A New Mechanism of Magnetic Field Generation in Supernova Shock Waves and its Implication for Cosmic Ray Acceleration

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Abstract: We discuss the diffusive acceleration mechanism in SNR shocks in terms of its potential to accelerate CRs to 10^{18} eV, as observations imply. One possibility, currently discussed in the literature, is to resonantly generate a turbulent magnetic field via accelerated particles in excess of the background field. We analyze some problems of this scenario and suggest a different mechanism, which is based on the generation of Alfven waves at the gyroradius scale at the background field level, with a subsequent transfer to longer scales via interaction with strong acoustic turbulence in the shock precursor. The acoustic turbulence in turn, may be generated by Drury instability or by parametric instability of the Alfven (A) waves. The essential idea is an A→A+S decay instability process, where one of the interacting scatterers (i.e. the sound, or S-waves) are driven by the Drury instability process. This rapidly generates longer wavelength Alfven waves, which in turn resonate with high energy CRs thus binding them to the shock and enabling their further acceleration.

Introduction

There is an increasingly popular view that the CR spectrum above the "knee" (about 10^{15}eV) is also produced in SNRs as its part below the knee by a *single mechanism* [10, 6]. The present paper along with the contribution OG 1.4.0759 suggests two separate new approaches to the problem of particle acceleration beyond the knee.

Approaches to Fast Acceleration

The magnetic turbulence confines accelerated particles to the shock front by pitch-angle scattering and is produced by the particles themselves, via CR-Alfven wave resonance. According to the quasi-linear theory (valid only for $\delta B \ll B_0$), pitch angle scattering of relativistic particles by the waves proceeds at the rate (e.g., [3]) $\nu \sim \Omega\,(mc/p)\,(\delta B/B_0)^2$, where Ω and p are the (non-relativistic) gyrofrequency and momentum. Resonance requires $kr_g\,(p) = \mathrm{const}$ (where k is the wave number and r_g is gyroradius) while particle diffusivity is $\kappa \sim c^2/\nu$. The acceleration time scale can be estimated as $\tau_{acc} \sim \kappa/u_s^2$, where u_s is the shock speed. The fluctuating part of the field is usually assumed to saturate at $\delta B \sim B_0$, ("Bohm diffusion limit"). Under these conditions the maximum particle energy is $E_{max} \sim (e/c)u_s B_0 R_s$ (about 10^{15}eV in a typical SNR shock) which is close to the "knee" but is three orders of magnitude below the "ankle".

One approach to reach the ankle is the generation of a fluctuating field δB significantly in excess of the unperturbed field B_0 [2], $(\delta B/B_0)^2 \sim M_A P_c/\rho u_s^2 \gg 1$ [8]. Here P_c is the partial pressure of CRs that resonantly drive the waves, $M_A = u_s/V_A \gg 1$ is the Alfven Mach number and ρu_s^2 is the shock ram pressure. For the resonantly driven waves $kr_g \sim 1$, and so if the waves grow nonlinearly until $r_g^* \ll r_g$, where r_g^* *is the particle Larmor radius calculated with the perturbed δB field, rather than* B_0, the particles that initially destabilize the waves are trapped by the wave, thus saturating the instability in the wave band corresponding to their energy. Earlier studies [8, 1] indicate saturation at $\delta B \sim B_0$.

Motivated by the problems in describing the observed high-energy spectra, we have suggested (OG 1.4.0759, [9]) a faster-than-Bohm rate acceleration which is also intimately related to the knee and other spectral break phenomena. The mechanism does not require $\delta B/B_0 \gg 1$ magnetic field fluctuations, since particles gain energy by bouncing between the scatterers convected with the gradually converging upstream flow in a nonlinearly modified shock precursor at a rate which is formally independent of the m.f.p. At the same time it is natural to ask whether it is indeed *possible* to achieve $\delta B \gg B_0$ or how to enhance the acceleration process otherwise.

