ICRC'07

Mérida, México

ICRC 2007 Proceedings - Pre-Conference Edition

# On the relation between the proton-air cross section and fluctuations of the shower longitudinal profile

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**Abstract:** The current status and prospects of deducing the proton-air cross section from fluorescence telescope measurements of extensive air showers are discussed. As it is not possible to observe the point of first interaction, $X_1$, directly, other observables closely linked to $X_1$ must be inferred from the measured longitudinal profiles. This introduces a dependence on the models used to describe the shower development. Systematic uncertainties arising from this model dependence, from the reconstruction method itself and from a possible non-proton contamination of the selected shower sample are discussed.

## Introduction

Indirect cosmic ray measurements by means of extensive air shower (EAS) observations are difficult to interpret. Models needed for a deeper understanding of the data have to be extrapolated over many decades in energy. This is the case for high energy (HE) interaction models, but also applies to the primary composition of cosmic rays. Unfortunately a changing primary composition and changes in the HE interaction characteristics can have similar effects on EAS development and are difficult to separate.

One of the key parameters for EAS development is the cross section $\sigma_{\mathrm{p-air}}$ of a primary proton in the atmosphere. Of course, only the part of the cross section leading to secondary particle production is relevant for EAS development, which we call for simplicity here $\sigma_{\mathrm{p-air}}$. But also the production cross section contains contributions which cannot be observed in EAS. As diffractive interactions of primary particles with air nuclei do not (target dissociation) or weakly (projectile dissociation) influence the resulting EAS, any measurement based on EAS is insensitive to these interactions. Therefore, we define an effective cross section to require an inelasticity $k_{\mathrm{inel}} = 1 - \frac{E_{\max}}{E_{\mathrm{tot}}}$ of at least 0.05

$$\sigma^*_{\mathrm{p-air}} = \sigma_{\mathrm{p-air}}(k_{\mathrm{inel}} \geq 0.05). \qquad (1)$$

In the following the amount of traversed matter before an interaction with $k_{\mathrm{inel}} \geq 0.05$ is called $X_1$. Taking this into account the reconstructed value of $\sigma^*_{\mathrm{p-air}}$ needs to be altered by a model dependent correction $\sigma^{\mathrm{model}}_{\mathrm{p-air}}(k_{\mathrm{inel}} < 0.05)$. This correction amounts to 2.4 % for SIBYLL [1], 3.9 % for QGSJETII.3 [2] and 5.5 % for QGSJET01 [3], resulting in a model uncertainty of $\sim$3 %.

All EAS simulations are performed in the CONEX [4] framework. To account for the limited reconstruction accuracy of a realistic EAS detector, $X_{\max}$ is folded with an Gaussian function having $20\,\mathrm{gcm}^{-2}$ width, which corresponds roughly to the resolution of the Pierre Auger Observatory [5].

## $X_{\mathrm{max}}$-distribution ansatz

The most prominent source of shower fluctuations is the interaction path length of the primary particle in the atmosphere. However the EAS development itself adds a comparable amount of fluctuations to observables like $X_{\max}$. This is mainly due to the shower startup phase, where the EAS cascade is dominated by just a few particles. Our approach to fit the full distribution of $X_{\max}$ does therefore handle the primary interaction point explicitly and
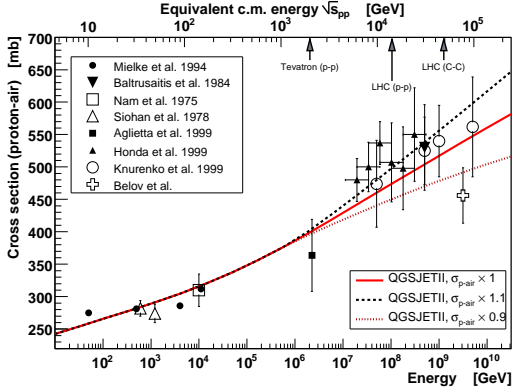
Figure 1: Impact of a 10 % change of $\sigma_{\mathrm{p-air}}$ in QGSJETII at 10 EeV. Data from [6, 8, 9, 10, 11, 12, 13].

