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Testing Transport Theories with Solar Energetic Particles

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Abstract: Based on numerical solutions of the focused transport equation we study the question whether pitch angle diffusion coefficients calculated from various suggested models for wave-particle interactions and different assumptions about the nature of magnetic fluctuations in the solar wind can lead to measurable differences in observables such as the rigidity dependence of the mean free path and the angular distributions of solar particles.

Introduction

Modeling of solar particle propagation offers the possibility to derive transport coefficients and to test the validity of different theories for the interaction of energetic charged particles with magnetic field fluctuations. Significant progress has been achieved in recent years towards a better understanding of the transport parallel to the average magnetic field which remedied some deficiencies of the classical quasi-linear theory of particle scattering (standard QLT, [5]). New approaches such as a dynamical quasi-linear theory (DQLT, [1, 2]) which take into account the dissipation range, decorrelation and damping effects as well as the three-dimensional geometry of the turbulence have shown to give better explanations for various aspects of the observations. Recent models for the diffusion of particles perpendicular to the magnetic field appear to require a non-linear treatment (e.g., [7]) which, when applied to parallel transport, gives results different than those obtained from DQLT. This again raises questions about particle transport which had been thought to be settled. In the present work we investigate whether different functional forms of pitch angle diffusion coefficients can lead to measurable differences in observable features of solar particle events, and whether we can use this information to discriminate between competing theories of particle scattering.

Interplanetary Transport

Methods to model solar particle propagation in the solar wind have been extensively described in the literature (for details see e.g., [2] and references therein). In this study we consider numerical solutions of the equation for focused transport:

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial s} + \frac{1-\mu^2}{2L} v \frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu}\left(D_{\mu\mu}(\mu) \frac{\partial f}{\partial \mu} \right) = q(s,\mu,t) \quad (1)$$

Here $f(s,\mu,t)$ is the particle's phase space density (proportional to the observed particle flux), s is the distance along the magnetic field line, $\mu = \cos\theta$ the particle pitch angle cosine, t the time, $L(z)$ the focusing length, and $D_{\mu\mu}(\mu)$ the pitch angle scattering coefficient. The injection of particles close to the sun is given by $q(s,\mu,t)$. Here we restrict ourselves to particles with sufficiently high energies so that the effects of convection and adiabatic losses can be neglected.

For practicle purposes often a product ansatz for the scattering coefficient of the form

$$\begin{aligned} D^r_{\mu\mu}(r,R,\mu) &\equiv D_{\mu\mu}/\cos^2\psi(r) \\ &= \kappa_0(r,R) \cdot \left\{ |\mu|^{q-1} + H \right\} (1-\mu^2) \quad (2) \end{aligned}$$

is made, which partially resembles the result of standard QLT and additionally introduces a pa-

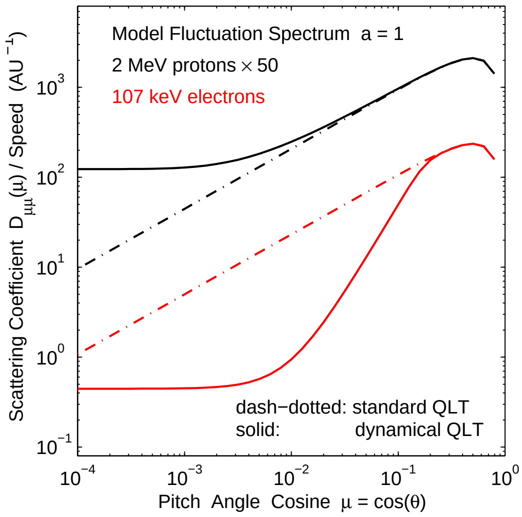

Figure 1: Pitch angle scattering coefficients calculated with DQLT from a model turbulence spectrum (details see text).

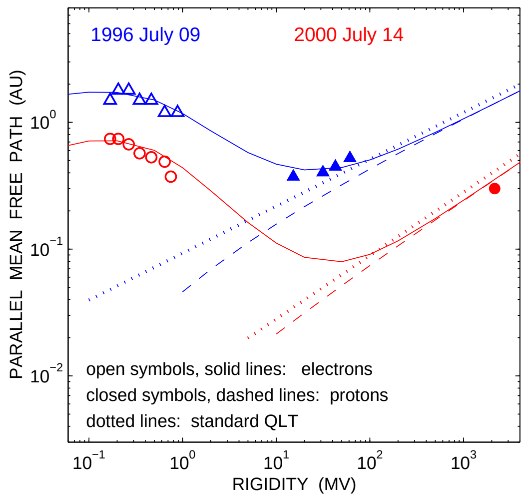

Figure 2: Mean free paths obtained from particle observations and predictions from DQLT (adopted from [3]).