Here we describe an acceleration enhancement mechanism based on the transfer of magnetic energy to longer scales via wave-wave interaction, which we call *inverse cascade* for short. This transfer is limited only by an outer scale L_{out}- likely the shock precursor size $\kappa(p_*)/u_s \sim r_g(p_*)c/u_s \gg r_g(p_{max})$. The conceptual picture of this process is simple. There are two populations of fluctuations naturally native to a CR accelerating shock. These are (i) Alfven waves, resonantly generated at small scales, i.e., $kr_g \sim 1$ (ii) acoustic waves and density fluctuations, at $kL_{out} > 1$ but not $\gg 1$. Acoustic modes can be generated by many processes (e.g., Drury instability). The density perturbation field naturally refracts the Alfven wave field, i.e.,

$$\frac{dk}{dt} = -\frac{\partial}{\partial x}\omega = -\frac{\partial}{\partial x}kV_A \simeq \frac{kV_A}{2}\frac{\partial}{\partial x}\frac{\widetilde{\rho}}{\rho_0}$$

where V_A is the Alfven velocity and $\tilde{\rho}$ is the density perturbation around the background level ρ_0. For a random array of scatterers, k evolves diffusively. This prevents the development of a narrow spectral band with $\delta B/B_0 \gg 1$. The Alfven wave population density flux is simply $\Gamma_k = -D_k \partial N/\partial k$, where N is the Alfven wave population density. Since $\partial N/\partial k > 0$ for $kr_g < 1$ (i.e., Alfven waves are excited at $kr_g \lesssim 1$), the flux is *toward* larger scales and lower k's. The advantage of an 'inverse cascade' for acceleration is that the turbulent field at the outer scale $\delta B(L_{out}) \equiv B_{rms}$ which necessarily must have long autocorrelation time can likely be regarded as an "ambient field", so far as accelerated particles of all energies are concerned. If $B_{rms} \gg B_0$, then the acceleration can be enhanced by a factor B_{rms}/B_0. Note that the resonance field $\delta B(r_g)$ remains smaller than B_{rms}, so that standard arguments about Bohm diffusion apply. Since the fluctuations around the new 'background' field B_{rms} remain relatively weak, they are not likely to dissipate so rapidly via nonlinear processes, such as induced scattering on thermal protons, as is to be expected in the case $\delta B \gtrsim B_0$.

Dynamics of Wave Interactions in the CR Shock Precursor

The growth rate of the CR-driven cyclotron instability is positive for the Alfven waves traveling in the CR streaming direction i.e., upstream, and it is negative for oppositely propagating waves. The wave kinetic equation for both types of waves can be written in the form

$$\frac{\partial N^{\pm}}{\partial t} + \frac{\partial \omega^{\pm}}{\partial k}\frac{\partial N^{\pm}}{\partial x} - \frac{\partial \omega^{\pm}}{\partial x}\frac{\partial N^{\pm}}{\partial k} \qquad (1)$$

$$= \gamma_k^{\pm} N^{\pm} + C^{\pm}\left\{N^+, N^-\right\}$$

Here $N^{\pm}(k, x, t)$ denote the population of quanta propagating in the upstream and downstream directions, respectively. Also, $\omega^{\pm}$ are their Alfven wave frequencies, $\omega^{\pm} = kU \pm kV_A$, where V_A is the Alfven velocity. The linear growth (γ^+) and damping (γ^-) rates are nonzero only in the resonant part of the spectrum, for which $kr_g(p_{max}) \geq 1$, i.e., $\gamma^{\pm} = \gamma^{\pm}(k)$. The last term on the r.h.s. of equation (1) represents nonlinear interaction of different types of quanta N^+ and N^-.

