ICRC'07 Mérida, México

Mathematical Model of the Forbush Effect of the Galactic Cosmic Ray Intensity

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Abstract: We propose a stationary and nonstationary three dimensional (3-D) models based on the transport equation to describe the temporal changes of the rigidity spectrum of the recurrent and sporadic Forbush effects of galactic cosmic ray intensity. We show that the temporal changes of the rigidity spectrum of the galactic cosmic ray intensity variations during the Forbush effects are generally shaped by the temporal changes of the interplanetary magnetic field turbulence, while for the magnitudes of the Forbush effects can be responsible the other parameters of solar wind and solar activity.

Introduction

Forbush effects [1] of galactic cosmic ray (GCR) intensity observed by super neutron monitors and ground muon telescopes can provide extremely useful information about the structure of the interplanetary magnetic field (IMF) turbulence [2-5]. The aim of this paper is to compose the various stationary and nonstationary models of the Forbush effect to explain the temporal changes of the rigidity spectrum of the Forbush effect observed by neutron monitors and muon telescopes. Our basic scenario is concerning with the assumption that the temporal changes of the rigidity spectrum ($\delta D(R)/D(R) \propto R^{-\gamma}$) exponent γ of the Forbush effect is strongly related with the temporal changes of the exponent ν of the power spectral density (PSD) of the IMF turbulence [2-5].

Experimental data

The exponent γ of the power law rigidity spectrum [6]

$$\frac{\delta D(R)}{D(R)} = \begin{cases} A R^{-\gamma} & for \quad R \le R_{\max} \\ 0 & for \quad R > R_{\max} \end{cases}$$

of Forbush effect was calculated by means of the daily average data of the neutron monitors and muon telescopes for the $R_{max} \le 100$ GV (the upper limiting rigidity beyond which the Forbush effect of the GCR intensity vanishes) [2-5]. We consider the recurrent (2-18 September 1996) and the sporadic (8-24 September 2005) Forbush effects of the GCR intensity. The changes of the GCR intensity by means of smoothed (over 3 days) daily data of few neutron monitors and muon telescopes for each Forbush effect and the corresponding temporal changes of the exponents γ of the rigidity spectra are presented in Fig. 1ab and Fig. 2ab for the recurrent and sporadic Forbush effects, respectively. For both analyzed Forbush effects the rigidity spectra during the beginning and recovery phases of the Forbush effects generally are relatively soft with respect the rigidity spectra in the minima and near minima phases of the GCR intensity. The temporal changes of the rigidity spectra exponent γ we ascribe to the changes of the structure of the IMF turbulence during the Forbush effects.

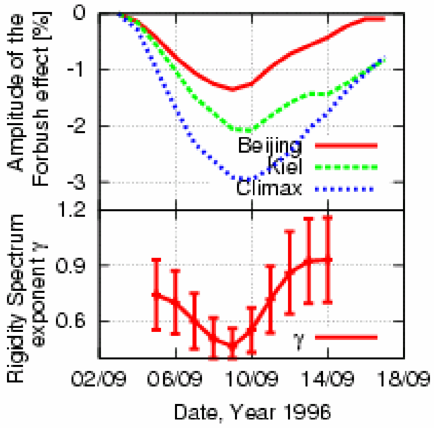

Figure 1ab. Temporal changes of the GCR intensities(top panel) and temporal changes of the rigidity spectrum exponent γ (bottom panel) for period of 2 – 18 September 1996

Particularly, the hardening of the rigidity spectrum $\delta D(R)/D(R) \propto R^{-\gamma}$ of the Forbush effect of the GCR intensity (the exponent γ gradually decreases) in the minimum and near minimum phases of the Forbush effect should be observed owing to the increase of the exponent ν of the PSD in the energy range of the IMF turbulence $(10^{-6} \div 10^{-5}\ Hz)$. We assume that the increase of the exponent ν during the Forbush effect is caused by the creation of new, relatively large scale inhomogeneities in the range of the frequencies $\sim 10^{-6} \div 10^{-5}\ Hz$ of the IMF turbulence [2-5, 7]. The new created large scale inhomogeneities can be connected with scattering of the convected structure of the IMF [E. Marsch, private discussion]. We assume that for the Forbush effects of the GCR intensity the changes of the rigidity spectra $\delta D(R)/D(R) \propto R^{-\gamma}$ are stipulated by the changes of diffusion coefficient K versus the rigidity R of the GCR particles [8]. It is of interest how the exponents ν of the PSD of the Bx, By and Bz components of the IMF changes with respect to the exponent γ of the rigidity spectra of both Forbush effects analyzed in this paper.

