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Simulations of time-dependent non-linear multiple diffusive shock acceleration

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Abstract: We present a new code (named MARCOS) aimed at the realistic simulation of diffusive shock acceleration in its full time-dependent non-linear developments, and more specifically at the simulation of multiple (ie successive) shocks acceleration as is believed to occur in many astrophysical places (most notably in superbubbles). We present briefly the numerical methods implemented, coupling the hydrodynamical evolution of a shock and the kinetic transport of the cosmic-rays momentum distribution function, as first done by Falle & Giddings. Following Kang and Jones and collaborators, we show how the adaptive mesh refinement technique (AMR) makes it possible to deal with the extremely demanding numerical requirements of realistic (Bohm-like) CR diffusion coefficients. We have also parallelized the code (in momentum space, using MPI), in order to be able to run multiple shocks simulations at the cost of single shock simulations. We make use of these numerical developments to present the first simulations of linear and non-linear multiple diffusive shock acceleration.

Introduction

In many astrophysical contexts cosmic-rays (CR) are likely to experience many successive shocks, notably inside superbubbles ([9]) and in the early cosmological flows ([6]). However multiple diffusive shock acceleration has not been studied to the same extent as single diffusive shock acceleration. From an analytical point of view the effect of multiple shocks has been well studied but mostly in the linear regime (see [10] and references therein). From a numerical point of view the full non-linear regime is quite well studied now (see [5] for a short review) but mostly in the single shock model.

In this paper we present a new code named *Marcos* (for *Machine à Accélérer les Rayons COSmiques*, the French for *COSmic-Rays Acceleration Machine*) aimed at the detailed study of CR acceleration by multiple shocks in the full time-dependent non-linear regime.

In section 2 we present the basics of the numerical methods implemented in our code, which couples the hydrodynamical evolution of a fluid with the kinetic transport of the CR. In section 3 we present the Adaptive Mesh Refinement (AMR) technique which allows us to resolve efficiently all the scales induced by CR diffusion. In section 4 we present parallelization of the code to be able to study in reasonable computing time the effects of a sequence of shocks.

Accelerating particles

In our approach the thermal (fluid being shocked) and non-thermal (CR) particles, although intimately coupled, are handled as two different populations. The fluid, described by its moments ρ, u, P, obeys the Euler equations (modified to include the CR pressure back-reaction), while the CR, described by their isotropic distribution function $f(x,p)$, follow a more general transport equation:

$$\frac{\partial f}{\partial t} + \frac{\partial u f}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x}\right) + \frac{1}{3p^2}\frac{\partial p^3 f}{\partial p}\frac{\partial u}{\partial x} \quad (1)$$

where p is the particle momentum (expressed in all this paper in $m_p c$ units), u is the fluid velocity and D is the CR diffusion coefficient (see more below).

We need to address the delicate problem of the injection of CR from the fluid. We simply consider here that a fraction η of the particles crossing the shock become CR at momentum $p_{inj} = \xi p_{th,2}$ where $p_{th,2}$ is the mean downstream thermal momentum (in this paper we use $\xi = 2$ and investigate the effect of various η).

To validate our code we have successfully reproduced the early work of [3], which proves that our code can handle well a strongly modified shock.

Diffusion scales

The Euler equations of hydrodynamics don't introduce any scale, but the kinetic transport equation introduces a wide range of scales, through the momentum-dependent diffusion of CR: the diffusion coefficient $D(p)$ is a growing function of p (we use in this paper $D(p) \propto p$, which is the relativistic limit of the Bohm diffusion) so that CR of increasing momenta explore broader regions around the shock front and change their energy over longer times. The space scales range from the microscopic scale where the CR decouple from the fluid (of the order of a few thermal gyration lengths) to macroscopic scales where CR escape from the system (of the order of the supernova remnant radius).

From a numerical perspective the resolution of the grid is then dictated by the diffusion of the lowest energy CR whereas the size of the grid is dictated by the diffusion of the highest energy CR. As p spans many orders of magnitude this would be extremely demanding in terms of memory requirements and computing time. To solve this problem we have implemented an Adapative Mesh Refinement (AMR) technique to allow the numerical resolution δx to vary according to the needs of the CR that are to be found at a given location at a given time, as pioneered by [2] and developed by [7] (see a different approach in [1]).

To validate and benchmark our AMR we have extended the case 2 of [3] to $D(p) \propto p$ (instead of $D(p) \propto p^{0.25}$). We have checked that AMR doesn't compromise the physical results whereas considerably lowering the numerical cost (with speed-ups of up to 3 orders of magnitude).

Multiple shocks

Even using AMR the computing time of realistic DSA simulations can still be very high, especially if one wants a precise description of the CR spectrum. This becomes an strong limitation if one wants to investigate the effects of multiple (that is successive) shocks. To lower the $p-$dimension numerical cost without any compromise regarding momentum resolution (see a different approach in [8]) we have parallelized our code in momentum (with MPI) to fully exploit the power of supercomputers. We have obtained fairly good scalings, allowing us to study multiple DSA as fast as one would study single DSA.

We present now the evolution of the previous tests when multiple shocks are run. We use the same hydrodynamical initial conditions for each shock, but an evolving CR upstream population. At the end of each shock we take the CR spectrum just downstream of the shock, shift it to lower energy to mimic (precisely) the adiabatic decompression, then inject it everywhere upstream of the next shock (considering that CR up to the energies considered haven't enough time to escape from the system between two successive shocks).

