Cherenkov Radiation from the Three –Dimennsional Cascade Shower for electron neutrino

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Abstract: In high energy neutrino astrophysics experiment, such as, NT-2000,AMANDA, ANTARES, we could get reliable information on electron neutrino events than that on muon neutrino due to muon, because the cascade showers due to the electron events are recognized as *Fully Contained Events,* while, the muon neutrino events are done as *Partially Contained Events*. We calculate the three-dimensional structure of the Cherenkov radiation from the cascade shower, by using the exact Monte Carlo technique which are expected to be available for the reliable analysis to the electron neutrino events.

1. Introduction

Following the terminology adopted by Super-Kamiokande, the neutrino events detected in high energy neutrino astrophysical experiments, such as, NT-2000 AMANDA, ANTARES and so on**(Ref.1)** , are classified as, *Fully Contained Events, Partially Contained Events, Stopping Muon Events* and *Upward Through Going Muon Events Fully Contained Events* among these events are regarded as the most qualified ones from which we could derive more ambiguity-free interpretation to neutrino events.
From such the point of view, we should be more interested in the neutrino events initiated by electron neutrino in high energy neutrino astrophysics experiment. Because the electron neutrino events are recognized as Fully Contained Events even up to 10^{21}eV for the absence of the LPM effect in the scale of 1cubic kilometer experiment, while we could not recognize muon neutrino as Fully Contsained Events beyond 5×10^{11}eV at most. In the presence of the LPM effect, the distance for total absorption of the cascade shower with 10^{21} eV attain at about 2 kilometer ,which are not recognized as the Fully Contained Events**(Ref.2).**

2 Three-dimensional treatment to cascade shower.

Three-dimensional treatment to the cascade showers are absolutely necessary for the accurate determination for the primary energies of the cascade showers and their arrival direction for the incident electron neutrinos.

Here, we treat the cascade shower within the-framework of [Approximation B] **(Ref.3)**
We treat rigorously Cherenkov radiation due to shower particles in the cascade shower which are deviated from the shower axis due to multiple scattering. Our sampling methods are as follows:

2-1 Free paths pf shower particles

Let us $\Psi_{rad}(E_0,E)dE$ and $\Psi_{pair}(E_0,E)dE$, the differential cross section for the bremssthalung and one for the pair creation, respectively, where E_0 denotes the primary energy of parent particles and E denote the daughter particle.

The free path for the electron due to the bremsstrahlung is given by

$$t=-\lambda_{rad}\ln\xi \qquad (1)$$

where ξ is a uniform random number between (0,1) and λ_{rad} denote the mean free path for the electron which is given as,

$$\lambda_{\mathrm{rad}} = 1 / \int_{E\,\min}^{E} \Psi_{rad}(E_0, E)dE \qquad (2)$$

In a similar way, the free path for the photon for pair creation is given as

$$t = -\lambda_{\mathrm{pair}} \ln \xi \qquad (3)$$

, where λ pair denotes the mean free path for photon which is defined as,

$$\lambda_{\mathrm{pair}} = 1 / \int_{E\,\min}^{E_0} \Psi_{pair}(E_0, E)dE \qquad (4)$$

2-2 The energy partition between the parent particles and the daughter particle

The energy of the emitted photon due to the bremsstrahlung is given as

$$E = \int_{E\,\min}^{E} \Psi_{rad}(E_0, E)dE \Big/ \int_{E\,\min}^{E_0} \Psi_{rad}(E_0, E)dE \qquad (5)$$

In a similar way, we give the energy of the emitted electron in the following.

$$E = \int_{E\,\min}^{E} \Psi_{pair}(E0, E)dE \Big/ \int_{E\,\min}^{E_0} \Psi_{\mathrm{pair}}(\mathrm{E}_0, \mathrm{E})\mathrm{dE} \qquad (6)$$

2-3 The ionization of the electron.

We assume the constant energy loss ε per the cascade unit.

The energy of the electron which travel with *t* due the bremsstrahlung, Ene is given as

$$E_{ne} = E - \varepsilon t \qquad (7)$$

2-4 The multiple scattering

We utilize the sampling method developed by Okamoto and Shibata **(Ref.4)** .