The coefficients in the wave transport part of eq.(1) depend on the parameters of the medium through U and V_A, perturbed by slow, large scale fluctuations. This usually results in parametric or modulational phenomena [11]. We will concentrate on the acoustic type perturbations (which may be induced by Drury instability), so that we can write for the density ρ and velocity U: $\rho = \rho_0 + \widetilde{\rho}$; $U = U_0 + \widetilde{U}$. The variation of the Alfven velocity $\widetilde{V}_A = V_A - V_{A0}$ is then $\widetilde{V}_A \simeq -V_A\widetilde{\rho}/2\rho_0$. Assuming the plasma $\beta < 1$ upstream, we neglect the magnetic field contribution to $\tilde{V}_A$. Note that fluctuations in U merely diffuse the *location* of the Alfven wave population in the precursor flow field. The perturbations of V_A induce perturbations of $N_k^{\pm}$, so we can write $N^{\pm} = \langle N^{\pm}\rangle + \widetilde{N}^{\pm}$, where $\langle N^{\pm}(k, x, t)\rangle$ is the quantity of interest, namely the mean wave population. This is obtained via quasi-linear theory, applied to to the wave kinetic equation in the same way it is usually applied to the particle kinetic equation. Given that our goal is to obtain an evolution equation for the average number of Alfven quanta $\langle N^{\pm}\rangle$, averaging equation (1) then finally yields

$$L\left\langle N^{\pm}\right\rangle = \gamma_k^{\pm}\left\langle N^{\pm}\right\rangle + \left\langle C\left(N^{\pm}\right)\right\rangle \tag{2}$$

where $L \equiv \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} - kU_x\frac{\partial}{\partial k} - \frac{\partial}{\partial k}D_k\frac{\partial}{\partial k}$

The diffusion in k represents *random* refraction by the acoustic perturbations $\widetilde{\rho}$ via the density dependence of V_A. The diffusion coefficient can be represented as

$$D_k = \frac{k^2V_A^2}{4C_s^2\rho_0}\sum_q q^2\omega_q^s\frac{\Delta\omega_k^{\pm}}{q^2V_A^2+\Delta\omega_k^{\pm 2}}N_q \tag{3}$$

where $W_q = C_s^2\widetilde{\rho}_q^2/\rho_0 = \omega_q^s N_q$ is the energy density of acoustic waves (with $\omega_q^s = qC_s$ being their frequency, number of quanta N_q and spectral density $\tilde{\rho}_q$). $\Delta\omega_k$ represents a nonlinear decay or damping rate while D_k is the rate at which the wave vector of the Alfven wave random walks due to stochastic refraction. Of course, such a random walk necessarily must generate larger scales (smaller k), thus in turn allowing the confinement of higher energy particles to the shock. Hence, confinement of higher energy particles is a natural consequence of random Alfven wave refraction in acoustic perturbations.

In contrast to the Alfvenic turbulence that originates in the shock precursor due to cyclotron emission from accelerated particles, there are (at least) two separate sources of long wave acoustic perturbations. One is related to parametric and modulational [11] processes undergone by the Alfven waves. These take the usual form of decay of an Alfven wave into another Alfven wave and an acoustic wave. The other source is the pressure gradient of CRs, which *directly* drives *linear (*Drury) *instability*. By analogy with equation (1), we can then write the following wave kinetic equation for the acoustic waves:

$$\frac{\partial}{\partial t}N_q + U\frac{\partial}{\partial x}N_q - qU_x\frac{\partial}{\partial k}N_q$$

$$= \left(\gamma_q^d + \gamma_D\right)N_q + C\left\{N_q\right\}$$

Here γ_D is the Drury instability growth rate $\gamma_D = \gamma_D\left(\nabla P_{CR}\right)$ and γ_q^d is the growth rate of the decay instability $\gamma_q^d = \gamma_q^d\left(N_k\right)$. The mechanism of the modulational instability is the growth of the density (acoustic) perturbations due to the action of the ponderomotive force on the acoustic waves by the Alfven waves. This force can be regarded as an effective radiation pressure term which must appear in the hydrodynamic equation of motion for the sound waves and results in the following growth rate of acoustic perturbations

$$\gamma^d = \frac{q^2}{4\rho_0}\frac{V_A}{c_s}\sum_k\frac{k\omega_k\Delta\omega_k}{q^2V_{gr}^2+\Delta\omega_k^2}\frac{\partial}{\partial k}\left\langle N_k^{\pm}\right\rangle$$