the EAS development in a parametric way

$$
\begin{aligned}
\frac{dP}{dX_{\max}^{\exp}} &= \int dX_{\max} \int dX_1 \, \frac{e^{-X_1/\lambda_{\mathrm{p-air}}^*}}{\lambda_{\mathrm{p-air}}^*} \\
&\times \ P_{\Delta X}(\Delta X + X_{\mathrm{shift}}, \lambda_{p-air}^*) \\
&\times \ P_{X_{\max}}(X_{\max}^{\exp} - X_{\max}),
\end{aligned} \quad (2)
$$

where $\Delta X$ was introduced as $X_{\max} - X_1$. Thus the $X_{\max}$-distribution is written as a double convolution, with the first convolution taking care of the EAS development and the second convolution handling the detector resolution. In this model we have two free parameters $\lambda_{\mathrm{p-air}}^*$, which is directly related to $\sigma_{\mathrm{p-air}}^*$, and $X_{\mathrm{shift}}$, needed to reduce the model dependence. Note that Eq. (2) differs from the HiRes approach [6] and that used in the simulation studies in [7] by explicitly including the cross section dependence in $P_{\Delta X}$.

The simulated $P_{\Delta X}$-distributions can be parametrized efficiently with the Moyal function

$$
P_{\Delta X}(\Delta X) = \frac{e^{-\frac{1}{2}(t+e^{-t})}}{\beta\sqrt{2\pi}} \ \text{and} \ t = \frac{\Delta X - \alpha}{\beta} \quad (3)
$$

using the two free parameters $\alpha$ and $\beta$.

## Impact of $\sigma_{\mathrm{p-air}}$ on EAS development

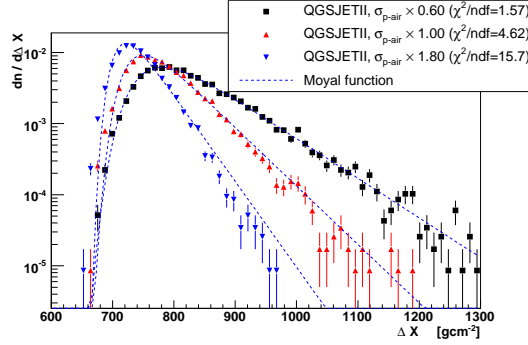To include the cross section dependence of $P_{\Delta X}$ in a cross section analysis at 10 EeV, we modified



Figure 2: Example fits of Eq. (3) to simulated $P_{\Delta X}$-distributions at 10 EeV.

CONEX for several HE models such that the cross section used in the simulation is replaced by

$$
\sigma_{\mathrm{p-air}}^{\mathrm{modified}}(E) = \sigma_{\mathrm{p-air}}(E) \cdot (1 + f(E)), \quad (4)
$$

with the energy dependent factor $f(E)$, which is equal to 0 for $E \leq 1$ PeV and

$$
f(E) = (f_{10\mathrm{EeV}} - 1) \cdot \frac{\log_{10}(E/1\,\mathrm{PeV})}{\log_{10}(1\,\mathrm{EeV}/1\,\mathrm{PeV})} \quad (5)
$$

for $E > 1$ PeV, reaching $f_{10\mathrm{EeV}}$ at $E = 10$ EeV. This modification accounts for the increasing uncertainty of $\sigma_{\mathrm{p-air}}$ for large energies (see Fig. 1). Below 1 PeV (Tevatron energy), $\sigma_{\mathrm{p-air}}$ is predicted within a given HE model by fits to the measured $p\bar{p}$ cross section.

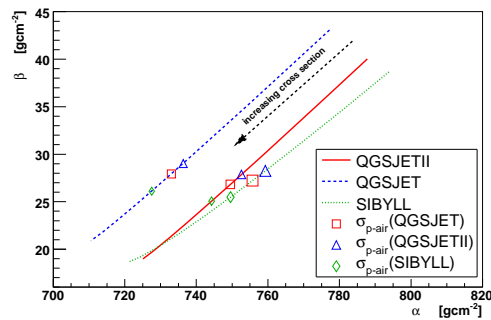The cross section dependence of $P_{\Delta X}$ and the corresponding parametrizations are shown in Fig. 2.



