On the origin of the knees in the cosmic-ray energy spectrum

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Abstract: Combining diffusion equation solutions with direct Monte-Carlo simulations of charged particle trajectories, the propagation of cosmic rays in the Galaxy is investigated. Different assumptions on the shape of the regular Galactic magnetic fields and source distributions are considered and their influence on cosmic-ray life times and the energy spectrum obtained at Earth is examined. The origin of the knee in the energy spectrum at $4 \cdot 10^{15}$ eV and the second knee at $4 \cdot 10^{17}$ eV is discussed. It is investigated whether the knee can be explained by propagation effects only and if the second knee is due to the end of the galactic component with a strong contribution of elements heavier than iron.

Introduction

The knee in the energy spectrum of cosmic rays (CRs) at $\sim (4-5) \cdot 10^{15}$ eV was first observed almost 50 years ago [9], but its origin is still under discussion and it is generally believed to be a corner stone in understanding the origin of CRs.

The verification of various hypotheses of the CR origin and the nature of the knee in their energy spectrum is complicated by the fact that the CR spectra at the sources and at the Earth are different. The change of the energy spectrum during propagation is related to the energy dependence of the CR life time in the Galaxy.

Using a combined approach, which includes the solution of a diffusion equation for the CR density in the Galaxy and a method of numerical calculations of trajectories, we have performed a calculation of the energy spectrum at Earth in the energy range $10^{12} - 10^{20}$ eV.

The method of numerical integration of trajectories is traditionally used for the calculation of the spectrum at high energies [14, 6, 5] but it is not too efficient at low energies as the calculation time needed is inversely proportional to particle energy. The diffusion model is more suitable at lower energies and the CR energy spectrum may be obtained by the solution of the diffusion equation for the CR density in the Galaxy [5, 11]. There are no computing difficulties inherent to the numerical simulation of trajectories, but the diffusion approach is limited by a certain energy boundary — the energy of a proton should not exceed 10^{17} eV [5].

In our calculations we tested different assumptions about the structure of the regular magnetic fields of the Galaxy, and also different spatial distributions of CR galactic sources, to determine to what degree this uncertainty will influence results of the calculations.

The results obtained were used to verify the hypotheses of the origin of the knee in the energy spectrum using experimental data as obtained at Earth.

Assumptions about magnetic fields

The magnetic field used in the calculations included a regular and a chaotic component

$$\vec{B} = \vec{B}_{reg} + \vec{B}_{chaot}. \quad (1)$$

For the regular magnetic field component a model from Ptuskin et al. [11] was used

$$B_z = 0, B_r = 0, B_\phi \propto \exp\left(-\frac{z^2}{z_0^2} - \frac{r^2}{r_0^2}\right), \quad (2)$$

where $z_0 = 5$ kpc, $R_0 = 10$ kpc, and

$$B_\phi(z=0) = 2.15\mu\mathrm{G} \cdot \sin\left(2\pi\frac{r - 9.35\ \mathrm{kpc}}{6.2\ \mathrm{kpc}}\right) \quad (3)$$

according to the Rand-Kulkarni model [13].

The poloidal component of the regular magnetic field [10] was also taken into account. The chaotic magnetic field in the simulation of the particle trajectories was defined according to the algorithm used in [15], generating irregularities on the scale $L = 100$ pc. In addition, we simulated interactions of charged particles with magnetic irregularities of smaller scales. For the spatial distribution of CR sources, a uniform distribution in the galactic disk and a radial distribution of supernovae remnants in the galactic disk [8] were used. Different models of the regular magnetic field and also different assumptions on the CR source distributions did not influence the main results of our calculations.

The knee in the energy spectrum

In the framework of the diffusion model, the knee in the CR energy spectrum at Earth can be explained by a change of the character of the dependence of the diffusion coefficient on energy. This dependence changes from

$$D_\perp \sim E^m, \quad (4)$$

where $m = 0.2 - 0.6$ is a parameter of the model, to

$$D_A \sim E, \quad (5)$$

since the Hall diffusion coefficient D_A is proportional to the Larmor radius of a particle (for more details, see [11]).