rameter H to phenomenologically describe an enhancement of scattering through $\mu = 0$ by non-resonant and non-linear effects. Information about the dependence of the scattering on radial distance r from the Sun and particle rigidity R is absorbed in $\kappa_0(r, R)$, ψ is the angle between the radial direction and the magnetic field, and q denotes the spectral index in the inertial range of the magnetic turbulence spectrum. The mean free path $\lambda_\parallel$ which relates the pitch angle scattering rate to the spatial diffusion parallel to the ambient magnetic field is given by

$$\lambda_\parallel = \frac{3v}{8} \int_{-1}^{+1} d\mu \, \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)} \qquad (3)$$

and for the radial mean free path we obtain $\lambda_r = \lambda_\parallel \cos^2\psi$. The mean free path is often used as a convenient parameter to characterize the varying degrees of scattering from one solar particle event to another, even when it adopts values close to or larger than the observers's distance from the Sun and the transport process can not be considered spatial diffusion. It has been noted though (e.g., [6]) that for weak scattering diffusion coefficients with different μ-dependences, all formally resulting in the same value of $\lambda_\parallel$, can lead to quite different solutions of Equation 1 for the time histories of the average intensity. Conversely, different values of $\lambda_\parallel$ can be obtained in modeling particle events with different assumptions about the μ-dependences of $D_{\mu\mu}$.

Numerical methods and results

Figure 1 shows scattering coefficients for 108 keV electrons and 2 MeV protons which were calculated from DQLT for the model turbulence spectrum of [1] assuming a decorrelation parameter a=1 and spectral indices of 5/3 and 3 in the inertial and dissipation range, respectively. The inclusion of dissipation range and dynamical effects lead to deviations in the μ-dependence of $D_{\mu\mu}$ with respect to standard QLT results which are distinctively different for electrons and protons at energies below a few MeV: at small μ the scattering of protons is enhanced noticeably whereas it is strongly suppressed for electrons. As a result, mean free paths calculated with Equation 3 become somewhat smaller for MeV protons but drastically larger for low energy electrons. This is ex-

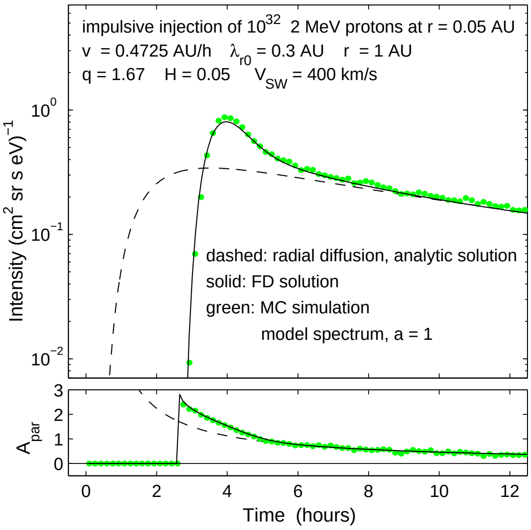

Figure 3: Comparison of time-intensity and anisotropy profiles for 2 MeV protons predicted from two different methods to solve Eq. 1.

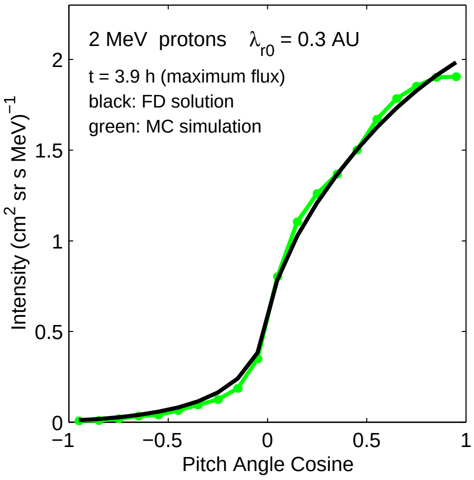

Figure 4: Comparison of 2 MeV proton pitch angle distributions predicted from two different methods to solve Eq. 1.

actly the rigidity dependence of λ which is observed in many solar particle events (cf., Fig. 2).

We like to point out that despite the good agreement between predictions from DQLT (together with the assumption that the fluctuations consist of a 10-20% slab and a 80-90% 2D component, where the latter does not contribute significantly to the scattering) and observations a major inconsistency has not been considered so far: whereas the theoretical mean free paths have been obtained with the exact scattering coefficients calculated from DQLT, only simplified forms of $D_{\mu\mu}$ as given in Equation 3 have been employed in numerical solutions of the transport equation used in the modeling of solar particle events.