Figure 3: Resulting $\sigma_{\mathrm{p-air}}$-dependence of the parametrized $P_{\Delta X}$-distribution. The markers denote the location of the original HE model cross sections.
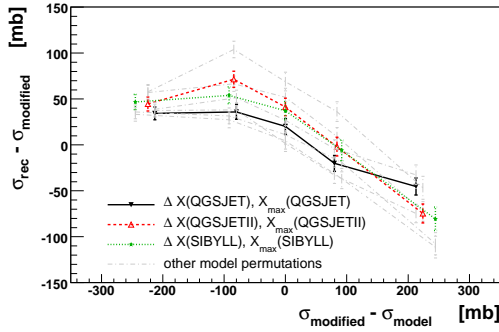
Figure 4: Sensitivity and HE model dependence of the $\sigma_{\mathrm{p-air}}$ reconstruction for a pure proton composition at 10 EeV.



Figure 5: Systematic caused by photon primaries at 10 EeV.
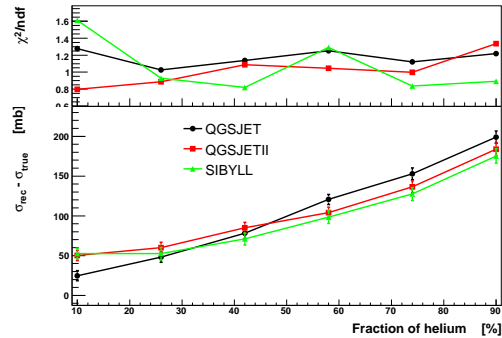


Figure 6: Systematic caused by helium primaries at 10 EeV.

At large $\Delta X$, the simulated distributions are not perfectly reproduced by the parametrizations. This effect worsens for large cross sections, as can be observed from the increasing $\chi^2/ndf$ (see Fig. 2). Also the deviation of the Moyal function from the $P_{\Delta X}$-distribution depends on the HE model. It is biggest for QGSJETII and smallest for SIBYLL. Unfortunately this disagreement produces a systematic overestimation of $\sim$30 mb for the reconstructed $\sigma_{\mathrm{p-air}}$. This is visible in all the following results and will be addressed in future work by making the parametrization more flexible.

The dependence of $\alpha$ and $\beta$ on $\sigma_{\mathrm{p-air}}$ can be interpolated with a polynomial of 2nd degree. Fig. 3 gives an overview of this interpolation in the $\alpha$-$\beta$ plane. Obviously the $P_{\Delta X}$ predicted by different HE model are not only a consequence of the different model cross sections.

## Results

### Pure proton composition
In Fig. 4 we show the reconstructed $\sigma_{\mathrm{p-air}}^{\mathrm{rec}}$ for simulated showers with modified high energy model cross section , $\sigma_{\mathrm{p-air}}^{\mathrm{modified}}$. The original HE cross section $\sigma_{\mathrm{p-air}}^{\mathrm{modified}} - \sigma_{\mathrm{p-air}}^{\mathrm{model}} = 0$ can be reconstructed with a statistical uncertainty of $\sim$10 mb, whereas the uncertainty caused by the HE models is about $\pm$50 mb. At smaller cross sections the reconstruction results in a slight overestimation ($<$ 50 mb). But for larger cross sections there occurs a significant underestimation of the input cross section. This is mainly due to the worse de-
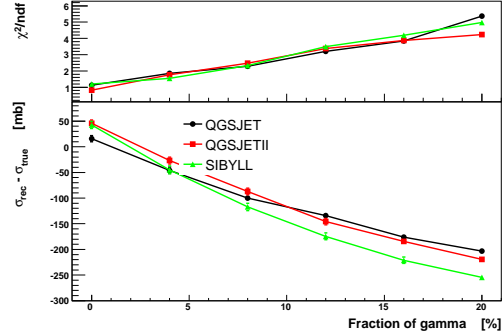
scription of $P_{\Delta X}$ by the used Moyal function for large values of $\sigma_{\mathrm{p-air}}$ (see last section).