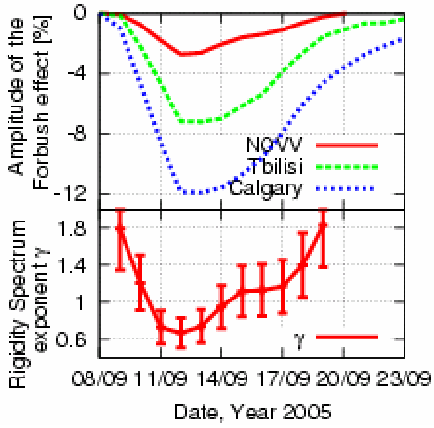

Figure 2ab. Temporal changes of the GCR intensities (top panel) and temporal changes of the rigidity spectrum exponent γ (bottom panel) for period of 8 – 23 September 2005

To obtain the values of the exponent ν of the acceptable accuracy there is needed the definite length of the using data series. So, we considered three periods, before (I), during (II) and after (III) the Forbush effects. Lengths of data series for I and III periods are predetermined by the duration of Forbush effect (length of the II period). The exponents ν of the PSD of the Bx, By and Bz components of the IMF were calculated using 1 hour IMF data from Ulysses for recurrent Forbush effect, and from ACE for sporadic Forbush effect. The values of the exponents ν for Bx, By and Bz components of the IMF for all three periods are presented in Table 1. Table 1 shows that during the Forbush effect (II period) the exponents ν are greater for all

Bx, By and Bz components, than before (I period) and after the Forbush effect (III period). Thus, the reversal dependence, $\gamma \approx 2-\nu$ [2-5] between the exponent ν of the PSD of the IMF turbulence and the exponent γ of the rigidity spectrum of the Forbush effects (when the ν increases the γ decreases) is related with the changes of the IMF turbulence during the Forbush effects.

Table 1

Forbush effect	IMF	Value of the exponent ν of the PSD of the IMF		
September 1996	Per-iod	I 19– 31 Aug	II 1 – 15 Sep	III 16-28 Sep
	Bx	1.65±0.05	1.85±0.06	1.35±0.05
	By	1.41±0.04	1.60±0.04	1.54±0.05
	Bz	1.36±0.04	1.68±0.05	1.44±0.05
September 2005	Per-iod	I 24 Aug – 8 Sep	II 9 – 25 Sep	III 25 Sep – 9 Oct
	Bx	1.63±0.05	1.65±0.05	1.53±0.06
	By	1.58±0.05	1.80±0.06	1.63±0.05
	Bz	1.25±0.04	1.87±0.05	1.59±0.05

Theoretical modeling

We describe the Forbush effect of the GCR based on the transport equation [9]:

$$\frac{\partial N}{\partial t} = \nabla_i \left(K_{ij} \nabla_j N\right) - \nabla_i (U_i N) + \frac{1}{3}\frac{\partial}{\partial R}(N R) \nabla_i U_i$$

Where N and R are density and rigidity of cosmic ray particles, respectively; U_i – solar wind velocity, K_{ij} is the anisotropic diffusion tensor of cosmic rays. For Forbush effect we accept that characteristic time of modulation is equal to the Sun's complete rotation period-27-days (~ 2.33 x 10^6 s), so the size of the vicinity where the Forbush effect take place equals ~ 6 astronomical unites (AU). Modulation region is taken ~ 30 AU, which is 5 times greater than the vicinity responsible for the Forbush effect.

As far the change of the exponent ν characterizes the state of the IMF turbulence, we assume that ν changes versus the heliolongitudes in the stationary model of the recurrent Forbush effect and versus the time in nonstationary model of the sporadic Forbush effect [3-5].