To address the above problem we present here for the first time, to our knowledge, solutions of the focused transport equation which are based on the full DQLT scattering coefficient. To take into account the large dynamic range in the μ-dependence of $D_{\mu\mu}$ a Monte Carlo (MC) simulation based on the method of stochastic differential equations [4] was used. The green dots in Figure 3 show the results for 2 MeV protons and a constant λ_r = 0.3 AU. For comparison, also the results of a solution based on a finite differences scheme (FD) and for a scattering coefficient of the form of Equation 2 for H = 0.05 are shown (solid line). Because of the similarity of the scattering coefficients in this case the intensity and anisotropy profiles as well as the pitch angle distribution at the maximum intensity (Fig. 4) are almost indistinguishable. A comparison of the two numerical approaches for 107 keV electrons and a constant λ_r = 0.5 AU is shown in Figures 5 and 6. Because of the strongly reduced (cf., Fig. 1) scattering below $|\mu|$ = 0.1 (and a consequential enhanced scattering at large positive and negative μ to match the chosen λ according to Eq. 3) the electrons arrive later in the MC simulation and show a sharper peak. For the same reason the electron pitch angle distribution at the maximum intensity exhibits a much stronger gradient around μ = 0 in the MC simulation than in the FD solution.

Discussion

Observed pitch angle distributions of $\sim$ 100 keV electrons typically are similar to the shape of the FD solution shown in Figure 6 and have little resemblance to the result of the MC simulation. Our preliminary conclusion is that there likely is additional scattering through μ = 0 which could be

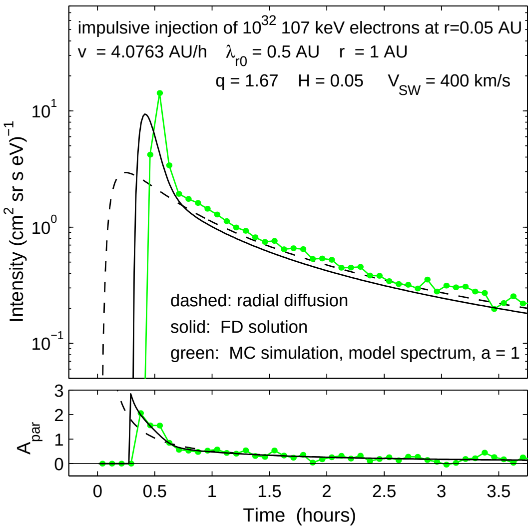

Figure 5: Comparison of time-intensity and anisotropy profiles for 107 keV electrons predicted from two different methods to solve Eq. 1.

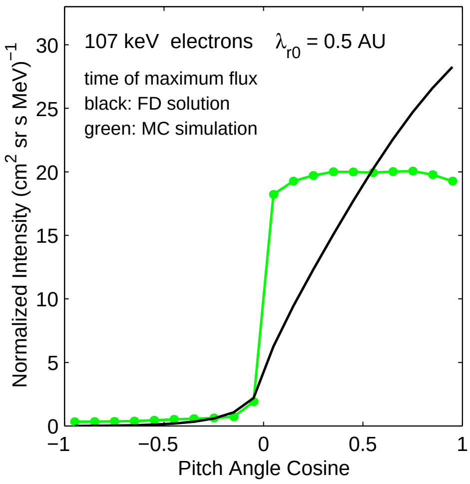

Figure 6: Comparison of 107 keV electron pitch angle distributions predicted from two different methods to solve Eq. 1.

caused by non-linear effects in the particle interaction with the slab component, or a contribution of the 2D component to pitch angle scattering and thus to the diffusion parallel to the magnetic field which is stronger than previously thought. Pitch angle distributions and intensity profiles of MeV protons determined from DQLT calculations and from the phenomenological ansatz for $D_{\mu\mu}$ are practically identical and in good agreement with observations and will probably not provide any new information regarding the above questions. For electrons it seems possible to add additional scattering to the DQLT coefficient until the observed features of the electron distribution functions are matched, and from this estimate the contribution of the above mentioned effects. This additional scattering could significantly reduce the calculated mean free paths for low-energy electrons and again lead to a disagreement with observed values of $\lambda_{\|}$. However, from the results of this study we find that for electrons in the case of weak scattering the mean free path is not a well defined parameter anymore and we suggest that the parameter considered for comparisons between theory and observations should rather be $D_{\mu\mu}(\mu)$. We hope that by the time of the conference we will be able to present such comparisons in greater detail.

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