Cosmic rays from thermal sources

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Abstract: The energy spectrum of cosmic rays (CR) exhibits very characteristic power-like behavior with the "knee" structure. We consider a generalized statistical model for the production process of cosmic rays which accounts for such behavior in a natural way either by assuming the existence of temperature fluctuations in the source of CR, or by assuming specific temperature distribution of the CR sources. Both possibilities yield the so called Tsallis statistics and lead to the power-like distribution.

Introduction

The origin of the characteristic structure of the energy spectrum of cosmic rays (CR), which has power-like behavior with "knee" structure, remains matter of hot debate (for survey of models proposed to explain the origin of CR see [1]). It could reflect different regimes of the diffusive propagation of CR in the Galaxy, but it could also be due to some property of the acceleration processes within the source of the CR itself. In this second case, a crucial question to answer is whether the sources of CR below the "knee" can also accelerate particles to much higher energies, so that a single population of astrophysical objects can explain the smooth spectrum of cosmic rays, as observed over many orders of magnitude in energy. We shall address this problem using a generalized statistical model specially adapted to this end. However, our work will concentrate more on the physics of CR than on the generalized statistics, providing therefore some physical explanations to ideas presented already in [2, 3].

Nonextensive statistics and results

We shall start with some basic information on nonextensive statistical mechanics as introduced by Tsallis [4], which have been already successfully applied to a variety of complex physical systems, including CR where, among others, the energy spectrum of cosmic rays have been analyzed from a nonextensive point of view [2, 3]. The idea is to maximize more general entropy measures than the Boltzman-Gibbs-Shanon (BGS) entropy, which depends on a new additional parameter q and which leads to generalized version of the statistical mechanics. If we optimize under appropriate constrains the BGS entropy,

$$S = -\int dE\, P(E)\, \ln P(E), \tag{1}$$

we obtain the equilibrium distribution in the usual form of

$$P(E) = \frac{1}{T}\,\exp\left(-\frac{E}{T}\right). \tag{2}$$

This distribution can alternatively be obtained as the solution of the following differential equation:

$$\frac{dP(E)}{dE} = -\frac{P(E)}{T}. \tag{3}$$

A more general formalism proposed in [4] and sometimes referred to as nonextensive statistical mechanics is based on the generalized entropy,

$$S_q = -\frac{\int dE\, P^q(E) - 1}{q-1}\,. \tag{4}$$

Its maximization under appropriate constrains yields a characteristic power-like distribution

$$P_q(E) = \frac{2-q}{T}\left[1 - (1-q)\frac{E}{T}\right]^{\frac{1}{1-q}}. \tag{5}$$

This equilibrium distribution can alternatively be obtained by solving the differential equation

$$\frac{dP(E)}{dE} = -\frac{P^q(E)}{T}, \qquad (6)$$

and precisely this equation has been used in [2] to describe the flux $\Phi(E)$ of cosmic rays. There is growing evidence that the nonextensive formalism applies most often to nonequilibrium systems with a stationary state that posses strong fluctuations of the inverse temperature parameter $\beta = 1/T$ [5, 6]. In fact, fluctuating β according to the gamma distribution with the variance $Var(\beta)$ results in a power like distribution (5) with nonextensivity parameter q being given by the strength of these fluctuations,

$$q - 1 = Var(\beta)/\langle\beta\rangle^2. \qquad (7)$$

This observation was applied in [3] to describe the flux $\Phi(E)$. The estimated temperature $T \sim 170$ MeV (comparable with the so called Hagedorn temperature known from description of hadronization processes) seems to be overestimated because author trays to describe also the low energy part of the CR spectrum which, in our opinion, is governed mainly by the geomagnetic cut-off and should be considered separately.

Energy spectrum

For relativistic particles (where the rest mass m can be neglected) the energy $E \sim p$ and the density of states of an ideal gas in three dimensions is given by $\omega(E) \propto E^2$. The flux $\Phi(E)$ can be obtained straightforwardly from $P(E)$ and we can write

$$\Phi(E) = N_0\, E^2\, P(E), \qquad (8)$$

where N_0 is normalization factor. For $E >> T$ we have the power spectrum $\Phi(E) \propto E^{-\gamma}$ with the slope parameter $\gamma = \frac{(3-2q)}{(q-1)}$.

Temperature fluctuations

There are at least two scenarios leading to gamma distribution in β (here $\mu^{-1} = \beta_0(q-1)$ and $\nu^{-1} = q-1$):

$$f(\beta) = \frac{\mu}{\Gamma(\nu)}\,(\mu\beta)^{\nu-1}\,\exp(-\beta\mu)\,, \qquad (9)$$

(a) temperature distribution of sources;

(b) temperature fluctuations in small parts of a source.

In what concerns the first point notice that gamma distribution (9) is the most probable outcome of the maximalization of Shannon information entropy (1) under constraints that $\int f(\beta)d\beta = 1$, $\langle\beta\rangle = \beta_0$ and (because distribution we are looking for is one sided, i.e., defined only for $\beta > 0$) that $\langle\ln(\beta)\rangle = \ln(\nu\beta_0)$.

To illustrate the second point let us suppose that one has a thermodynamic system, in a small part of which temperature fluctuates. Let $\xi(t)$ describes the stochastic changes of temperature in time and let it be defined by the white Gaussian noise ($\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t+\Delta t)\rangle = 2D\delta(\Delta t)$). The inevitable exchange of heat which takes place between the selected regions of our system leads ultimately to the equilibration of temperature. As we have advocated in [5], the corresponding process of the heat conductance leads eventually to the gamma distribution $f(\beta)$ (9) with variance (7) related to the specific heat capacity C_V by

$$q - 1 = C_V^{-1}. \qquad (10)$$

Actually, the temperature fluctuations in astrophysics is much-discussed problem nowdays. Its effect on temperatures empirically derived from the spectroscopic observation was first investigated in [7] whereas in [8, 9, 10, 11] it was shown that temperature fluctuations in photoionized nebulea have great importance to all abundance determinations in such objects.