Okamoto and Shibata started from the fundamental equation for multiple scattering derived Eyges (Ref.5), and extend it in more generalized way and finally obtain the distribution function for electrons in the following.

$$\Psi(r,\theta)drd\theta = \{\frac{dR}{\pi R_0^2} e^{-R^2/R0^2}\} \cdot \{\frac{d\Theta}{\pi\Theta_0^2} e^{-\Theta^2/\Theta0^2}\} \qquad (8)$$

With the use of the polar coordinates R(R, ϕ_R) and $\Theta(\Theta, \phi_\theta)$, we get

$$R = R_0 \cdot \sqrt{-\ln w},\ \phi_R = 2\pi w \qquad (9\text{-}1)$$

$$\Theta = \Theta_0\sqrt{-\ln w},\ \phi_\theta = 2\pi w \qquad (9\text{-}2)$$

The component of the lateral and angular parts for the location of shower particles are given as,

$$\theta x = -u_2 R\cos 2\pi\xi_1 + u_1\theta\cos 2\pi\xi_2 \qquad (10\text{-}1)$$

$$\theta y = -u_2 R\sin 2\pi\xi_1 + u_1\theta\sin 2\pi\xi_2 \qquad (10\text{-}2)$$

$$x = u_1 R\cos 2\pi\xi_1 + u_2\theta\sin 2\pi\xi_2 \qquad (10\text{-}3)$$

$$y = u_1 R\sin 2\pi\xi_1 + u_2\theta\cos 2\pi\xi_2 \qquad (10\text{-}4)$$

The further details are give in the original papers(Ref.4). In the sampling procedure of the shower particles under the Okamoto-Shibata formalism, they assume that the lateral and angular deviation of the shower particles concerned are sampled from the original direction of the incident particle. Here, we sample these deviations for the next generation shower particles from the direction for the present generation shower particles. Then, they are given as follows:

Let us suppose that shower particle exist with the follow in direction cosines:

$$l = \sin\theta \cdot \cos\phi, m = \sin\theta \cdot \cos\phi, \quad n = \cos\theta \qquad (11)$$

and its scattering angle and the azimuthal angle are and ,respectively, then ,the new direction cosines(L,M,N) arw given as,

$$\begin{pmatrix} L \\ M \\ N \end{pmatrix} = \begin{pmatrix} \cos\theta_i \cos\phi_i & -\sin\phi_i & \sin\theta_i \cos\phi_i \\ \cos\theta_i \sin\phi_i & \cos\phi_i & \sin\theta_i \sin\phi_i \\ \sin\theta_i & 0 & \cos\theta_i \end{pmatrix} \times \begin{pmatrix} \sin\theta_s \cos\phi_s \\ \sin\theta_s \sin\phi_s \\ \cos\theta_s \end{pmatrix} \quad (12)$$

The relation between the direction cosines and the component are given as,

$$\begin{aligned} L &= \sin\theta\cos\phi = \theta_x \\ M &= \sin\theta\sin\phi = \theta_y \\ N &= \cos\theta \end{aligned} \quad (13)$$

Also, the lateral and longitudinal component for shower particle after the traverse t, is given as,

$$\begin{pmatrix} Xs \\ Ys \\ Zs \end{pmatrix} = \begin{pmatrix} \cos\theta_i \cos\phi_i & -\sin\phi_i & \sin\theta_i \cos\phi_i \\ \cos\theta_i \sin\phi_i & \cos\phi_i & \sin\theta_i \sin\phi_i \\ \sin\theta_i & 0 & \cos\theta_i \end{pmatrix} \times \begin{pmatrix} \sin\theta_s \cos\phi_s \\ \sin\theta_s \sin\phi_s \\ \cos\theta_s \end{pmatrix} \quad (14)$$

2-5 Cherenkov radiation from a segment of the electron track

In our Monte Carlo simulation, we simulate every segment of the electron track due to bremsstrahlung in an exact way. In our calculation, we calculate the Cherenkov radiation from such a segment of the electron track whose energy is E ,its coordinate (x,y,x) and its direction cosines are (*l*,*m*, *n)* and its length ,*t*. Such a Cherenkov radiation is registered on the ring whose center is coincided with the axis of the cascade shower.

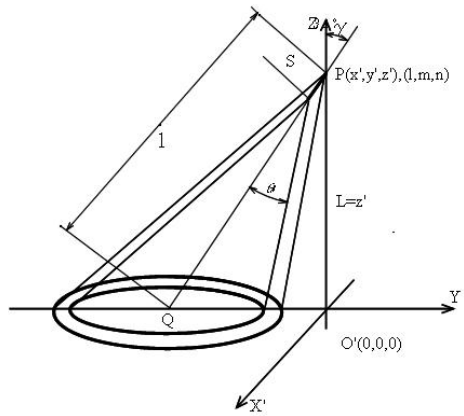

Figure 1: The pattern on xy-plane(observed plane) of Cherenkov light from radiated electron track (it's length is S) .

The area of photon reached at observed plane from electron track(s) is given by

$$S = \pi L^2 \left(\frac{\sin 2\theta}{\cos 2\gamma + \cos 2\theta} \right)^2 \sqrt{1 - \frac{\sin^2\gamma}{\cos^2\theta}} \times \{1 - (\frac{l-s}{l})^2\} \quad (15)$$

We assume that an attenuation length in the water of cherenkov light is 30m and light path is taken average length *l*.

References

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