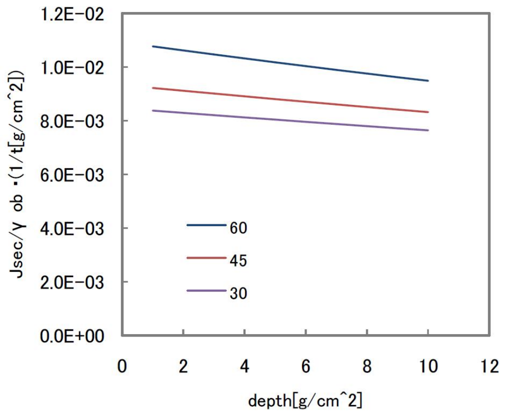

Figure 2: (Secondary electron / Gamma ray) ratio $\times$ $(1/t \text{ g/cm}^2)$: The lines show the value of $\xi/\eta_2 \cdot (1/t)$ measured within the zenith angle of 30°, 45°, 60° respectively.

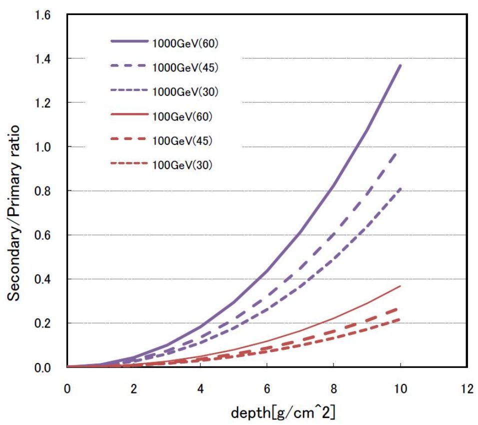

Figure 3: Secondary/Primary electron ratio: The curves are in the case of electron energy of 100 GeV and 1 TeV with the zenith angle of 30°, 45°, 60° respectively.

The total secondary electrons ($E_1 < E < E_2$) of all flights with the parameters ξ_i/ζ_i and $(S\Omega T)_i$ at the ith flight becomes

$$N_{sec}(E_1, E_2) = \int_{E_1}^{E_2} dE \, \gamma_{ex}(E) \sum_i \frac{\xi_i}{\zeta_i} (S\Omega T)_i \; .$$

The correction of energy reduction by bremsstrahrung is given by the parameter ζ. We do not know which observed electrons are primary, so that all of them are firstly corrected by bremsstrahrung loss energy and next subtracted secondary electrons statistically.

The results of secondary/primary electron ratio shows that balloon experiments of primary electrons in a few TeV region have to be performed at high altitude ($< 8 \text{ g/cm}^2$). In the next, we will perform the comparison of these calculations with Monte Carlo Simulations.

References

[1] Geant4, http://wwwasd.web.cern.ch/wwwasd/geant4/G4UsersDocuments/UsersGuides/PhysicsReferenceManual/html/node27.html

[2] Nishimura, J., Handbuch der Physik, 46, II, 1, Springer, 1967

[3] Nishimura, J. et al., Astrophys. J., 238, 394, 1980

[4] Yoshida, K., Ohmori, R., Kobayashi, T., Komori, Y., Sato, Y. and Nishimura, J., Phys, Rev, D 74, 083511, 2006

ICRC 2007 Proceedings - Pre-Conference Edition