The parallel diffusion coefficient K_{II} of cosmic ray particles has a form: $K_{II} = K_0\, K(r)\, K(R)$, where

$K_0 = 4.5 \times 10^{21}\, cm^2/s$ (for the rigidity of 10 GV), $K(r) = 1 + 0.5(r/1AU)$, in the case of the stationary model of the recurrent Forbush effect:

$$K(R) \equiv K(R, \nu(r,\varphi)) = R^{\alpha(r,\varphi)} = R^{2-\nu(r,\varphi)},$$

$$\nu(r,\varphi) = 0.8 - 0.35 * (\cos(\varphi) - 0.2) * Exp\left(\frac{-(r-1AU)}{3AU}\right),$$

and in the case of the nonstationary model of the sporadic Forbush effect:

$$K(R) \equiv K(R, \alpha(\tau)) = R^{\alpha(\tau)} = R^{2-\nu(\tau)},$$

$$\nu(\tau) = 0.8 + 40 * \left(\frac{5\tau - 0.5}{3}\right)^{1.5} * Exp(-12 * (\tau - 0.1)).$$

The expected changes of the density of the GCR for the rigidity of 10 GV during the Forbush effects are presented in Fig.3ab for stationary model of the recurrent and nonstationary model of the sporadic Forbush effect, respectively.

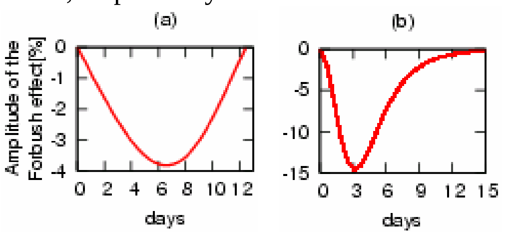

Figure 3ab. Changes of the expected amplitudes of the Forbush effect of the GCR intensity for the rigidity of 10 GV based on the solutions of the stationary model of the recurrent Forbush effect (a), and of the nonstationary model of the sporadic Forbush effect (b).

We calculated the expected power law rigidity spectrum of the Forbush effect as

$\frac{\delta D(R)}{D(R)} = \frac{1}{f}\frac{df}{dR} \propto R^{-\gamma}$ for the both Forbush effects (Fig.3ab). In Fig.4ab are presented the changes of the expected exponent γ, the exponent $\nu(r,\varphi)$ and $\nu(\tau)$; the later is included in the transport equation as a parameter responsible for the changes of the condition of the IMF turbulence for the stationary model of the recurrent and nonstationary model of the sporadic Forbush effect, respectively. Fig.4ab shows that the rigidity spectrum is hardening during the decrease phases of the Forbush effects; the exponents γ is inversely correlated with the parameters $\nu(r,\varphi)$ and $\nu(\tau)$, by the same token confirming our assumption.

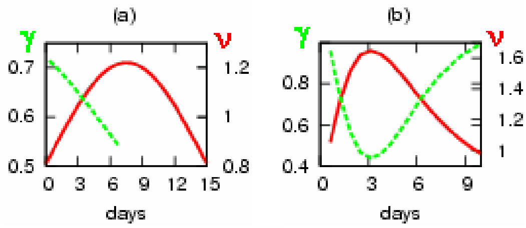

Figure 4ab. Temporal changes of the expected rigidity spectrum exponent γ and exponent ν of the PSD of the IMF turbulence for the stationary model of the recurrent Forbush effect (a), and for the nonstationary model of the sporadic Forbush effect (b)

Conclusions

1. The hardening of the rigidity spectrum $\frac{\delta D(R)}{D(R)} \propto R^{-\gamma}$ of the Forbush effect of the GCR intensity (the exponent γ gradually decreases) in the minimum phase of the Forbush effect takes place owing to the increase of the exponent ν caused by the creation of the new, relatively large scale inhomogeneities in the $10^{-6} \div 10^{-5}$ Hz range of IMF turbulence. This process can be taken place due to the non linear interactions of the high speed solar wind streams with the background solar wind.
2. The relationship between the exponent γ and the exponent ν is observed owing to the dependence of the diffusion coefficient K of GCR particles on the rigidity R as, $K \propto R^{\alpha}$ where, according to the quasi linear theory, $\alpha = 2 - \nu$.
3. The time dependent three dimensional model satisfactorily describes the behaviour of the exponent γ during the sporadic Forbush effect, whereas the stationary model is successfully acceptable for describing of the recurrent Forbush effect; the theoretical calculations are compatible with the results obtained based on the neutron monitors and ground muon telescopes experimental data.

Acknowledgements

Authors thanks the investigators of the Alma-ata, Apatity, Beijing, Calgary, Fort Smith, Irkutsk, Kerguelen, Kiel, Oulu, Pothefstroom, Rome, Tbilisi neutron monitors, Nagoya muon telescope and Ulysses and Advanced Composition Explorer spacecrafts for possibility to use their data.

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