Note that modulational instability requires an inverted population of Alfven quanta i.e., $\partial\left\langle N\right\rangle/\partial k > 0$. As Alfven waves are generated by high energy resonant particles in a limited band of k space at short wavelength (i.e., $kr_g \sim 1$), such an inversion clearly can occur. The growth rate γ_D has been calculated in [5]:

$$\gamma_D^{\pm} = -\frac{\gamma_C P_C}{\rho\kappa} \pm \frac{P_{Cx}}{C_s\rho}\left(1+\frac{\partial\ln\kappa}{\partial\ln\rho}\right) \tag{4}$$

Here P_C and P_{Cx} are the CR pressure and its derivative, respectively, and γ_C is the CR adiabatic index. There are a variety of nonlinear processes that can lead to the transfer of magnetic energy (generated by accelerated particles in form of the resonant Alfven waves) to longer scales. First, as it can be seen from equation (2), scattering of the Alfven waves in k space due to acoustic perturbations transfers magnetic fluctuation energy away from the resonant excitation region to smaller (and also to larger) k, and also amplifies the long wavelength acoustic scattering field. Second, the nonlinear interaction of Alfven waves and magnetosonic waves represented by the wave collision term on the r.h.s. can drive such a process. It was shown [4] that the main parameter that determines the behaviour of the solution for $\langle N_k\rangle$ is

$$S = \frac{L_p}{u_1 t_s} = \frac{t_{conv}}{t_s}. \tag{5}$$

Here $t_{conv} = L_p/u_1$ is the precursor (of length L_p) crossing time and $t_s \simeq \langle D_k/k^2 \rangle^{-1}$ is the refractive scattering time for the wave. In the limit $S \to 0$ the solution for $\langle N_k \rangle$ simply follows the unstable driver, $\langle N_k \rangle \propto P$, where P is the particle partial pressure at the resonant momentum, $kr_g(p) \sim 1$. In the more interesting case $S \gg 1$, a reduction in the wave spectral density by a factor $S \gg 1$ due to diffusion of the Alfven waves occurs along with a significant spreading in k of the excited spectrum. The parameter S can also be represented as:

$$S \sim q_0 L/M_A$$

Here we have assumed for simplicity that the acoustic shocks in the ensemble have characteristic strength $\Delta\rho \sim \rho_0$, so that S roughly represents the averaged number of such shocks reduced by $M_A \gg 1$, i.e., , $S \sim N_s/M_A$. Therefore, S may vary significantly, with an uncertainty related to the number of shocks N_s. One guideline to determination of N_s is provided by numerical studies of acoustic instability of CR modified precursors e.g., [7]. According to this study the number of shocks is not large, about $5-7$. However, this value of S is severely restricted by constraints related to computational feasibility. In particular, the maximum particle momentum is about $100mc$ and particle diffusivity $\kappa(p)$ has been taken to grow with momentum more slowly than in the Bohm case. Therefore, the actual precursor length, and thus N_s, can be much larger in cases of higher maximum momentum and values of κ with more realistic scaling. On the other hand, coalescence of shock would reduce this number leaving the parameter S large, but not very large. A precise value of N_s requires a detailed study of the kinetics of the shock population in the turbulent precursor. Scaling of the results found in [7] suggests that N_s falls in a range $10^3 < N_s < 10^4$. Since $10 < M_A < 10^3$, this assures us that $S \gg 1$.

Concluding Remarks

The results of this paper have several implications for CR acceleration. First, given that density fluctuations are present via the Drury instability, waves at $kr_g \sim 1$ will not grow to large amplitude ($\delta B \gtrsim B_0$), but rather will have their energy diffused in k to a broad band of larger and smaller scales. Thus, theories which predict the generation of strong, small scale fields without considering scattering by precursor fluctuations probably have significantly *overestimated* the strength of the field at $kr_g \sim 1$. Second, the modulational mechanism presented here, while not a "dynamo", in a strict sense, is a robust and universal means to scatter wave energy to larger scales and so confine higher energy particles (i.e., $p > p_{max}$). Hence, it constitutes a novel means for enhanced acceleration. Third, it increasingly seems that the most energetic particles result more from precursor dynamics than from the traditionally invoked crossing of the subshock discontinuity (see also OG 1.4.0759).

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