Heat capacity

The total specific heat of the crust, C, is the sum of the contribution from the relativistic degenerate electrons, from the ions and from degenerate neutrons. In temperature that can be reached in the crust of an acreating neutron star (which is of the order of $T \sim 5 \cdot 10^8$ K and is below the Debaye temperature $T_D \sim 5 \cdot 10^9$ K) we have $C_{ion} < C_e < C_n$. When the temperature drops below the critical value $T = T_C$ the neutrons become superfluid and their heat capacity C_n^{sf} increases [12, 13],

$$\frac{C_n^{sf}}{C_n} \simeq 3.15\frac{T_C}{T}\exp\left(-1.76\frac{T_c}{T}\right)\cdot \cdot\left[2.5-1.66\left(\frac{T}{T_C}\right)+3.68\left(\frac{T}{T_C}\right)^2\right]. \quad (11)$$

At $T \sim 0.7 \cdot T_C$ we have $C_n^{sf} \sim 1.1 \cdot C_n$ what correspond the changes of spectral index $\Delta\gamma \sim 0.5$. Changing nonextensivity parameter from q_1 to q_2 at temperature T_{cut} we obtain the flux

$$\Phi(E) = N_0\, E^2 \cdot \cdot\left[P_{q_1}(E) - \alpha_1(E)P_{q_1}(E) + \alpha_2(E)P_{q_2}(E)\right], \quad (12)$$

where $P_q(E)$ is given by eq. (5) and $\alpha_i = \Gamma\left[\frac{1}{q_i-1}, \frac{1-(1-q_i)E/T}{(q_i-1)T_{cut}/T}\right]/\Gamma\left(\frac{1}{q_i-1}\right)$. Nucleon superfluidity was predicted already in [14] and today pulsar glitches provide strong observational support for this hypothesis [15]. Nucleon superfluidity arises from the formation of Cooper pairs od fermions (actually in [16] also quark superfluidity from cooling neutron stars were investigated). Continuous formation and breaking of the Cooper pairs takes place slightly below $T = T_C$ (critical temperature T_C is in the order $10^9 - 10^{10}$ K).

Neutron stars are born extremely hot in supernova explosions, with interior temperatures around $T \sim 10^{12}$ K. Already within a day, the temperature in the cental region of the neutron star will have dropped down to $\propto 10^9 - 10^{10}$ K and reach 10^7 K in about 100 years [17]. The first measurement of the temperature of a neutron star interior (core temperature of the Vela pulsar is $T \sim 10^8$ K, while the core temperature of PSR B0659+14 and Geminga exceeds $2 \cdot 10^8$ K) allows to determine the critical temperature $T_C \sim 7.5 \cdot 10^9$ K [18].

Acceleration

If the energy growth is given by

$$\frac{\partial E}{\partial t} = a + bE \quad (13)$$

than from master equation

$$\frac{\partial P(E)}{\partial t} = -c\,P(E) \quad (14)$$

one gets the evolution equation

$$\frac{\partial P(E)}{\partial E} = -cP(E)\,\frac{\partial t}{\partial E} = -\frac{P(E)}{T(E)} \quad (15)$$

in the form of eq.(3) with energy dependent temperature parameter

$$T(E) = \frac{a+bE}{c} = \frac{T_0 + (q-1)E}{q}, \quad (16)$$

where we have used: $c = T_0^{-1}$, $a = q^{-1}$ and $b = (1-q^{-1})T_0^{-1}$. Energy dependent temperature parameter $T(E)$ in this form immediately leads to the energy distribution given by Eq.(5). Notice that q^{-1} plays the role of the weight with which we select the constant (thermal) and linear (accelerating) terms in the equation describing the growth of energy.

Conclusions

The spectrum of cosmic rays has the shape of broken power law $E^{-\gamma}$, with the slope $\gamma_1 \simeq 2.7$ at energies below $\sim 10^{15}$ eV and $\gamma_2 \simeq 3.1$ at energies between the knee and $E \sim 10^{18}$ eV. This slopes correspond to the nonextensivity parameters (taking into account that $q = (3+\gamma)/(2+\gamma)$) $q_1 = 1.213$ and $q_2 = 1.196$, respectively, what, equivalently, means that the change of heat capacity is $C_2/C_1 = 1.09$. Nonextensive statistics successfully describes the smooth power-law spectrum and its origin can be traced back to the gamma distribution of inverse temperature, $f(\beta)$. Out of the two possible scenarios leading to such distribution of inverse temperature we prefer the temperature fluctuation in the source rather than the temperature distribution of sources. The point is that in the second case some cut temperature T_{cut} is expected which would result in the rapid break in the energy spectrum, which is not observed. The temperature T (not essential in the high energy region, $E >> T$) seems to be of the order of MeV, i.e. of the order of the interior stars temperature (if stars are born extremely hot in supernova explosions, with interior temperatures around $T \sim 100$ MeV, already within a day the temperature in the cental region of the star will have dropped down to $\propto 0.1 - 1$ MeV and reach the 1 keV in about 100 years). The critical temperature (corresponding to the nucleon superfluidity) is $T_C \sim 0.1 - 1$ MeV. It means then that the origin of changes of the nonextensivity parameters at temperature $T_{cut} \simeq 10^{15}$ eV $\simeq 10^{19}$ K is still open question. The nonextensive formalism leads to production (injection) spectrum and the acceleration processes ($dE/dt \sim E$, which does not change the shape of power spectrum) and allows the shift of this spectrum to high energies.

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