Linear results

On figure 1 we have switched CR back-reaction off. In that case we run each shock until the time required for CR to be accelerated from their injection momentum $p_{inj} = 0.1$ to the maximum momentum considered $p_{max} = 10$. We clearly see the convergence of the spectrum from an initial power-law of slope $s = 3r/(r-1)$ (the well-known linear solution for a single shock) to a final power-law of slope $s = 3$ (the well-known limit in the case of multiple shocks). In between the spectrum is never a simple power-law, as the asymptotic convergence to $s = 3$ is all the more slow since the momentum is high. The way the spectrum hardens at different momenta agrees with linear theory within roughly 1%. This validation of our code in the linear regime gives us confidence to explore the unknown non-linear regime.

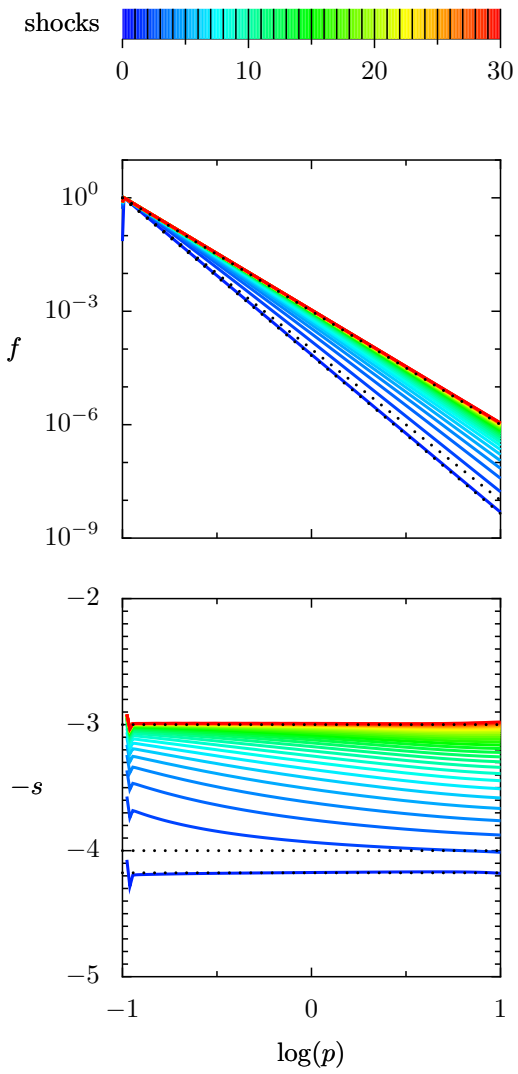

Figure 1: Time evolution of the final downstream CR spectra for a sequence of successive linear shocks. Each colored line shows the CR distribution function $f(p)$ and its logarithmic slope $s = \partial \log(f)/\partial \log(p)$ just at the end of each shock. Added dotted are the three remarkable power-laws of slope $s = 3$, $s = 4$ and $s_1 = 4.18$ the theoretical linear slope for the compression ratio $r = 3.55$ of the shocks.

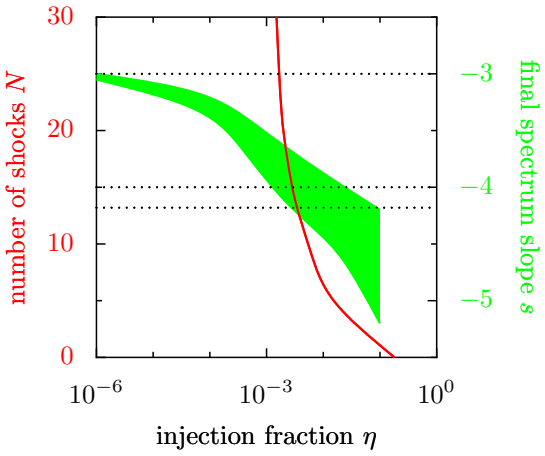

Figure 2: Evolution of the number of shocks N that reach a quasi-steady state before one shock gets fully smoothed and of the range of final CR spectra slopes s as a function of the injection fraction η in the non-linear regime.

Non-linear results

On figure 2 we let the CR back-react on the shock. In that case we run each shock until the CR pressure has converged downstream of the shock. We count how many successive shocks N run before one eventually gets fully smoothed by CR back-reaction. At the last "complete" shock N we measure the range of spectra slopes s. We investigate how this two quantities evolve with respect to the injection fraction η (a parameter which is quite poorly constrained). Above $\eta = 10^{-1}$ the very first shock is smoothed by CR. As η is lowered the number of shocks that reach a quasi-steady state raises exponentially. At around $\eta_c = 1.5 \times 10^{-3}$ we reach $N = 30$ which is the maximum number of shock we allow to run (for both physical and numerical reasons). Regarding the CR distribution slopes we observe two evolutions as η decreases. First the spectra globally harden, which is expected as more shocks can run. Note that below η_c the slopes would get closer to the $s = 3$ limit if one would allow for a higher number of shocks; anyway we see that in the non-linear regime the building of the $s = 3$ spectrum within 30 shocks (as on figure 1) requires an injection fraction lower

than $\eta = 10^{-6}$. Secondly the range of slopes gets constantly narrower, especially below η_c.

Thus this simulations suggest the existence of two regimes with respect to the injection ratio η: there seems to be some critical η_c above which CR dictate the fate of the shocks (producing soft and irregular spectra) and below which CR are almost transparent to the successive shocks (producing harder and more regular spectra). We have observed the same global picture with other simulations involving a constant diffusion coefficient D.

Conclusion

We have presented a new code aimed at the simulation of time-dependent non-linear acceleration by shocks – especially by a sequences of shocks. Our goal here was to explain our method and have a first look at the physics behind multiple DSA. A forthcoming paper will present the code techniques and tests in more details ([4]). We will then study more precisely the role of various key physical parameters. The simulations presented here all derived from the seminal work of [3] which was used as a basis to validate our own work. We intend now to run simulations with physical parameters adapted to the superbubbles environment.

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