In order to get a sharp steepening $\Delta\gamma \approx 0.8$ in the elemental spectra of CRs at energies around $4 \cdot 10^{15}$ eV, we assumed $m \approx 0.2$, as suggested in [11]. Furthermore, the intense rise of the diffusion coefficient with energy (if we assume $m = 0.4 - 0.8$) leads to excessive anisotropy — it is more than 10% at an energy of 10^{16} eV for $m = 0.6$ [2].

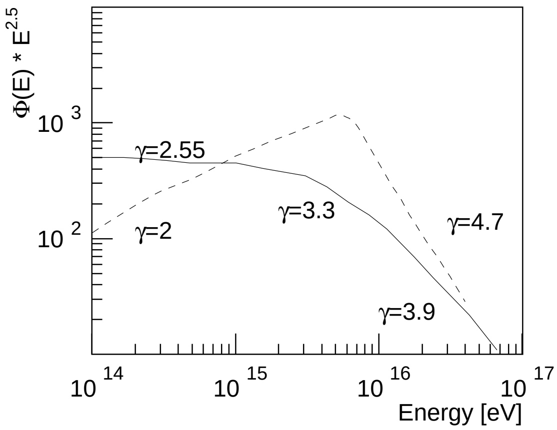

Figure 1: The energy spectra of protons in sources. The curves are normalized at 10^{15} eV. The solid line represents the spectrum obtained from the KASCADE spectrum, the dashed line is the source spectrum according to the standard picture of CR acceleration [12].

If one takes into account the complex CR mass composition, the value of $\Delta\gamma$ decreases to $\approx 0.4 - 0.5$ in the all-particle spectrum [11], which is consistent with experimental data [7]. Thus, the knee in the all-particle spectrum at an energy of about $4 \cdot 10^{15}$ eV can be explained as the result of the changes in conditions of the propagation in the Galaxy (from the diffusion to the drift in the large scale magnetic field of the Galaxy). But the situation with elemental spectra of CR is more problematic.

The energy spectra for various nuclear groups obtained by KASCADE and other air shower experiments can be approximated by the ansatz (the poly gonato model) [3, 4]

$$I_z(E) = I_0(Z)E^{-\gamma_Z} \cdot \left(1 + \left(\frac{E}{E_k(Z)}\right)^\zeta\right)^{-\frac{\Delta\gamma}{\zeta}}, \quad (6)$$

where Z is the charge of the particle, γ_Z the exponent before the knee which is obtained from direct measurements, $E_k(Z) = Z \cdot E_k(Z=1)$ the energy corresponding to the knee; as well as $\zeta \approx 2$ and $\Delta\gamma = 2$ characterizing the shape of the knee structure in the spectra. The observed change of the exponent of the spectrum should be compared to the value $1 - m \approx 0.8$. This value follows from a diffusion model to explain the origin of the knee [11]. It is obvious that the experimental value of $\Delta\gamma$ is essentially greater; hence, at least a part of the observed $\Delta\gamma$ should stem from the peculiarities of the energy spectrum at the sources. It is instructive to point out that at higher energies (above 10^{17} eV) the diffusion coefficient becomes proportional to E^2 and, formally, one could get a sharp knee. But the diffusion approach produces wrong results at such energies and an essentially more complicated transport equation is needed.

Taking the spectra measured at Earth as parameterized with equation 6 and taking into account the dependence of the CR life time on energy, obtained by numerical calculation of trajectories the spectra at the sources can be estimated. The result is presented in Fig. 1 as solid line. It represents the proton spectrum at the sources. The result indicates that the relatively sharp knee in the elemental spectra at the Earth (see e.g. [4]) can not be explained in the context of the diffusion model only, and it is necessary to assume a change of spectra in sources at corresponding energies.

For a final conclusion it is necessary to ultimately establish the exact shape of the spectra for elemental groups.

The second knee in the spectrum

Using spectra at the sources similar to the one shown in Fig. 1 the spectra at Earth have been estimated [5]. As source composition, the abundances of elements from hydrogen to uranium as measured in the solar system [1] have been weighted with $Z^{3.2}$. This choice is arbitrary to a certain extent, but may be motivated by a higher efficiency in the injection or acceleration processes for nuclei with high charge numbers. The abundances are scaled with a factor which is identical for all elements to obtain approximately the absolute values as expected at the Earth according to the poly gonato model. At the source, a power law $\propto E^{-2.5}$ has been assumed for all elements with a knee, caused e.g. by the maximum energy attained during the acceleration, at $Z \cdot 4.5$ PeV, with a power law index -3.5 above the respective knee. Using the derived propagation path length and interaction length, the amount of interacting particles has been determined. Secondary products generated in spallation processes are taken into account, assuming that the energy per nucleon is conserved in these reactions. They are added to the corresponding spectra with smaller Z. The spectra thus obtained are compared to spectra according to the poly gonato model in Fig. 2.