### Photon primaries
Primary photons generate deeply penetrating showers. Even a small fraction of photon showers has a noticeable effect on the tail of the $X_{\mathrm{max}}$-distribution [7]. Fig. 5 demonstrates how much a few percent of photons could influence the reconstructed $\sigma_{\mathrm{p-air}}$. The current limit on the photon flux is 2 % at 10 EeV [14]. Note that there is a clear trend of an increasing $\chi^2/ndf$ with increasing photon fraction, meaning the photon signal is not compatible with the proton model.

### Helium primaries
On the contrary, helium induced EAS are very similar to proton showers. Therefore their impact on $\sigma_{\mathrm{p-air}}$ is significant and very difficult to suppress, see Fig. 6. Interestingly, even for large helium contributions there is no degradation of the quality of the pure proton model fit ($\chi^2/ndf$ is flat). Thus it

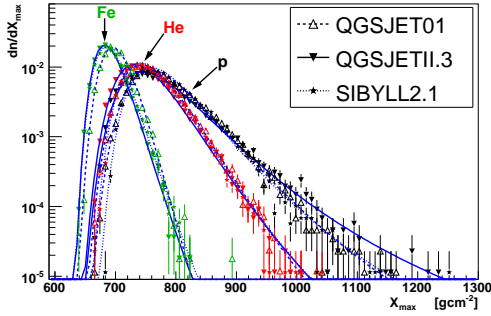ICRC 2007 Proceedings - Pre-Conference Edition

Figure 7: Composition impact on $X_{\max}$ at 10 EeV.

is not possible in a simple way to distinguish between a 25% proton / 75% helium mixture or just a pure proton composition with a cross section increased by about 150 mb.

**Outlook: Mixed primary composition**

Fluctuations and the mean value of the $X_{\max}$-distribution are frequently utilized to infer the composition of primary cosmic rays [15]. It is well understood how nuclei of different mass $A$ produce shower maxima at different depth $X_{\max}(A)$ and how shower-to-shower fluctuations decrease with $A$ (semi-superposition model).

The relative change of the $X_{\max}$-distribution from a pure proton to a pure mass $A$ primary composition can be evaluated using CONEX. To fit $X_{\max}$-distributions we use the formula [16]

$$\frac{dP}{dX_{\max}}(A) = N \cdot e^{-\left(\frac{\sqrt{2}(X_{\max}-X_{\mathrm{peak}})}{\gamma \cdot (X_{\max}-X_{\mathrm{peak}}+3\cdot\delta)}\right)^2}$$

(6)

with four parameters $N$, $X_{\mathrm{peak}}$, $\gamma$ and $\delta$. The normalization constant $N$ was not fitted, but set to reproduce the known number of events. Fig. 7 shows how the $X_{\max}$-distributions for proton, helium and iron primaries are positioned relative to each other for several HE models. This relative alignment can be utilized during $\sigma_{\mathrm{p-air}}$-fits to reduce the composition dependence. The total mixed composition $X_{\max}$-distribution is then the weighted sum of the individual primaries

$$\frac{dP}{dX_{\max}^{\mathrm{mix}}}(X_{\max}) = \sum_i \omega_i \, \frac{dP}{dX_{\max}}(A_i, X_{\max})$$

(7)

where the weights $\omega_i$ are additional free parameters to be fitted together with $X_{\mathrm{shift}}$ and $\lambda^*_{\mathrm{p-air}}$. The shape of $\frac{dP}{dX_{\max}}(A)$ for $A > 1$ is always assumed to change relative to the proton distribution.

First studies indicate that the correlation between the reconstructed composition and the corresponding $\sigma_{\mathrm{p-air}}$ does not allow a measurement of the cross section. The situation is expected to be more promising if the parameter $X_{\mathrm{shift}}$ is fixed, however, the model dependence of the analysis will then be larger than shown here.

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