Two features should be noted: The absolute fluxes at Earth are predicted quite well, especially when considering that only a simple scaling law has been introduced for the abundances at the sources, starting with the composition in the solar system. More important for the present discussion is the shape of the spectra. As expected, the shape of the proton spectrum is not influenced by the (few) interactions during propagation and the difference of the spectral index at the source and at Earth $\gamma = -2.71$ [3] can be explained by the energy dependence of the escape path length $\propto E^{-0.2}$. On the other hand, it can be recognized that due to nuclear interactions the spectra for heavier elements are flatter. The slopes obtained with the simple approach for the CNO, silicon, and iron groups agree well with the steepness as expected from the poly gonato model. For heavy elements at low energies secondary products generated in spallation processes play an important role for the shape of the spectrum. At low energies many nuclei interact due to the large escape path length and the small interaction length, thus, the spectra of nuclei without any interaction deviate from power laws. However, the spallation products of heavier elements at higher energies compensate the effect and the resulting spectra are again approximately power laws, as can be seen in Fig. 2.

Summary and Conclusion

The propagation pathlength and escape time of cosmic rays in the galaxy has been calculated in a combined approach solving a diffusion equation and numerically calculate the trajectories of particles in the Galaxy. To explain the relatively steep fall-off of the observed energy spectra for elemental groups at their respective knees, the modulation of the spectrum due to propagation solely is not sufficient. An additional steepening of the spectra at the source is necessary, e.g. caused by the max-

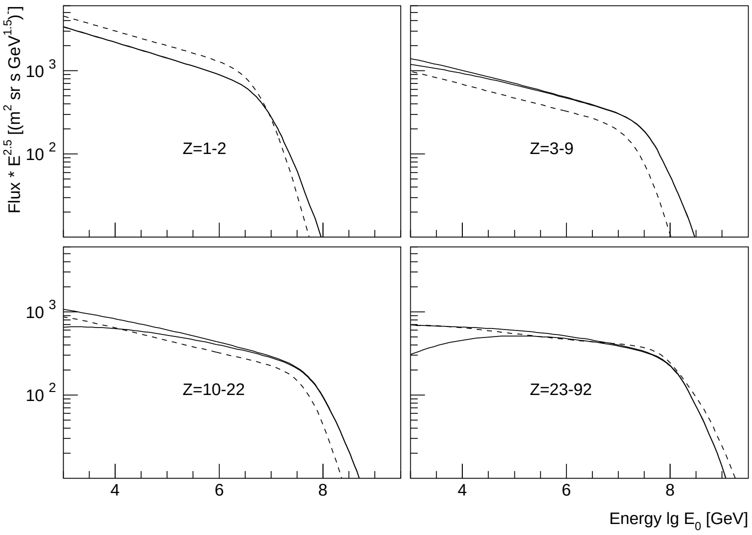

Figure 2: Energy spectra at Earth for elements with nuclear charge Z as indicated. The dashed lines represent spectra according to the poly gonato model, the solid lines are expected from the diffusion model discussed, see text. Two solid lines are shown in each panel, representing an estimate for the uncertainties [5].

imum energy attained during acceleration. It can be concluded that the knee in the energy spectrum of cosmic rays has its origin most likely in both, acceleration and propagation processes.

It seems to be reasonable that the second knee around 400 PeV $\approx 92 \cdot E_k(p)$ is due to the cut-off of the heaviest elements in galactic cosmic rays. Considering the calculated escape path length and nuclear interaction length within the diffusion model, it seems to be reasonable that the spectra for heavy elements are flatter as compared to light elements. The calculations show also that even for the heaviest elements at the respective knee energies more than about 50% of the nuclei survive the propagation process without interactions. This may explain why ultra-heavy elements could contribute significantly ($\sim 40\%$) to the all-particle flux at energies around 400 PeV and thus explain the second knee in the energy spectrum.

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